DIRECTORATE OF DISTANCE EDUCATION
KURUKSHETRA UNIVERSITY
KURUKSHETRA—136119

B.C- 204
B.Com. Part II
(Business Statistics)

External : 80
Internal : 20
Time : 3 Hours

Note : Ten questions shall be set in the question paper with at least three questions from each unit. The candidates shall be required to attempt five questions in all, selecting at least one question but not more than two from each unit.

Unit-I Introduction : Statistics as a subject; Statistical Data: meaning and types, Collection and Rounding of data, Classification and Presentation of data.

Analysis of : Univariate Data: Construction of a frequency distribution; concept of central tendency and dispersion-and their measures; Skewness and measures; Kurtosis and measures.

Analysis of Bivariate Data: Regression and Correlation Analysis.

Unit-II Index Numbers: Meaning, types and uses; Methods of constructing price and quantity indices (simple and aggregate); Tests of adequacy; chain-base index numbers;Base shifting, splicing, and deflating; problems in contructing index numbers; Consumer price index.

Time Series: causes of variations in time series data; Components of a time series; Decomposition-additive and multiplicative models; determination of trend, Moving averages method and method of least squares (including linear second degree, parabolic and exponential trend); Computation of seasonal-indices by sample averages, ratio-to-trend, ratio-to moving agerage and link relative methods.

Unit-III Theory of Probability: Probability as a concept; approaches of probability; addition and multiplication laws of probability; Conditional probability; Bayes Theorem.

Probability distributions: Binomial, Poisson and Normal distributions-their properties and parameters.

Suggested Readings :

1. Hooda, R.P.: Introduction to Statistics, Macmillan, New Delhi, 2002.

2. Hooda, R.P.: Statistics for Business and Economics, Macmillan, New Delhi, 1999.

3. Hole & Jassen: Basic Statistics for Business and Economics; John Wiley and Sons, New York, 1992.

4. Lewin and Rubin: Statistics for Management; Prentice-Hall of India, New Delhi, 2000.
5. Sancheti, D.C. and Kapoor, V.K.: Statistics (Theory, Methods & Application); Sultan Chand 86 Sons, Delhi, 2000.
6. Ya-Lin Chau: Statistical Analysis with Business and Economics: Applications, Holt, Reinhart & Winster, 1997.

Directorate of Correspondence Courses
Kurukshetra University.
Kurukshetra-136 119

B.Com. II
BUSINESS STATISTICS

BC - 204

Directorate of Distance Education

Kurukshetra University, Kurukshetra – 136 119

B.Com. Part-II

Paper - BC–204 : Business Statistics *Revised By :* **Dr. Heera Lal Sharma**

Lesson No. : 1

Introduction : Statistics as a Subject, statistical Data, Classificatin and Presentation of Data

lajpuk

1. ifjp;
2. mís';
3. fo"k; dk iLrqrhdj.k
   - 3.1 lkf[;dh dh ifjHkk"kk ,o vFk
   - 3.2 lkf[;dh dh iÑfr ,o {k=k
     - 3.2.1 lkf[;dh foKku d :i e
     - 3.2.2 lkf[;dh dyk d :i e
     - 3.2.3 lf[;dh dk {k=k
   - 3.3 lkf[;dh dh lhek,
   - 3.4 vkdMk dk ldyu
     - 3.4.1 ikFkfed vkdM
     - 3.4.2 f}rh;d vkdM
   - 3.5 vkdMk dk oxhdj.k
     - 3.5.1 vkdMk d oxhdj.k d e[; y{k.k
     - 3.5.2 oxhdj.k d mís';
     - 3.5.3 oxhdj.k dh lhek
   - 3.6 lkj.kh;u dk vFk ,o ifjHkk"kk
     - 3.6.1 lkj.kh;u dk egRo
   - 3.7 ledk dk in'ku
     - 3.7.1 fpUgk }kjk in'ku dk egRo
   - 3.7.2 fpUgk }kjk in'ku dh lhek,
   - 3.8 ledk dk fcUnj[kh; in'ku
4. lkjk'k
5. iLrkfor iLrd
6. vH;kl d fy, ç'u

1. ifjp; (Introduction)

lkf[;dh 'kCn dk i;kx teu fo}ku xkV iQkbM vkduokty (Gott Fried Achenwall) u loiFke 1749 e fd;k FkkA blfy, bUg lkf[;dh dk tUenkrk (Father of Statistics) dgk tkrk gA bl lkf[;dh (Statistics as data) l lkf[;dh; fof/;k }kjk lkf[;dh; eki dh lx.kuk djr gA lkf[;dh l vfHkik; g l[;kRed lpuk ;k mul lEcfU/r ifjek.kkRed rF; ,o fu"d"kA (Statistis means quantitative information or quantification of the facts and findings relating to different phenomena) lkf[;dh fo"k; l vfHkik; dN fu"d"kk dk Kkr dju d fy, lpukvk ;k vkdMk dk bdVBk djuk] oxhdj.k djuk] fo'y"k.k ,o fuopu lEcU/h rduhdh ,o mik; l gA lkf[;dh d foLrr vkdkj d dkj.k gh bldh ifjHkk"kk ,dopu rFkk cgopu d :i e ifjHkkf"kr djuh iMrh gA ljyre :i e dgk tk ldrk g fd lkf[;dh l[;kRed lpukvk dk Hk.Mkj gA

2. mí'; (Objectives)

bl vè;k; dk i<u d ckn vki tku ldxA

(i) lkf[;dh dh ifjHkk"kk ,o vFkA

(ii) lkf[;dh dh iÑfr ,o {k=kA

(iii) vkdMk dk ldyu] oxhdj.k ,o in'ku dl djuk gA

(iv) lkf[;dh dh lkoHkkfed mi;kfxrk dl gA

(v) lkf[;dh dk fu.k;u e i;kx djukA

3. fo"k; dk iLrrhdj.k (Presentation of Contents)

3.1 lkf[;dh dh ifjHkk"kk ,o vFk

lkf[;dh cgopu d :i eµ bldk vFk vdk d :i e 0;Dr fd, x, vkdMk l gkrk g] tl jktxkj] tul[;k o lkotfud vk; ,o 0;; d vkdM bR;kfnA ijUr ;gk ;g le>u ;kX; g fd ,d l[;kRed rF; tl ';ke dk 100 #i, ifr ekg tc [kp feyrk gA lkf[;dh ugh dgyk,xhA lkf[;dh lpukvk ;k vkdMk d leg dk dgr gA tl B.Com I e 100 fo|kFkh gA Hkkjr e 20 o"k l Åij d cPpk dh vklr Åpkb 5'–8" g tcfd uiky e 5'–2" gA

vr% fu"d"k d :i e lHkh lkf[;dh vkdM g ijUr lHkh vkdM lkf[;dh ugh gkr gA

,- ,y- ckÅy d vulkj] ¶vkdM vul/ku d fdlh foHkkx e rF;k d l[;k d :i e ,l fooj.k gkr g] ftUg ,d nlj d lEcfU/r :i e iLrr fd;k tkrk gA¸

(Statistics are numerical statements of facts in any department of enquiry placed in relation to each other)—A.L. Bowley

;y ,o d.Mky d vulkj] ¶vkdMk l gekjk vfHkik; mu l[;kRed rFok l tk i;klr lhek rd vud idkj d dkj.kkk l iHkkfor gkr gA¸

(By statistics we mean quantative data affected to a marked extent by multiplicity of causes —Yule and Kendall)

lkf[;dh dh vkdMk d :i e fo'k"krk, **(Important characteristics of Statistics as Numercial Data)**

1. **rF;k dk leg (Aggregate of Facts)**

   vkdMk ;k lpukvk d leg ;k vkIr dk gh lkf[;dh dgk tk ldrk gA dkb vdyh l[;k ;k vkdMk lkf[;dh ugh cu ldrh] D;kfd mlI dkb fu"d"k ugh fudkyk tk ldrk] bIfy, dgk tk ldrk g fd lHkh lkf[;dh vkdM g] ijUr lHkh vkdM lkf[;dh ugh cu ldrA

2. **l[;kvk e 0;Dr (Numerically Expressed)**

   l[;k d :i e 0;Dr vkdMk dk lkf[;dh dgk tk ldrk gA x.kkRed :i l 0;Dr vkdM lkf[;dh ugh cu ldr] tl jke NkVk g] ';ke cMk gA jke ,d vehj vkneh g tcfd ';ke xjhc 0;fDr gA

3. **vud dkj.k (Multiplicity of Causes)**

   vkdMk ij vud dkj.kk dk iHkko iMrk gA ;fn vkdM fdlh ,d dkj.k l iHkkfor gk rk mI dkj.k dk gVku dk mudk egRo [kRe gk tk,xkA ftII vkdMk dk lkekU; gku ij lkf[;dh d :i e egRo [kRe gk tkrk g] bIfy, vkdM ,d dkj.k I ugh cfYd vudk dkj.kk I iHkkfor gkr gA

4. **fo'k"k mí'; (Specified Objectives)**

   vkdM ;k lpuk, fdlh mí'; d fy, gh bdVB fd; tkr gA fcuk fdlh mí'; d ,df=kr vkdMk dk dkb egRo ugh gkrkA ;fn tuI[;k lEcU/h vkdMk dk bdVBk fd;k tk, rk mudk dkb mí'; gkrk g rHkh o lkf[;dh cur gA

5. **ijLij lEcfU/r ryukRed (Mutually Related and Comparable)**

   lkf[;dh cuu d fy, vkdM ,d nlj l lEcfU/r ,o ryuk ;kX; gku pkfg,A ;fn vkdMk dk vkil e lEcU/ ugh gkxk tk mudh ryuk Hkh ugh gkxhA ftI idkj ehuk dh yEckb 6 iQV g] lhek dk otu 50 fdyk gA budk dkb lEcU/ ugh g] u gh dkb ryuk dh tk ldrh gA bu nkuk e yEckb o otu ,d lkFk fn; gk rk ryuk lEHko gA

lkf[;dh&,dopu d :i e (Statistics - A Singular Noun)

ifjHkk"kk (Definition)

ØkDlVu rFkk dkmVu d vulkj] ¶lkf[;dh dk l[;kRed vkdMk dk lxg dju] iLrrhdj.k] fo'ys"k.k rFkk mud fuopu l lEcfU/r foKku dgk tk ldrk gA¸

("Statistics may be defined as the collection, presentation, analysis and interpretstion of numerical data")—Croxton and Cowden

lfyxeu d vulkj] lkf[;dh og foKku g tk fdlh fo"k; ij idk'k Mkyu d mí'; l lxg fd; x, vkdMk d lxg.k] oxhdj.k] in'ku] ryuk vj 0;k[;k dju dh fof/;k dk foppu djrk gA

("Statistics is the science which deals with the methods of collecting, classifying, presenting, comparing and interpreting numerical data collected to throw some light on any sphere of enquiry")—Seligman lkf[;dh dh ,d mi;Dr ifjHkk"kk bl idkj gk ldrh gA

lkf[;dh ,d foKku vkj dyk g tk lkekftd] vkfFkd] ikÑfrd o vU; leL;kvk l lEcfU/kr ledk d lxg.k] oxhdj.k] lkj.kh;u] iLrrhdj.k] lEcU/&LFkkiu] fuopu vkj iokueku l lEcU/ j[krh g rkfd fu/kfjr mís'; dh ifr gk ldA

lkf[;dh led (Statistical Data)

LVfVfLVDl 'kCn dh ifjHkk"kk l rkRi;l lkf[;dh led l gh gA

dkujµled fdlh ikÑfrd vFkok lkekftd ?kVuk dh eki d vueku g tk ikjLifjd lEcU/ dk in'ku dju d fy, fdlh i¼fr d vulkj j[k tkr gA

("Statistics are mesurements, enumerations or estimates of natural and social phenomena, systematically arranged so as to exhibit their inter relations.") — Connor

gkjl lØkbVµ¶led l gekjk vfHkik; rF;k d mu legk l g tk vuxf.kr dkj.kk l i;kIr lhek rd iHkkfor gkr g] tk l[;kvk e 0;Dr fd; tkr gA ,d mfpr ek=kk dh 'k¼rk d vulkj fxu ;k vuekfur fd; tkr g] fdlh io&fuf'pr mís'; d fy, ,d 0;ofLFkr <x l ,df=kr fd; tkr g vkj ftUg ,d&nlj l lEcfU/r :i e iLrr fd;k tkrk gA¸

ledk dh ;g ifjHkk"kk dkiQh mfpr yxrh gA bl e ledk dh lHkh fo'k"krkvk dk lekos'k g] tk gkuk pkfg,A

3.2 lkf[;dh dh iÑfr ,o {k=k (Future and Scope of Statistics)

lkf[;dh dh iÑfr dk le>u d fy, ;g vè;;u vko';d g fd og foKku g ;k dykA

3.2.1 lkf[;dh foKku d :i e

foKku fdlh Kku dk Øec¼ leg g (Science is a body of systematised knoweldge)

fdlh Kku dh 'kk[kk dk ^foKku* rHkh dgk tkrk g tc bl e ;g x.k fo|eku gkµ

1. Kku dk Øec¼ vè;;u gk rFkk mldh jhfr;k 0;fLFkr gkA
2. og dkj.k vkj iHkko (Cause and Effect) d lEcU/k dk fo'y"k.k djrk gkA
3. mld fu;e v[k.M] loekU;] 0;kid rFkk lkoHkke gkA
4. iokueku dh {kerk ,o og lno xfr'khy gkA

; lc x.k lkf[;dh e ik, tkr gA foKku d :i e ;g fofHkUu fl¼kUrk rFkk i¼fr;k dk Hk.Mkj gA

dN fo}kuk dk dguk g fd lkf[;dh foKku ugh gA ;g ,d oKkfud fof/ gA bu fo}kuk e ØkDLVu rFkk dkmMu dk uke mYy[kuh; gA okfyl o jkcVl dk fopkj g fd lkf[;dh LorU=k ,o eyHkr Kku dk leg ugh g] cfYd Kku ikIr dju dh jhfr;k dk leg gA

3.2.2. lkf[;dh dyk d :i e

^dyk* dk vfHkik; fØ;k l gA foKku ge fdlh fo"k; dk Kku inku djrk gA dyk ge fdlh dk; dk dju dk lokÙke <x crkrh gA dyk mu fØ;kvk dk leg g] ftudh lgk;rk l ge vHkh"V ifj.kke ij igpr gA dyk fu/kfjr y{; ij igpu dk mik; Hkh crkrh gA dyk dh lk/uk d fy, fo'k"k Kku] vuHko o vkRel;e dh vko';drk gkrh gA lkf[;dh e Hkh dyk

d y{k.k fo|eku gA lkf[;dh dyk d :i e fof'k"V leL;kvk d lUrk"ktud lek/ku d fy, fu;e] jhfr;k rFkk l=kk dk i;kx djuk crykrh gA okLro e lkf[;dh foKku rFkk dyk nkuk gh gA bld l½kfUrd rFkk O;kogkfjd nkuk igy gA bldk i;kx doy Kku ikIr dju d fy, ugh] cfYd rF;k dk le>u rFkk fu"d"k fudkyu d mí'; l fd;k tkrk gA

3.2.3. lkf[;dh dk {k=k (Scope of Statistics)

oreku le; e lkf[;dh dk {k=k cgr vf/d fodflr gk pdk gA bldk ifjHkkf"kr djuk u doy dfBu g cfYd cf½ekuh Hkh ugh gA lkf[;dh dk i;kx Kku dh iR;d 'kk[kk e vko';d gk pdk g tl vFk'kkL=k O;olk; ,o okf.kT;] fu;ktu] i'kklu] Kku&foKku vkfn {k=kk e gkrk gA bl idkj vkt d oreku ;x e lkf[;dh dk egRo dkiQh vf/d c< pdk gA

3.3. lkf[;dh dh lhek, (Limitations of Statistics)

lkf[;dh dk vkt d ;x e gj {k=k e i;kx gku l bldk {k=k O;kid gk pdk gA lkf[;dh fof/;k d i;kx dju l rF;k e fuf'prrk ,o Li"Vrk vk tkrh g] fdUr bldh dN lhek, Hkh g ;fn budk è;ku e ugh j[kk tk,xk rk lkf[;dh vkdMk l fudky x, ifj.kke xyr o Hkei.k gk ldr gA lkf[;dh dh lhek, fuEufyf[kr gµ

1. lkf[;dh doy l[;kRed rF;k dk vè;;u djrh g
(Statistics Studies only Quantitative Facts)

lkf[;dh doy mu leL;kvk dk vè;;u djrh g] ftudk vdk e O;Dr fd;k tk ldrk g] tl vk;&O;;] mRiknu&fcØh] vk; vkfn yfdu x.kkRed rF;k dk vè;;u ugh djrh] tl U;k;] fe=krk] lH;rk] lUnjrk vkfnA

2. lkf[;dh O;fDrxr bdkb;k dk vè;;u ugh djrh
(Statistics does not deal with Individual Items)

;g leg dk vè;;u djrh gA O;fDrxr bdkb;k dk ughA MCY; vkb- fdx d vuqlkj] ¶lkf[;dh viu fo"k; dh iÑfr d dkj.k gh O;fDrxr bdkb;k ij fopkj ugh dj ldrh vkj u dHkh djxhA ;fn o egRoi.k gk rk mud vè;;u d fy, vU; lk/uk dk i;kx djuk pkfg,A mnkgj.k d fy, fdlh 'kgj dh ifr O;fDr vkl vk; 10 ifr'kr c<h g] fiNy o"k dh ryuk e] rk gk ldrk g fd fdlh O;fDr dh vk; fLFkj ;k de Hkh gb gk rk ge bl dkb egRo ugh nxA

3. lkf[;dh ifj.kke doy vkSlr :i l lR; gkr g
(Statistics are True on an Average)

lkf[;dh d fu;e ,o fu"d"k iR;d ifjfLFkfr e i.k :i l lR; ugh gkrA o vkSlr :i e gh lR; gkr gA ,d 'kgj d ifjokjk dh vkSlr ekfld vk; 5,000 #i, g rk bldk eryc ;g ugh fd lHkh ifjokjk dh vk; 5,000 #i, d cjkcj gA

4. lkf[;dh led e ,d:irk ,o ltkrh;rk gkuk vko';d g
(Uniformity and Homogeneity is Essential in Statistics Data)

vkdMk dh vkil e ryuk dju d fy, mue ,d:irk o ltkrh;rk gkuk vko';d gA ;fn led fotkrh; gk rk mudh ryuk ugh dh tk ldrhA mnkgj.k d rkj ij Nk=kk dh vk; o vxth fo"k; e ikIr vd dh ryuk djuk lEHko ughA

5. lkf[;dh doy lk/u ek=k g] leL;vk dk lek/ku ugh
(Statistics is a means but not solution of the Problems)

;g lhek Mk- ckÅy d dFku ij vk/kfjr gA Mk- ckÅy d vulkj] ¶lkf[;dh dk dk; ,d lrd i;kxdrk dh HkkfUr ledk dk ldyu] in'ku rFkk o.ku djuk gkrk g] mll fu"d"k fudkyuk ugh dk;&dkj.k lEcU/ d vul/ku e Hkh iek.k iLrr djuk gkrk g] fu"d"k fudkyuk ugh¸ ijUr dN fo}ku bl dFku l lger ugh gA okLro e lkf[;dh dk dk; fcuk fdlh i{kikr fd, leL;k l lEcfU/r vkdMk dk ldyu djuk ,o mudk fo'y"k.k djd leL;k dk okLrfod :i iLrr djuk gA ; fu"d"k vPN ;k cj g] vkfn i'uk dk mÙkj nuk lkf[;dh dk dk; ugh gA

mi;Dr foopu l ;g Li"V g fd lkf[;dh dh fof/;k dk i;kx djr le; mudh lhekvk dk è;ku j[kuk vko';d g] vU;Fkk ledk l Hkei.k fu"d"k fudy ldr gA

fu"d"k d :i e bldh lhekvk dk è;ku e j[kuk pkfg,A yfdu bud Mj l bld i;kx e deh ugh gkuh pkfg,A D;kfd lkf[;dh dh lkoHkkfed mi;kfxrk gA (Statistics has Universal Utility)

Mk- okÅy d vulkj] ¶lkf[;dh dk Kku fdlh fon'kh Hkk"kk vFkok chtxf.kr d Kku d leku g] tk fdlh Hkh le; fdlh Hkh ifjfLFkfr e mi;kxh gk ldrk gA¸

3.4 vkdMk dk ldyu (Collection of Data)

oreku ;x e vkdMk dk ldyu lkf[;dh dk vk/kj gA ,df=kr led 'k¼ ,o i;klr rFkk mí';k d vulkj gku pkfg,] rkfd vulU/ku d iQyLo:i fu"d"k fo'okl dju ;kX; gkA

vè;;u dh nf"V l ledk dk nk Hkkxk e ckVk tk ldrk gµ

1. ikFkfed led (Primary Data)

2. f}rh; led (Secondary Data)

3.4.1. ikFkfed led (Primary Data)

ikFkfed vkdM ekfyd :i l ,df=kr vkdM gkr gA ;g vulU/kudrk }kjk igyh ckj ,df=kr fd, tkr gA gkjl lØkbLV d vulkj] ¶ikFkfed ledk l vk'k; g fd o ekfyd g vFkkr ftudk legdj.k cgr gh de ;k ugh gvk gA ?kVukvk dk vdu ;k x.ku mlh idkj fd;k x;k g] tlk fd ik;k x;k gA e[; :i l ; dPp inkFk gkr gA mnkgj.k d rkj ij ;fn vulU/kudrk O;fDrxr :i l feydj ;k i'ukoyh djok dj vkdM bdVB djrk g rk o ikFkfed vkdM dgyk,xA

3.4.2. f}rh; led (Secondary Data)

f}rh; vkdM o gkr g tk igy l gh fdlh vulU/kudrk d }kjk fdlh vU; vulU/ku d fy, bdVB fd; tk pd gA f}rh; vkdM idkf'kr ,o vidkf'kr nkuk gh :ik e miyC/ gk ldr gA Cy;j d vulkj] ¶f}rh; led o g tk igy gh vfLrRo e g vkj tk oreku i'uk d mÙkj e ugh cfYd fdlh nljs mí'; d fy, ,df=kr fd, x, gA¸ f}rh; ledk dk ,df=kr dju dh fof/;k dk f}rh;d fof/;k dgr gA

ikFkfed ,o f}rh; ledk d vUrj dk Li"V djr g, oly] foyV o lkbeu u fy[kk g fd vulU/ku dh fof/ e ekfyd :i e ldfyr led ikFkfed led dg tkr gA ftudk ldyu vU; O;fDr;k d }kjk gkrk g] f}rh;d led dgykr gA

vUrj dk vk/kj	izkFkfed vkdM	f}rh;d vkdM
1. mís';	; vkdM vuql/kku d mís'; d vuqdy gkr gA bue l'kk/u dh t:jr ugh iMrhA	; fdlh vU; mís'; d fy, ,df=kr fd, tkr g rFkk i;kx fdlh vU; mís'; d fy, fd;k tkrk gA
2. lkr	izkFkfed led izkFkfed jhfr;k }kjk ,df=kr fd, tkr gA	budk lkr vU; 0;fDr ,o lLFkk, gkrh gA ; ik;% io&idkf'kr gkr gA
3. le; ,o 0;;	bu ledk dk bdVBk dju e èku o le; vf/d ek=kk e yxkuk iMrk gA	vi{kkÑr ble le; o /u dki{h de yxrk gA
4. lrdrk	bld i;kx dju e lrdrk dh vko';drk ugh gkrhA	bld i;kx dju e lko/kuh j[kuh iMrh gA

izkFkfed led ,df=kr dju d fy, e[;r% bu fof/;k dk i;kx fd;k tkrk gA

1. iR;{k 0;fDrxr vuql/kku (Direct Personal Investigation)
2. viR;{k ekf[kd vuql/kku (Indirect Oral Investigation)
3. LFkkuh; lkrk o loknnkrkvk }kjk lpuk
   (Information through Local Sources or Correspondents)
4. i'ukoyh Hkjokdj (Information through)

f}rh;d led Hkh e[;r% nk idkj l lkrk l bdVB fd, tkr gA

1. idkf'kr lkr (Published Sources)

ble e[;r% ljdkjh idk'ku] vUrjk"Vh; idk'ku] v¼&ljdkjh idk'ku] lfefr;k rFkk vk;kxk d ifronu ,o idk'ku] if=kdk] lekpkj&i=k] vuqlU/kku l[;k, vkfn idkf'kr vkdM miyC/ djokrh gA

2. vidkf'kr lkr (Unpublished Sources)

vuql/kku lLFkk,] fo'ofo|ky;] Je&dk;ky;] 0;kikfjd lxBu vkfn Hkh vkdM ,df=kr djr gA ik;% ; vkdM idkf'kr ugh djokrA bul izkFkuk djd vkdM fy, tk ldr gA ; f}rh; vkdMk d :i e tku tkr gA

3.5. vkdMk dk oxhdj.k (Classification of Data)

,df=kr vkdMk dk lf{kIr] ljy o lgt cuku dh ifØ;k dk gh vkdMk dk oxhdj.k dgk tkrk gA fuEu ifjHkk"kkvk e vkj Hkh Li"V fd;k tk ldrk gµ

dkujµoxhdj.k ledk dk (;FkkFk vFkok HkkoukRed :i l) lekurk o lkn';rk d vèkkj ij legk ;k foHkkxk e Øekulkj 0;Dr dju dh ifØ;k g vkj 0;fDrxr ink dh fHkUurk d eè; mud x.kk dh ,drk dk 0;Dr djrk gA

(Classification is the process of arrnaging things 'either actually or nootional in groups of classes according to their resemblances and affinities and given expression to the unity of attributes that may subsist among a diversity of individual' —Conner

Lij o fLeFkµ¶oxhZdj.k lEcfU/r rF;k dk fofHkUu oxk e foHkkftr dju dh ifØ;k gA¸

okLro e oxhZdj.k ledk dh legk vFkok oxk e lekurk o ,d:irk d vk/kkj ij 0;Dr dh ifØ;k gA oxhZdj.k dju l igy mldk mí';, vo'; fofnr gkuk pkfg,A oxhZdj.k dju l nk l vf/d ledk dh ryuk vklkuh l dh tk ldrh gA oxhZdj.k lkj.kh;u (Tabulation) dh ikjfEHkd voLFkk gA

3.5.1 vkdM+k d oxhZdj.k d e[; y{k.k (Main Features of Classification of Data)

1. oxhZdj.k d vUrxr lkf[;dh; vuql/ku d mí'; {k=k ,o Lo:i d vuqlkj ,df=kr vkdM+k dk fofHkUu oxk e ckVk tkrk gA

2. oxhZdj.k x.k ;k fo'k"krk d vk/kj ij gkrk gA

3. oxhZdj.k okLrfod ;k dkYifud gk ldrk gA

4. oxhZdj.k ink e fofHkUurk e ,drk (Unity in diversity) Li"V djuk gA

5. oxhZdj.k d fy, dkb fuf'pr o dBkj fu;e fu/kfjr ugh gkrA

3.5.2. oxhZdj.k d mí'; (Objects of Classification)

1. **vkdM+k dk ljy o lf{klr cukuk**

oxhZdj.k dju l ledk dk vklkuh l le>k tk ldrk gA dkiQh cM+ ledk dk doy dN gh Jf.k;k e oxhZdj.k }kjk foHkkftr djd lf{klr :i fn;k tk ldrk g] ftll ledk dk le>u d fy, dkiQh de ekufld Je djuk iM+rk gA vkdM+ Hkh tYnh le> vk tkr gA

2. **ledk dh lekurk o vlekurk dk iadV djuk**

oxhZdj.k dh lgk;rk l lkf[;d rF;k dh lekurk Li"V :i l idV gkrh g rFkk le>u e Hkh lgk;rk feyrh gA

3. **ryuk e lgk;rk djuk**

oxhZdj.k l ledk dk ryukRed vè;;u lEHko gkrk g] tlkfd fuEu mnkgj.k l Li"V gµ

Marks in Accountancy	Class A	Class B
0–10	7	5
10–20	9	14
20–30	13	11
30–40	1	0
Total	30	30

4. **vkdM+k dk fo'y"k.k ;kX; cukuk**

oxhZdj.k d ckn vkdM+k dk lkf[;dh; fof/;k }kjk fo'y"k.k Hkh fd;k tk ldrk g] ftll ledk dh vfHkdj.k vFkok fo"kerk ekih tk ldrh gA fo'y"k.k ifØ;k dk oxhZdj.k dh iFke fLFkfr dgk tk ldrk gA

5. **ledk dh mi;kfxrk c<kuk**

ledk dk oxhZdj.k dju l tulk/kj.k d fy, mi;kfxrk e of¼ gkrh g] ftll fo'y"k.k Hkh vklkuh l fd;k tk ldrk gA

6. ledk dk iHkkoh cukuk

oxhdj.k ledk dk laf{kIr ,o mi;kxh cukdj fo'y"k.k ;kX; cuk nrk g] ftl ls led iHkkoh cu tkr gA

oxhdj.k ,o Øec¼ ledk dk egRo t-vkj- fgDl u bu 'kCnk e idV fd;k gµ ¶oxhdj.k ,o Øec¼ rF; Lo;a ckyr g] vO;ofLFkr :i e o ekl d leku er gkr gA¸

"(Classified and arranged facts speak themselves, unarranged they are as dead as mother) —J.R. Hicks

3.5.3. oxhdj.k dh lhek (Limits of Classfication)

vkdMk dk oxhdj.k dju dh ifØ;k e dN fooj.k lekIr gk tkr g] vkdMk dk ftruk vfèkd laf{kIr fd;k tk,xk mruk gh vf/d fooj.k NV tku dh lEHkkouk cuh jgrh g] blfy, vkdMk dk laf{kIrhdj.k cgr gh lko/kuh l djuk pkfg,A

3.6 lkj.kh;u (Tabulation)– vFk ,o ifjHkk"kk

oxhÑr vkdMk dk ljy ,o laf{kIr dju d fy, lkjf.k;k e iLrr dju dh ifØ;k dk lkj.kh;u dgk tkrk gA

dkuj d vuqlkjµ¶lkj.kh;u fdlh fopkj/hu leL;k dk Li"V dju d mí'; l fd;k tku okyk lkf[;dh; rF;k dk Øec¼ ,o lO;ofLFkr iLrrhdj.k gA¸

("Tabulation involves the orderly and systematic presentation of numerical data in a form desired to elucidate the problem under consideration") – J.R. Cornor

Cy;j d vuqlkjµ ¶foLrr vFk e vkdMk d [kkuk (;k dkyek) vkj ifDr;k e fdlh Øec¼ O;oLFkk dk lkj.kh;u dgr gA¸

("Tabulation in its broadest sense, is an orderly arrangement of data in column and rows") —Blair

ljy 'kCnk e lkf[;dh; ledk dk [kkuk o ifDr;k e Øec¼ :i l iLrr dju dh ifØ;k dk lkj.kh;u dgk tkrk gA oxhdj.k ,o lkj.kh;u dh ifØ;k lkFk&lkFk lEiUu dh tkrh g] blfy, lkj.kh;u d mí'; Hkh ogh g] tk oxhdj.k d gA

3.6.1 lkj.kh;u dk egRo (Importance of Tabulation)

lkj.kh;u ledk d ladyu o oxhdj.k rFkk mld fuopu d chp dh ,d egRoi.k ifØ;k g ftll vkdMk l fu"d"k fudkyu e lgk;rk feyrh gA bldk egRo bld ykHkk o x.kk l Li"V gk tkrk g] tk bl idkj gµ

1. vkdMk dk le>u e ljyrk

lkj.kh;u tfVy vkdMk dk Li"V iLrr dju e lgk;d gkrh g] ftl le>u e ljyrk dh tkrh gA

2. ryuk dju e lgk;d

rF;k dk ijLij fudVorh LrEHkk o ifDr;k e j[ku l ryukRed vè;;u djuk ljy gk tkrk gA

3. LFkku o le; dh cpr

lkj.kh;u l vkdMk dk U;ure LFkku e iLrr fd;k tkrk g] ftldk ,d nf"V e n[kdj le>k tk ldrk g] ftll le; o LFkku dh cpr gkrh gA

4. **vkd"kd in'ku**

fofHkUu idkj dh j[kkvk dh lgk;rk l inf'kr vkdM vkd"kd cu tkr gA

5. **v'kf¼;k dh tkp e vklkuh**

v'kf¼;k dh tkp cgr ljy gk tkrh gA

6. **fuopu e lgk;d**

rF;k dk j[kk fp=kk o vky[kk e in'ku lxerk l fd;k tk ldrk g] ftll fuopu dj lfoèkktud gk tkrk gA

lkj.kh dh viuh dN lhek, Hkh gkrh g] tl lkj.kh e doy led gkr g] mudk fooj.k ugh gkrkA lkj.kh dk le>u dh fo'k"k Kku dh vko';drk gkrh gA lkj.kh fdlh fo'k"k egRo d ox dk dkb fo'k"k egRo ugh n ikrh gA

3.7 ledk dk in'ku (Presentation of Data)

vkdMk dk inf'kr dju e oxhdj.k o lkj.kh;u dk viuk fo'k"k LFkku gA fiQj Hkh ;g fof/ vkdMk dk bruk vkdf"kr ugh dj ikrhA ;fn bUgh ledk dk fp=kk ,o xkiQ d ekè;e l inf'kr fd;k tk, rk ; tulk/kj.k dk vf/d vkdf"kr djx] D;kfd fp=kk dk iHkko eku liVy ij dkiQh nj rd jgrk gA

3.7.1 fp=kk }kjk in'ku dk egRo (Significance of Diagramatic Presentation)

1. **ledk dk ljy cukuk**

fp=kk d }kjk tfVy ledk dk Hkh ljy cuk;k tk ldrk gA fp=kk d in'ku l fo'y"k.k Hkh vPNh idkj l fd;k tk ldrk gA

2. **fp=ke; in'ku l vf/d Lej.kh;**

fp=kk dk iHkko efLr"d ij dkiQh le; rd jgrk gA iu% ;kn dju dh lEHkkouk Hkh vf/d jgrh g] blfy, ledk dk fp=kk }kjk inf'kr fd;k tkrk gA

3. **fp=k vkdMk d in'ku dh vkd"kd fof/**

fp=kk }kjk vkdMk dk in'ku dju dk mudk vkd"kd c<rk g D;kfd fp=k ekuo efLr"d ij vf/d iHkko NkMr gA fp=kk dk i;kx lekpkj&i=kk] i=k&if=kdkvk e Hkh blh dkj.k l gkrk gA vfèkd vkd"kd c<ku d fy, fp=kk e fofHkUu idkj d jxk viuk fpUgk dk i;kx Hkh fd;k tkrk gA

4. **ryuk dju e lgk;d**

fp=kk }kjk vkdMk dh ryuk djuk vf/d vklku gkrk gA] fp=kk dk rk doy n[ku ek=k l ryuk dh tk ldrh gA

5. **le>u e le; dh cpr**

fp=kk }kjk inf'kr led vklkuh l le> e vk tkr gA bld le>u e vf/d le; ugh yxkuk iMrk gA

6. **foKkiu ekè;e e i;kx**

vktdy fp=kk dk i;kx foKkiu d ekè;e l cgr vf/d fd;k tku yxk gA dEiuh fcØh vkfn dh lpuk fofHkUu foHkkxk dh fdruh g] ;g tkudkjh Hkh fp=kk d ekè;e l nh tkrh gA bldk egRo rc gh ikIr fd;k tk ldrk g c'kr fd mfpr fp=k dk i;kx mfpr LFkku ij fd;k tk, vr% iLrrdrk dk fp=kk d i;kx dh tkudkjh gkuh pkfg,A

3.7.2 fp=kk }kjk in'ku dh lhek, (Limitations of Diagrammatics Presentation)

,e-t- ekjku (M.J. Moroney) d vuqlkj] ¶fdlh fp=k dk vè;;u dju d fy, ^i;kIr pkdUuk jguk vko';d gkrk gA ;g bruk ljy] Li"V rFkk euHkkoh gkrk g fd vUu;ku 0;fDr cMh vklkuh l e[k cu tkrk gA bl rduhd dk i;kx djr le; rFkk fu"d"k fudkyr le;] fo'k"k lkoèkkuh dh vko';drk gkrh gA

1. **ryukRed vè;;u lEHko**

   fp=kk dh lgk;rk l doy ryukRed vè;;u gh lEHko gk ikrk gA vdy fp=k dk dkbZ fo'k"k vFk ugh gkrk] tc rd mldh ryuk vkdMk oky fp=kk l u dh tk,A

2. **l{e vUrj fn[kkuk lEHko ugh**

   ;fn vkdMk e l{e vUrj g rk bl fp=kk }kjk inf'kr djuk lEHko ugh gA

3. **l[;kRed in'ku vlEHko**

   fp=kk d ekè;e l vkdMk dk 'k¼ :i e in'ku lEHko ugh gkrk gA fp=k vuekfur :i l gh vkdMk dk in'ku djr gA

4. **ljyrkiwod n#i;kx**

   v'k¼ fp=k cukdj mudk n#i;kx fd;k tk ldrk gA Hkked fp=k cukdj foKkiu vkfn e vklkuh l n#i;kx fd;k tkrk gA

5. **vk[kk dk /k[kk**

   ;fn vkdMk vu#i fp=k cuk, tk, rk blls vk[kk dk Hkkjh /k[kk gk ldrk gA

6. **vkx fo'y"k.k vlEHko**

   fp=kk dh ;g Hkh lhek g fd budk vkx fo'y"k.k lEHko ugh gA

   lkf[;dh e fuEu idkj d fp=kk dk i;kx fd;k tkrk gA

   1. ,d foLrkj oky fp=k (One Dimensional Diagram)
   2. nk foLrkj oky fp=k (Two Dimensional Diagram)
   3. rhu foLrkj oky fp=k (Three Dimensional Diagram)
   4. eku fp=k (Cartograms or Map Diagram)
   5. fp=k y[k (Pictograms)

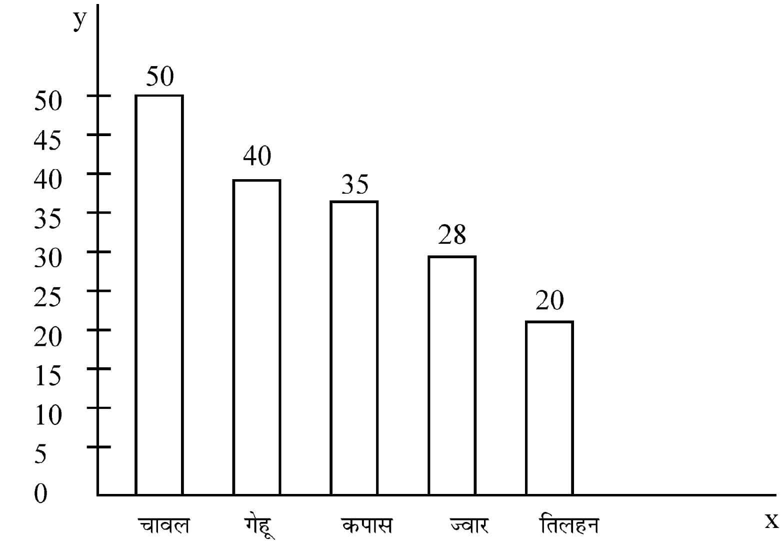
   mnkgj.k d rkj ij fp=k dk le>k tk ldrk gA

   Hkkjr e fofHkUu iQlyk d vUrxr {k=kiQy d fuEu vkdMk e ljy n.M fp=k cukb;A

iQly	{k=kiQy Area (Million Acres)
pkoy (Rice)	50
xg (Wheat)	40
dikl (Cotton)	35
Tokj (Jawar)	
frygu (Oilseeds)	20

fofHkUu iQlyk d vUrxr {k=kiQy

iekuk : I lVhehVj = 5 yk[k ,dM



fp=k }kjk iQlyk d {k=kiQy dk n[kr gh le>k tk ldrk g fd fdldk {k=kiQy T;knk g vkj fdldk gA

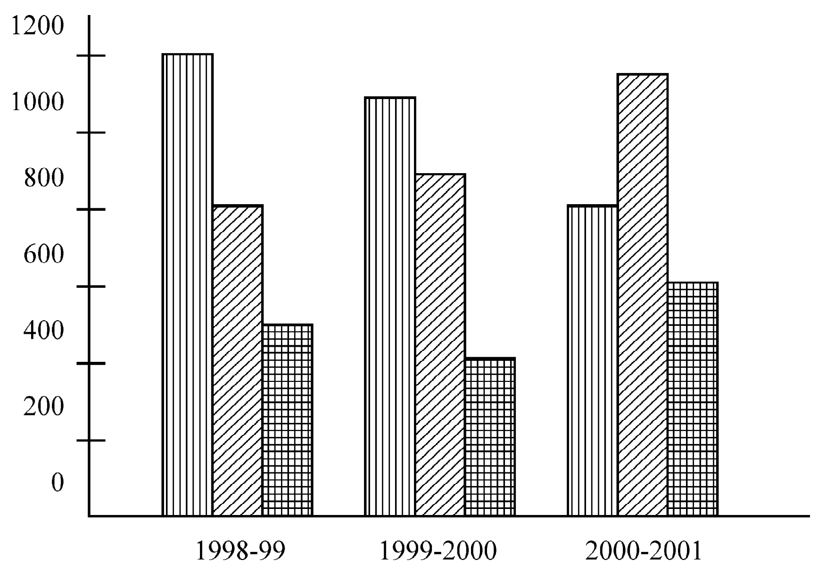
fuEu vkdMk dk cgn.M fp=k }kjk inf'kr dhft,A

Represent the following data by means of Multiple bars

(Facul) ldk;	(No. of Students) Nk=kk dh l[;k		
	1998-99	1999-2000	2000-2001
dyk (Arts)	1200	1100	900
okf.kT; (Commerce)	800	900	1200
foKku (Science)	500	400	600

ldk; ckj Nk=kk dh l[;k

iekuk % 1 lVhehVj = 200 Nk=k



cgn.M fp=k

3.9 leadk dk fcUnj[kh; in'ku (Graphic Presentation of Data)

vFk ,o ifjHkk"kk (Meaning and Definition)

lkf[;dh; vkdMk dk xkiQ iij ij in'ku fcUn j[kk dgykrk gA lkf[;dh; rF;k ij fcUn j[kh; in'ku mUg le>u ;kX; cuku dh ljy ,o iHkkoh fof/ gA ,e-,e- Cy;j d vu lkj] ¶le>u e o jpuk e ljyre] lokf/d py vkj lcl vf/d i;kx e yk;k tku okyk fp=k fcUnj[k gA¸

vkb-vkj-olyk d dFku l Li"V g fdµl[;kRed ikBu dh lcl ljy ,o lkekU; fof/ fcUnj[k gA ;g l[;kvk dk fp=k bl idkj iLrr djrh g fd u=kk dk mud lEcU/ rRdky irk yx tkrk gA l[;kvk dk Li"V cuku e bldk lokf/d egRo gA

xf.kr dh nf"V l fcUnj[k dk ^chtxf.kr&T;kfefr* dh o.kekyk dgk x;k gA

fcUnj[kh; in'ku d dk;

1. ;g fof/ fo'y"k.k dk x.kuk rFkk fu;ktu e ekxn'ku djrh gA
2. ;g xf.kr e le; o Je cpku d fy, i;kx gkrh gA
3. xf.krh; oØ d LFkku ij enÙk gLr oØ (Free hand curve) lek dh iÑfr o vf/d vu:i cukb tk ldrh gA
4. lEcfU/r rF;k dk ikl&ikl inf'kr djd ryuk ;kX; o ljy cuk nrk gA

blfy, fcUnj[kh; in'ku le>u e ljy] le; o Je dh cpr] ryukRed vè;;u lfoèkk] LFkk;h iHkko] lg&lEcU/ dk vueku] ,frgkfld ,o dkfyd lpuk, inku dju e lgk;rk inku djrk gA

fcUnj[kh; in'ku d nk"k (Demerits of Graphic Presentation)

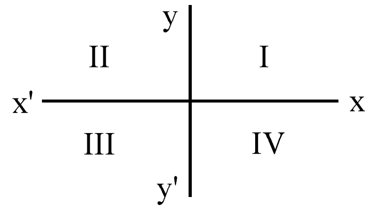
fcUnj[kh; in'ku i.k:i l nk"k jfgr ugh gA t-vkj- fjxyeu rF; vkb-,u- fiQiQLdoh d vu lkj] ;fn fcUnj[kh; in'ku dk lokf/d iHkkoh cukuk g rk tk bll vufHkK g] mUg bld vkèkkjHkr <kp ij fo'k"k è;ku nuk pkfg,A ljy j[kkfp=kk l Hkh mUg i.kr% le> fcuk xyr fu"d"k fudky tk ldr gA

bld e[; nk"k bl idkj gµ

1. bld i.kr% 'k¼rk dh tkp ugh gkrhA
2. fcUnj[k rd&lxr u gku d dkj.k efLr"d dk iHkkfor ugh dj ikrkA
3. fofHkUu ekin.Mk dk ydj fofHkUu <x l iLrr djd n#i;kx fd;k tk ldrk gA
4. fdlh rF; dh if"V d fy, fcUnj[kk dk iek.k d :i e iLrr ugh fd;k tk ldrk gA
5. fcUn j[kk lHkh idkj dh leL;kvk dk lek/ku dju e lgk;d ugh g] blfy, bldh lpuk, vi;klr gkrh gA
6. lk/kj.kr% fcUnj[k fdlh fo"k; dh iofÙk dk rk inf'kr djrk g] ijUr okLrfod ,o 'kr&ifr'kr 'k¼ ifj.kke ugh n ldrkA

fcUnj[k dh jpuk (Construction of Graph)

fcUnj[kh; i=k (Graph Paper) ij vfdr fd; tku oky fcUnvk dk vkil e feyk nu l bldh jpuk gkrh gA j[kkvk ij L;kgh ;k ifUly iQjdj mUg ekVh o Li"V dj nr gA



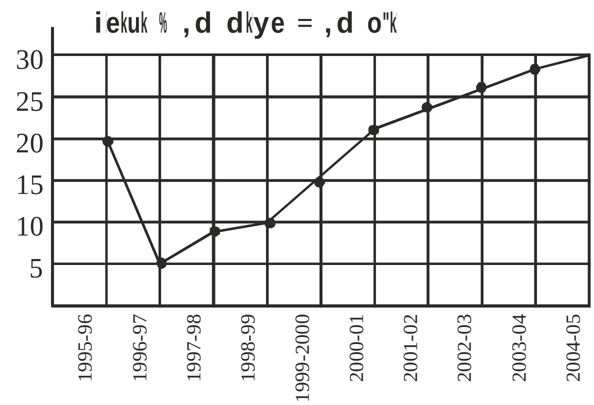
bl idkj fcUnj[kh; i=k pkj Hkkxk e cV tkrk gA ftue l iR;d Hkkx dk pj.k (Quardrant) dgr gA

mnkgj.k (Example) d rkj ij fcUnj[kh; in'ku dk le>k tk ldrk gA

fuEu vkdMk dh lgk;rk l ,d mi;Dr j[kk fp=k cukb,A

With the help of the figures given below, prepare a suitable graph

Year	Production (Rs. in Crores)
1995-96	20
1996-97	5
1997-98	8
1998-99	10
1999-2000	15
2001-2002	21
2001-2002	23
2002-2003	25
2003-2004	28
2004-2005	30



fcUnj[kh; in'ku l vkdMk dh iÑfr dk vueku vklkuh l yxk;k tk ldrk gA bldk le>u e le; o Je dki0h de yxrk gA j[kk fp=k l lkyk dk mRiknu l lEcU/ irk yx tkrk g fd fdl lky e mRiknu e of¼ gb rFkk fdl lky e deh gbA bl idkj fcUnj[kh; in'ku gekjh tfVy leL;kvk dk vlkuh l le>u o ljy cuku e lgk;d gkrk gA

4. lkjk'k (Summary)

li0yrk dk eye=k lkf[;dh e Nik gvk gA vk/fud le; e thou d iR;d {k=k e l[;kRed rF;k dk i;kx fd;k tkrk gA lkf[;dh dk Kku vdk ij vk/kfjr g ftUg ge led (Statistics or Data) d uke l tkur gA lkf[;dh foKku ,o dyk nkuk g] tk lkefgd] vkfFkd rFkk ikÑfrd leL;kvk d vè;;u ,o lek/ku d mí'; l ledk dk lxg.k] oxhdj.k] lkj.kh;u in'ku] fo'y"k.k ,o fuopu djrh g vkj bul lEcfU/r oKkfud ,o jhfr;k d i;kx l vU; Hkfrd ,o lkekftd foKkuk dk le>u e enn djrh g rkfd fu/kfjr mí';k dh ifr gk ldA 0;kolkf;d lkf[;dh dk {k=k foLrr g] ble 0;kolkf;d ledk dk lxg.k djd mUg lkjf.k;k ,o fp=kk }kjk iLrr dju d lkFk gh ,lh ifØ;kvk dk mi;kx djr g ftll ;=k ,o Je dh d'kyrk rFkk mRiknu foi.ku rFkk foKkiu vkfn dh ub i.kkfy;k dk eY;kdu djd mi;Dr fof/ d lEcU/ e fu.k; fy;k tk ldrk gA leL;kvk d vè;;u vkj fo'y"k.k d fy, vc lkf[;dh dk i;kx vifjgk; gk x;k gA fo"k; Hky gh Hkkfrd ;k jlk;u foKku dk gk ;k xf.kr dk] leL;k vkfFkd] jktufrd o lkekftd gk ledk dk i;kx lHkh {k=kk e gA vr% dgk tk ldrk g fd ledk d fcuk foKku fu"iQy g rFkk foKku d fcuk led vk/kjghu gA

5. iLrkfod iLrd (Recommended Books)

1. Introduction to Statistics- by Dr. R.P. Hooda, Macmillan India Limited
2. Statistical Methods- by S.P. gupta, Sultan Chand & Sons.
3. Business Statistics- by T.R. Jain & S.C. Aggarwal, V.K. India Enterprises.
4. Business Statistics- by Prof. M.L. Oswal, Prof. N.P. Aggarwal, Dr. H.L. Sharma, Ramesh Book Depot, Jaipur
5. Business Statistics- by S.C. Sharma, R.C. Jain, Arya Book Depot New Delhi.

6. vH;kl d fy, i'u (Self Assessment Questions)

1. ledk dh ifjHkk"kk nhft, rFkk bld dk;k ,o ifjlhekvk dk Li"V :i l o.ku dhft,A
2. lkf[;dh foKku ,o dyk nkuk dgh tkrh g D;k \ lkf[;dh dk vU; fo}kuk l lEcU/ g rk D;k g\
3. lkf[;dh dk l[;kRed ledk d ldyu] iLrrhdj.k] fo'y"k.k rFkk fuopu :i e ifjHkkf"kr fd;k tk ldrk gA le>kb,A
4. ikFkfed rFkk f}rh;d vkdMk e vUrj Li"V djA ikFkfed vkdMk d ldyu fofHkUu fof/;k le>kb,A iR;d d x.k o nk"kk dh foopuk dhft,A
5. f}rh;d vkdM fdl dgr g\ bld fofHkUu lkr dku l g\ f}rh;d vkdMk dk i;kx djr le; dku&lh lko/kfu;k è;ku e j[kh tkuh pkfg,\
6. lkf[;dh; vkdMk d j[kk fp=ke; in'ku dh mi;kfxrk rFkk lhekvk dk o.ku djA

B.Com. Part-II

Paper - BC–204 : Business Statistics *Revised By :* **Dr. Heera Lal Sharma**

Lesson No. : 2

dUæh; iofÙk d eki

(Measures of Cenral Tendency)

ljpuk (Structure)

1. ifjp;
2. mí';
3. fo"k; dk iLrrhdj.k
   - 3.1 dUæh; iofÙk dk vFk ,o egRo
   - 3.2 vkn'k ekè; d vko';d rRo
   - 3.3 lkf[;dh ekè;k d idkj
     - 3.3.1 lkekUrj ekè;
     - 3.3.2 cgyd
     - 3.3.3 efè;dk
     - 3.3.4. x.k'oj ekè;
     - 3.3.5 gjkRed ekè;
   - 3.4 foHkktu eY;
4. lkjk'k
5. iLrkfod iLrd
6. vH;kl d fy, i'u

Meaning, Properties and Measuring of Central Tendency

1. (Introduction)

Central Tendency

lkdfyr ledk d oxhdj.k rFkk lkj.kh;u }kjk ledk dh leLr egRoi.k fo'k"krkvk ij idk'k ugh Mkyk tk ldrk gA vr% fo'kky led lkexh dh dN l{e fo'k"krkvk ij idk'k Mkyu d fy; dN vU; egRoi.k ekik dh enn yuh gkrh gA bu ekik e l iFke egRoi.k eki dUæh; iofÙk dk eki dk lkf[;dh ekè; (Statistical averages) gA

2. mí'; (Objective)

bl vè;k; dk vè;;u dju d ckn vki tku ldxµ

(i) lkexh dk lf{klr :i e dl iLrr djuk gA

(ii) ledk dh ogn k[kyk dk ljy ,o ryukRed dl cuk;k tkrk gA

(iii) dUæh; iofÙk dk vFk ,o ifjHkk"kk D;k gA

(iv) lkf[;dh ekè; dk Kkr dju d fy; fofHkUu jhfr;k d ckj eA

(v) lekukUrj ekè; d chtxf.krh; x.k D;k gA

3. fo"k; dk iLrrhdj.k (Presentation of Contents)

3.1 vFk ,o egRo (Meaning and Importance)

lkf[;dh ekè; ik;% ldfyr ledk dk ifrfuf/Ro inku dju okyh ,d led gA ;g ,d ,lh led g ftld pkjk vkj ckdh lc led ik; tkr gA vkj bl dkj.k o'k ;g dUæh; iofÙk dk eki Hkh dgykrh gA

lkf[;dh fo'y"k.k e ekè;k dk egRoi.k LFkku gA ekè;k dh lgk;rk l fofHkUu led legk d chp ryuk dh tk ldrh gA ekè; lk/kj.k ckypky e Hkh vR;f/d i;kx gkrk gA mnkgj.kkFk gekj ;gk vkSlru (Inaverage) yky jx dk diMk fcdrk gA bl lIrkg vkSlru 10 leh- o"kk gb vkfn lkf[;dh fo'y"k.k vU; fof/;k dk Hkh vk/kj inku djrk gA Mk- ckmy u lkf[;dh dk ^ekè;k dk foKku* dgk FkkA

ey o d.Mky d 'kCnk e fdlh vkofÙk forj.k dh vofLFkfr ;k fLFkfr d eki ekè; dgykr gA

lkf[;dh e ekè; dk egRo nr g, Mk- ckmy u Li"V fd;k gµ ¸lkf[;dh dk okLro e ekè;k dk foKku dgk tk ldrk gA

3.2 vkn'k ekè; d vkoj.k rRo (Essential of an Ideal Average)

vc i'u mBrk g fd dku&lk ekè; mfpr jgxkA ,d vPN ,o vkn'k ekè; e fuEufyf[kr x.k pkfg,A

1. fLFkj ifjHkk"kk (Regility defined)

ekè; bl idkj l ifjHkkf"kr djuk pkfg, fd mldk ge'kk ,d gh vFk fudy vU;Fkk vyx&vyx 0;fDr fofHkUu vFk fudkyxA chtxf.krh; l=k e 0;Dr ekè; lUrk"ktud jgrk gA bld ifj.kke ,d l ikIr gkr gA

2. le>us e ljy (Simple in Understanding)

ekè; ,lk gkuk pkfg, ftl lk/kj.k O;fDr le> ldA gekjk mí'; ledk dk ljy cuku dk g u fd tfVyA

3. x.kuk e ljy (Easy to Compute)

ekè; ,lk gkuk pkfg, ftldh vklkuh l x.kuk dh tk ldA dN ekè;k dh x.kuk dfBu g ftUg i;kx e ykus e cpuk pkfg,A

4. lHkh eY;k ij vk/kfjr (Should depend on all items)

vkn'k ekè; cgh gkxk ftldh x.kuk e leg d lHkh ink dk i;kx gkA vU;Fkk] ekè; d nk ifrfuf/Ro djxk vkj u gh ifj.kke lrk"ktud gkxA

5. chtxf.krh; foopu lHko (Algebraic treatmeat is possible)

tk ekè; bl x.k dk j[kr g og vkx d lkf[;dh fo'y"k.k e cgr gh lgk;d gkr gA

6. U;kn'k l U;ure iHkkfor (Minimum affected by Sampling)

,d vPNk ekè; U;kn'k d ifjoru l de l de iHkkfor gkuk pkfg,A

7. pje eY;k }kjk de iHkkfor

,d vPNk ekè; U;ure rFkk vf/dre ink vFkkr pje eY;k }kjk de l de iHkkfor gkuk pkfg,A

3.3 lkf[;dh ekè;k d idkj (Kinds of Statistical Averages)

ekè;k dk fuEufyf[kr 'kh"kd d vUrxr vè;;u fd;k tk ldrk gA

1. xf.krh; ekè; (Artithmetic Averages)

(a) lekUrj ekè; ;k ekè;d (Arithmetic Averages or mean)

(b) x.kkÙkj ekè; (Geometric Mean)

(c) gjkRed ekè; (Harmonic Mean)

(d) f}?kkrh; ekè; (Quandratic Mean)

2. fLFkfr lEcU/h ekè; (Averages of Position)

(a) Hkfe"Bd ;k cgyd (Mode)

(b) efè;dk (Median)

3. O;kikfjd ekè; (Business Averages)

(a) py ekè; (Moving Average)

(b) ixkeh ekè; (Progressive Average)

Nk=kk dk lfo/k vkj ekè;k dh viuh mi;kfxrk dk è;ku e j[kr g, vè;;u dju d fy; ekè;k dk Øe cny fn;k x;k gA

3.3.1 lekUrj ekè; (Arithmetic Averages or Mean)

:ikUrj ekè; vR;f/d egRoi.k vkj ykd'kh; ekè; g vkj bl lkekU;r% vklr Hkh dgk tkrk gA

ifjHkk"kkµ lekUrj ekè; fdlh J.kh d lHkh ey;k dk ml J.kh ink dh l[;k e Hkkx nu ij ikIr gkrk gA

0;fDrxr J.kh (Individual Series)

x.kukµ1 iR;{k jhfr (Direct Method)

;fn J.kh d N in $X_1, X_2 .... X_a$ gk rk lekUrj ekè; ($\overline{X}$) fuEu l=k e fudkyk tk ldrk gA

$$\overline{X} = \frac{X_1 + X_2 + ... + X_a}{N}$$

lekUrj ekè; dk x (iMk tkrk g x-ckj) ;k 'a' l inf'kr djr gA (Σ flxek) xhd v{kj g ftldk vFk g eY;k dk tkM (The sum of) gkrk gA

$$\text{or } \overline{X} = \frac{\Sigma x}{N}$$

2. y?k jhfr (Short-Cut Method)

;fn ink dh l[;k vf/d gk rk y?k jhfr vf/d mfpr jgrh gA

$$\text{l=k } \overline{X} = \text{A} + \frac{\Sigma dx}{N}$$

(i) A dfYir ekè; gA ;g lcl NkV vkj lcl cM ink d chp dk dkb eY; gkuk pkfg,A

(i) $d_x = x - \text{A}$

Illustration 1

Calculate the arithmetic average by Direct Method and Short-cut Method of values given below :

15, 18, 22, 26, 28, 30, 32

Solution :

S. No.	Values	Deviation from Assumed means $d_x = (x - 25)$
1	15	– 10
2	18	– 7
3	22	– 3
4	26	– 1
5	28	+ 3
6	30	+ 5
7	32	+ 7
N = 7	Σx = 171	Σdx = – 4

Direct Method $\overline{X} = \frac{\sum fx}{n}$

$$\overline{X} = \frac{171}{7} = 34.429$$

Short-Cut Method $\overline{X} = \frac{\sum dx}{N}$

$$= 25 + \frac{-4}{77} = \frac{171}{7} = 24.429$$

Problem :

1. Use Direct Method as well as Short Cut Method to find the arithmetic means of following date :

S. No.	1	2	3	4	5	6	7	8
Income (Rs.)	525	610	515	805	607	907	600	410

($\overline{X}$ = Rs. 622.375)

2. Calculate mean mark of the following :

S. No.	1	2	3	4	5	6	7	8	9	10
Income (Rs.)	10	25	80	72	60	15	40	55	30	35

($\overline{X}$ = Rs. 42.2)

[kf.Mr J.kh (Discrete Series)

1. **iR;{k jhfr (Direct Method)**

iR;d in rFkk mudh vkofÙk (Frequency) d x.kuQy k d ;kx dk vkofÙk;k d ;kx l Hkkx ndj leLrj ek/; Kkr fd;k tkrk gA

$$\overline{X} = \frac{\sum fx}{N} \quad [N = \Sigma f]$$

tgk] x in dk eY; rFkk f mlh in dh vkofÙkA

2. **y?kq fof/ (Short-Cut Method)**

;fn A dkfYir gk vkj dx iR;d eY; dk dkfYir ek/; l fopyu (deviation) gk vFkkr $dx = X - a$, tc

$$\overline{X} = \frac{\sum fdx}{N}$$

Σfdx, vkofÙk;k vkj fopyuk d x.kuQyk dk ;kx $N = \Sigma f$ dy vkofÙkA

Illustration 2.

Find out mean marks secured by standarts

Marks (x)	3	4	5	6	7	8
No. of Students	7	12	35	32	8	6

Solution (Direction Method)

Mark x	No. of Students	*fx*
3	7	21
4	12	48
5	35	175
6	32	192
7	8	56
8	6	48
	Σ*f* = 100	Σ*fx* = 540

$$\overline{X} = \frac{\Sigma fx}{N}$$

$$= \frac{540}{100} = 5.4 \text{ Marks}$$

Short Cut Method (dfYir ekè; A = S)

Marks x	No. of students f	A l foi.ku *dx* (– *x* – *a*)	*fdx*
3	7	–2	–14
4	12	–1	–12
5	35	–0	0
6	32	1	321
7	8	2	16
8	6	3	18
	N = 100		Σ*f dx* = 40

$$\overline{X} = \frac{\Sigma fx}{N}$$

$$= \frac{540}{100}$$

$$= 5 + 0.4 = 5.4 \text{ marks}$$

vfHkfPNUu J.kh (Continues Series)

vfHkfNUu J.kh e lekUrj ekè; [kf.Mr J.kh dh Hkkfr gh fudkyk tkrk gA vfHkfNUu J.kh e igy iR;d ox (Class) dk eè; eY; (Mid value "*x*") mu J.kh dk eku ekuk tkrk gA bl idkj J.kh dk [kf.Mr J.kh e cny yr gA rRi'pkr [kf.Mr J.kh dh Hkkfr dh ey dk i;kx djr gA

Illustration 3

ekè; (Airthmetic biean) Kkr djk

Marks	0–5	5–10	10–15	5–20	20–25
No. of Students	4	14	16	8	8

Solution : dfYir ekè; A = 12.5

Marks	mid-value	No. of students	*fx*	dfYir ekè; *A* l fopyu *dx* = (*x* – A)	*fdx*
0–5	2.5	4	10	–10	–50
5–10	7.5	14	105	–5	–70
10–15	12.5	16	200	0	0
15–20	17.5	8	140	5	40
20–25	22.5	8	180	10	80
		Σ*f* = 50	Σ*fx* + 635		Σ*fdx* = 10

Direct Method

$$\overline{X} = \frac{\Sigma fx}{N}$$

$$= \frac{635}{50} = 12.7 \text{ marks}$$

Short-Cut Method :

$$\overline{X} = A + \frac{\Sigma fdx}{N}$$

$$= 125 + \frac{10}{50} = 12.7 \text{ marks}$$

in&fopyu fof/ (Step-Deviation Method)

;fn vfHkfPNUu J.kh e ox&foLrkj leku g rk fopyu (*dx*) dk ox&foLrkj (2) l Hkkx n] djd *ds* (= *dxfi*) Kkr dj yr g vkj fuEu l=k dk i;kx djd lekUrj eè; Kkr djr gA

$$\overline{X} = A + \frac{\Sigma fdx}{N} xi$$

Illustration 4

fiNyk mnkgj.k (Illustration 3) e in fopyu (Step deviation) fof/ l lekUrj eè; Kkr djkA

Solution—(Assumed mean A = 12.5, Step deviation i = 5)

Marks	mid-value x	No. of Students f	Deviation from assumed mean dx (= x – A)	dx = $\frac{dx}{i}$	*fdx*
0–5	2.5	4	–10	–2	–8
5–10	7.5	14	–5	–1	–14
10–15	12.5	16	0	0	0
15–20	17.5	8	5	1	8
20–25	22.5	8	10	2	16
		Σ*f* = 50			Σ*f ds* = 2

$$A + \frac{\Sigma fdx}{N}$$

$$\overline{X} = 12.5 + \frac{2 \times 5}{50}$$

$$= 12.5 + 0.2$$

$$= 12.7 \text{ marks}$$

Hkkfjr lekUrj eè;

;fn fofHkUu eY;k dk lkif{kr egRo (relative importane or weights) vyx&vyx gk rk Hkkfjr lekUrj eè; fuEufyf[kr l=k }kjk fudkyk tkrk gA

ekuk ?kj d eY;k (X_1, X_2............X_3 weight w_1, w_2, ...w_N gk rk

Hkkfjr lekUrj ekè; $= \dfrac{W_1X_1 + W_2X_2 + ...W_NX_N}{W_1 + W_2 + ...W_N}$

Problems

3. Calculate Arithmetic Average from the following data

Value	8	13	18	23	28
Frequency	20	30	50	40	10

($\bar{X}$ = 17.67 Units)

4. Calculate arithmetic medan from the data given below

Class interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of Plots	216	210	456	598	357	331	213	117	112	110

($\bar{X}$ = 41.7)

5. Calculate mean marks from the following table

Marks (less than)	10	20	30	40	50	60	70	80
No. of Students	25	40	60	75	95	125	190	240

($\bar{X}$ = 49.58)

6. Determine mean marks from the following table

Marks more than	5	15	25	35	45	55	65	75	85
No. of Students	100	97	89	74	54	29	19	10	4

($\bar{X}$ = 47.6)

lkefgd lekUrj ekè; (Combined Arithmetic mean)

;fn vyx&vyx legk d lekUrj ekè; rFkk mud ink dh l[;k Kkr gk rk lkefgd lekUrj ekè; vFkkr le legk d ink dk feykdj cuk;k x, ,d u; leg dk lekUrj ekè; fuEu l=k }kjk fudkyk tk;xkA

Combined Arithmetic Mean $\bar{X} = \dfrac{W_1 + \bar{X}_1 N_2 \bar{X}_2 + ...Nr\bar{X}r}{N_1 + N_2 + ...Nr}$

tgk $\bar{X}_1$] $\bar{X}_2$]$\bar{X}_3$ --------- $\bar{X}r$ = fofHkUu legk d lekUrj ekè;

N_1, N_2, N_3...........Nr = fofHkUu legk d ink dh l[;k

Illustration 5 :

The mean wage of 100 workers working in a factory running two shifts, of 60 and 40 workers respectively is Rs. 38. The mean wage of 60 workers working in the morning shift is Rs. 40 Final the mean wage of 40 workers in evening shift.

Solution :

Number	Arithmetic Average	Combined Arithmetic Average
$N_1 = 60$	$X_1 = 4$	$\overline{X} = 38$
$N_2 = 40$	$X_2 = ?$	

Now $\overline{X} = \dfrac{N_1X_1 + N_2X_2}{N_1 + N_2}$

Q $\quad 38 = \dfrac{60 \times 40 + 40x_2}{60 + 40}$

or $\quad 38 = \dfrac{24,000 + 40x_2}{100}$

or, $\quad 3800 = 24,00 + 40x_2$

or, $\quad 40x_2 = 1,400$

Q $\quad \dfrac{x_2 = 1400}{40} = \text{Rs. } 35$

Problem :

7. The mean rainfall in a certain place from Monday to Saturday in a particular week was 4.5 cm. Owing to heavy rains, on sunday, the average rainfall to the whole week inceased to 6 cm. Calculate the rainfall on Sunday. (15 cm.)

8. The mean annual salary paid to all employees of a company was Rs. 5000. The mean annual salaries paid to make and female employees were Rs. 5200 and 4200 respectively. Determine the percentage of males and females by employed the company. (males = 80%)
(Females = 20%)

vKkr eY; vFkok vKkr vkofÙk;k dk fu/kj.k (Location of Missing Values or Frequency)

lekUrj ekè; dh n'kk e ge ekye g fdµ

$$x = \frac{\Sigma x}{N} \text{ or } \frac{\Sigma fx}{N}$$

;fn l=k e $\overline{X}$, x vkj f e l dkb nk dk eY; Kkr gk rk rh l jk vk lkuh l fudkyk tk ldrk gA

Illustration 6

Find the missing frequency in the table given below :

X :	10–20	20–30	30–40	40–50	50–60
f	5	7	?	10	5

Arithmetic mean is given x = 35.75 units.

Solution : Let the missing frequency is a

Size	Mid Value x	f	fx
10–20	15	5	75
20–30	25	7	175
30–40	35	a	35a
40–50	45	10	450
50–60	55	5	275
		$\Sigma f = a + 27$	$\Sigma fx = 35a + 975$

$$x = \frac{\Sigma fx}{N}$$

$$\therefore \quad 35.75 = \frac{35a + 975}{a + 27}$$

$$\text{or} \quad 35.75a + 965.25 = 35a + 975$$

$$\text{or} \quad 35.75 - 35a = 975 - 965.25$$

$$\text{or} \quad 0.75a = 9.75$$

$$a = \frac{9.75}{0.75} = 13$$

'k¼ ekè; (Corrected Arithmetic mean) Kkr djukµ

dHkh&dHkh v'k¼ eku d fy[k tku ij v'k¼ lekUrj ekè; vklkuh l fudky fy;k tkrk g yfdu v'kf¼ dk irk yx tku d ckn 'k¼ eku dh lgk;rk l 'k¼ ekè; vklkuh l fudkyk tk ldrk gA ;g fuEufyf[kr mnkgj.k l le>k tk ldrk gA

Illustration 7 :

In a frequency distribution of 100 items of mean 40 it was found that due to mistake a value 46 was wrongly written as 64. Find the correct mean.

Solution :

$$\overline{X} = \frac{\Sigma x}{N}$$

$$\therefore \quad 40 = \frac{\Sigma x}{100}$$

or $\Sigma x = 4000$

Since 64 is wrong value and 46 is correct value, hand correct value of Σ x = 4000 – 64 + 46 = 3982

$\therefore$ Corrected mean is given by

$$\overline{X} = \frac{\sum x}{N} = \frac{3982}{100} = 39.82 \text{ units}$$

lekUrj ekè; d x.k (Merits of Arithmetic Mean)

1. **ljy (Simple)–** lekUrj ekè; lcl vf/d i;kx e vku okyh ekè; g rFkk ljyrk l bldh x.uk Hkh dh tk ldrh gA

2. **lHkh eY;k ij vk/kfjr (Based on all observation)–** lekUrj ekè; J.kh d lHkh eY;k dk ydj gh Kkr fd;k tk ldrk gA

3. **fuf'pr (Regid)–** lekUrj ekè; dk eY; fuf'pr gkrk gA

4. **fLFkjrk (Stable)–** lekUrj ekè; ij U;kn'k d ifjoru dk lcl de iHkko iMrk gA

5. **xf.krh; foopu (Mathematical Analysis)–** lekUrj ekè; J.kh d lHkh eY;k ij vkèkkfjr gkrk g vkj bldk xf.krh; foopu lHko g ftld dkj.k bldh mi;kfxrk cgr c< tkrh gA

lekUrj ekè; d vox.k (Demerits of Arithmetic mean)

1. **pje eY;k l iHkkfor (Affected by extreme observation mean)**

   lekUrj ekè; lHkh eY;k ij vk/kfjr gkrk g vkj ;fn J.kh e dN cgr cM g rk mudk iHkko iMrk gA mnkgj.kkFk] ;fn fdlh J.kh d eY; g 8, 10, 12, 14, 15, 100, 212 rk lekUrj ekè; 53 vk;xk tk J.kh dk ifrfuf/Ro ugh djrk gA

2. **voykdu l x.kuk lHko ugh (Cannot calculated by inspection)**

   voykdu ;k xkiQ d }kjk lekUrj ekè; Kkr ugh fd;k tk ldrk gA

3. **cukoVh ekè; (Fletition average)**

   dHkh&dHkh lekUrj ekè; dk eku ,lk vkrk g tk fd lHko ugh gA mnkgj.k] ifr 0;Ld L=kh 2.2 cPpk dh l[;k gkL;kLin irhr gkrh gA

4. **ylh lhekvk J.kh e x.kuk lHko ugh (It cannot be calculated in the open end classes)**

   ;fn J.kh e 'kyh lhek, nh gb g tl fd 20 l de ;k 50 l vf/d rk 'k¼ lekUrj ekè; Kkr ugh fd;k tk ldrk gA

5. **v|foÙkh; fooj.k e (In asymmetrical distribution)**

   ;fn J.kh dk vkofÙk fooj.k vjkferh; g tk lekurj ekè; forj.k dk mfpr ifrfuf/Ro ugh djxkA

Problems :

9. There were 500 workers working in a factory. Their mean wage was calculated at Rs. 20 Later on, it was discovered that the wages of two workers were misread as 180 and 20 in place of 80 and 220. Find the corret wage

10. From the following calculate the mean marks—

Marks	No. of Students
Less than 10	30
Less than 20	70
20–30	50
20–40	98
40 and above	332
50 and above	308
60–70	132
70 and above	14

11. The following on the monthly salaries in rupees of 30 employees of a factory

139	123	99	133	132	100
80	148	108	116	77	123
148	114	95	114	134	103
62	106	69	126	104	129
140	118	88	85	63	142

The factory gone bonds of Rs. 10, 15, 20, 25, 30 and 35 for individuals in the repective salaried groups.

Exceeding 60 but not exceeding 75; exceeding 75 but not exceeding 90 and so on up to exceeding 150. Find out the average bonds paid per employee.

(x = 24.50)

cgyd ;k Hkfe"Bd (Mods)

Hkfe"Bd ;k cgyd ml eY; dk dgk tkrk g tkfd ledekyk e lokf/d ckj vk;k gkA og lokf/d ?kuRo okyk fcUn g ftld pkjk vkj vf/dre led ik; tkr gA 'Mode' iQp Hkk"kk d 'kCn 'Lamode' l cuk g ftldk vFk lokf/d iQ'ku ;k fjokt l gA

;fn fdlh led ekyk e nk ;k nk l vf/d in leku :i l leku ckj vf/d :i e vkr g rk og forj.k cg&Hkfe"Bd (Multi-Modal) gkrk gA ;fn Hkfe"Bd dh l[;k ,d g rk forj.k ,dkdh Hkfe"Bd (Unit-modal) nk g rk f}µHkfe"Bd (Bi-modal) rFkk rhu gk rk f=k&Hkfe"Bd dgykr gA

Hkfe"Bd dh x.kuk (Calculation of Mode)

1. 0;fDrxr J.kh (Individual Series)

0;fDrxr J.kh dk [kf.Mr J.kh e cny dj Hkfe"Bd Kkr fd;k tk ldrk gA vf/dre vkofÙk dk in&eY; Hkfe"Bd gkxkA

Illustration 8 :

Find out the mode of the following series

11, 14, 16, 13, 15, 19, 15, 14, 20, 19, 18, 17, 16, 14, 15, 11, 12, 14, 18, 16, 15, 16, 18, 15

Solution :

nh gb O;fDrxr J.kh dk fuEufyf[kr [kf.Mr J.kh e cnyk tk ldrk gA

Values	11	12	13	14	15	16	17	18	19	20
Frequency	2	2	1	4	5	4	1	3	2	1

in&eY; 15 dh lokf/d vkofÙk 5 g vFkkr 15 Hkfe"Bd gkxkA

2. [kf.Mr J.kh (Discrete Series)

[kf.Mr J.kh e Hkfe"Bd Kkr dju dh nk fof/;k gA

(i) fujh{k.k fof/ (Inspection Method)

[kf.Mr J.kh dh vkofÙk;k fu;fer gk] igy vkofÙk;k fujUrj cM vkj ckn e fujUrj ?kV] rk ekè; e fLFkr vf/dre vkofÙk d lkeu okyk in Hkfe"Bd gkxkA

Illutration 9.

The following are the marks secured by 80 students. Find the mode.

Marks	4	5	6	7	8	9	10
No. of Students	8	10	15	17	13	10	7

Solution :

nh gb J.kh e vf/dre vkofÙk 17 d lkeu okyk in 7 Hkfe"Bd gkxkA

(ii) lkegu fof/ (Grouping Method)

tc vkofÙk;k vfu;fer gk vkj vf/dre vkofÙk dk irk yxkuk dfBu gk rk Hkfe"Bd lkegu fof/ }kjk Kkr fd;k tk ldrk g tkfd fuEu fof/ gµ

loiFke ,d lkj.kh (Table) cukb tkrh g ftle 7 dkye (Columns) gkr gA

dkye (i)—in eY;k dk fy[kk tkrk gA

dkye (ii)—i'u e nh xb vkofÙk;k fy[kh tkrh gA

dkye (iii)—nk&nk vkofÙk;k dk tkM fy[kk tkrk gA

dkye (iv)—igyh vkofÙk dk NkMdj nk&nk vkofÙk;k dk tkM fy[kk tkrk gA

dkye (v)—rhu&rhu vkofÙk;k dk tkM fy[kk tkrk gA

dkye (vi)—igyh vkofÙk dk NkMdj] rhu&rhu vkofÙk;k dk tkM fy[kk tkrk gA

dkye (vii)—igyh nk vkofÙk;k dk NkMdj] rhu&rhu vkofÙkk dk tkM fy[kk tkrk gA

bl idkj lkegu dju d i'pkr iR;d dkye dh vf/dre vkofÙk;k d lkeu oky in eY; dk fxu fy;k tkrk gA tk in&eY; vf/dre vkrk g] ogh Hkfe"Br gkrk gA

Illustration 10 :

Calculate mode from the data given below :

Values	5	6	7	8	9	10	11	12
Frequency	10	12	18	20	20	16	6	14

Solution :

vkofÙk;k vfu;fer gksu d dkj.k legu fof/ }kjk Hkfe"Br Kkr djxA

Values	Frequences					
X	I	II	III	IV	V	VI
5	18	] 30				
6	12					
7	18		] 30			
8	20	38		] 48		
9	20		] 40		] 50	
10	16	] 36		] 56		] 58
11	6					] 36
12	14	20			] 42	

Columns	Analysis Table Values having Maximum frequency							
	5	6	7	8	9	10	11	12
I				1				
II			1	1				
III				1	1			
IV				1	1	1		
V		1	1	1				
VI			1	1	1			
Total	–	1	3	6	3	1	–	–

mi;Dr lkj.kh l Kkr gkrk g fd in&eY; 8 lcl vf/d 6 ckj vk;k gA vFkkr Hkfe"Br eY; gkxkA

v[kf.Mr ;k vfofPNUu J.kh (Continuous Series)

v[kf.Mr J.kh e Hkfe"Br fuEu idkj l Kkr fd;k tkrk gA

iFke fujh{kd fof/ ;k legu fof/ }kjk Hkfe"Br ox (Modal Class) Kkr fd;k tkrk gA ;g og gkrk g ftld vkx vf/dre vkofÙk gkrh gA

rRi'pkr fuEu l=kk dh lgk;rk l Hkfe"Br dk eY; Kkr fd;k tkrk gA

$$Z = 1_1 + \frac{If_1 - fol}{If_1 - fol_1 + If_1 f_2 1} + i$$

when, Z : Mode

I_1 = Lower limit of modal class

fo = Frequency of the class preceeding the modal class

f_1 = Frequency of the model class

f_2 = Frequency of the class succedding the modal class

$u(= I_2 - I_1)$ = Class interval of the modal class

vU; l=kµ

;fn Hkfe"Br ox dh Åijh lhek I_2 i;kx e ykuh gkA

$$Z = 1_1 + \frac{If_1 - fol}{If_1 - fol_1 + If_1 f_2 1} \times i \quad ...(2)$$

$$\Delta_1 = D_1 - fol$$

$$\Delta_2 = IF_1 - f_2 I$$

$$Z = 1_1 + \frac{\Delta_1}{\Delta_1 + \Delta_2} \quad ...(3)$$

and $$Z = 1_2 - \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i \quad ...(4)$$

Illustration 11

Find out mode of the following data :

Values	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Frequency	15	28	30	47	40	34	30	15	8	10

Solution :

fujh{k.k fof/ ;k legu fof/ }kjk ;g Kkr fd;k tk ldrk g fd (30–40) Hkfe"Br ox gA fuEufyf[kr l=k dk i;kx dju ij

$$Z = 1_1 + \frac{f_1 - fo}{2 f_1 - fo - f_2} \times i$$

$$= 30 + \frac{47 - 30}{2 \times 47 - 30 - 40} \times 10 = 37.083 \text{ units}$$

Illustration 12

Calculation mode in the following frequency distribution

Values	3–6	6–9	9–12	12–15	15–18	18–21	21–24	24–27	27–28
Frequency	4	15	18	20	30	29	27	17	12

Solution :

J.kh 15-18 dh vkofÙk vf/dre g] ijUr bl J.kh rd vkofÙk;k c<rh gA vr% legu fof/ }kjk Hkfef"Br J.kh Kkr dh tk;xhA

	Frequences					
Class	I	II	III	IV	V	VI
3–6	4	19				
6–9	15		33	37	53	
9–12	18	38				68
12–15	20		50			
15–18	30	59		79		
18–21	29		56		86	
21–24	27	44		56		73
24–24	17		39			
27–30	12					

Analysis Table

Values having maximum frequency

Columns	3–6	6–9	9–12	12–15	15–18	18–28	28–24	24–27	27–30
I					I				
II					I	I			
III						I	I		
IV				I	I	I			
V					I	I	I		
VI						I	I	I	
Total	–	–	–	–	4	5	3	1	

Modal Class is 18–21

By grouping method, the modal class is (18–21) Mode can be calculated by using the following formula.

$$= I_1 + \frac{If_1 - fol}{IF_1 - fol + If_1 - f_2 I} \times i$$

$$= 18 + \frac{(29-30)}{(29-30)+(29-27)} \times 3$$

$$= 18 + \frac{1}{1+2} \times 3 - 19$$

vleku oxkUrjk okyh J.kh e Hkkfe"Bd Kkr djuk (Calculation of mode in unequal class interval)

;fn nh xb J.kh e oxkUrj vleku g rk igy J.kh dk lekUrjk okyh J.kh e ifjoru dj yuk pkfg,A

;fn J.kh lekos'kh (Inclusive) g rk mldh mioth (exclusive) e cny yuk pkfg,A

Illustration 13 :

Find the value of mode from the given below

Weight	93–97	98–102	103–107	108–112	113–117	118-122	123–127	128–132
No. of students	2	5	12	17	14	6	3	1

Solution :

Since given series is inclusive continuous series, firstly it will be converted into exclusive, modal class legu fof/ }kjk Kkr dh tk;xhA

	Frequences					
Class	I	II	III	IV	V	VI
92.5-97.5	2			19		
97.5.102.5	5	7				
102.5.107.5	12		7		34	
107.5.112.5	17	29				43
112.5.117.5	14		31			
117.5.122.5	6	20		37		
122.5.127.5	3		9		23	
127.5.132.5	1	4				10

Column								
	92.5.97.5	97.5.102.5	102.5.107.5	107.5.112.5	112.5-117.5	117.5.122.5	122.5-127.5	127.5-132.5
I			I					
II		I	I					
III			I					
IV			I	I				
V	I	I	I					
VI		I	I	I				
Total	1	3	6	3	1	–	–	–

∴ Modal class is 107.5–112.5

Then modal class in exclusive series will be [107.5–112.5], Now, using the formula for mode

$$Z = I_1 + \frac{f_1 - fo}{2F_1 - fo_1 - f_2} \times i$$

$$= 107.5 + \frac{17 - 12}{2 \times 17 - 12 - 14} \times 5$$

$$= 110.625 \text{ kgs.}$$

1. **ljy (Simple)**

   Hkfe"Bd dh x.kuk ljy g rFkk ljyrk l le>k Hkh tk ldrk gA dHkh&dHkh fujh{k.k }kjk gh Kkr fd;k tk ldrk gA mnkgj.kkFk] vklr ykx vkB uEcj d tr igur g] vkfn e Hkfe"Bd ekè; dk mi;kx fd;k x;k gA

2. **xkiQ }kjk Kkr fd;k tk ldrk g (Can be determined by graph)**

   vkofÙk&fooj.k e Hkfe"Bd xkiQ Hkh Kkr fd;k tk ldrk gA

3. **[kyh] lhekvk okyh J.kh e x.kuk lHko (in a series of open end class intervals calculation is possible)**

   Hkfe"Bd dh x.kuk [kyh lhekvk okyh J.kh e fd;k tk ldrk gA

4. **lokÙke ifrfuf/Ro (Most representative)**

   Hkfe"Bd ij vkofÙk;k dk vf/dre ?kuRo gkrk g rFkk bld pkjk vkj vkofÙk;k dfUær gA vr% Hkfe"Bd J.kh dk ifrfuf/Ro djrk gA

nk"kµ

1. **xf.krh; foopu lHko ugh (Mathematical analysis is not possible)**

   Hkfe"Bd Kkr dju e J.kh d iR;d in dk i;kx ugh gkrk g vkj bl dkj.ko'k bldk xf.krh; foopu lHko ugh gA

2. **vfuf'pr (Uncertain)**

   ,d fooj.k e ;fn ink dh l[;k vf/d ugh gkxh] Hkfe"Bd ugh Hkh gk ldrk g vkj lekU;r% lfuf'pr HkfePBd ugh gkxkA

3. **fun'ku dk vHkko (Affect of sampling)**

   Hkfe"Bd ij lekUrj ekè; dh vi{kk fun'ku dk vf/d iHkko iMrk gA

Problems :

13. Calculate mode of the following individual series

Values	13	18	17	20	16	18	22	15	14	14
	18	15	16	18	21	20	18	14	18	19

(Z = 18)

14. Determine mode of the following date

Observations	10	12	14	16	18	20	22	24	26
Frequency	18	22	18	30	34	36	35	15	17

(Z = 20)

15. Compute the mode

Size	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45
Frequency	20	24	32	28	20	16	34	10	8

(Z = 13.33)

16. Determine mode of the following

Marks	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79
No. of Students	6	29	87	181	247	263	133	43

(Z = 47.6)

17. Calculate the modal value from the following data–

Income (Rs.)	No. of persons	Income (Rs.)	No of persons
Less than 100	8	Less than 400	60
– 200	22	500	67
– 300	35	– 600	70

(Z = Rs. 340)

efè;dk (Median)

efè;dk led J.kh dk og in eY; g tk leg dk nk leku Hkkxk e bl idkj foHkDr djrk g fd ,d Hkkx e leLr eY; efè;dk l vf/d vkj nlj Hkkx e leLr eY; mll de gA efè;dk forj.k dk eè; ;k dUæ dk eY; gkrk gA

efè;dk dh x.kuk

1. 0;fDrxr J.khµiFke J.kh dk vkjkgh (ascending) ;k vojkgh (decending) Øe e fy[k yr g rRi'pkr fuEufyf[kr l=k dh lgk;rk l efè;dk (M) Kkr djr gA

$$M = \text{Size of the } \left(\frac{n+1}{2}\right)^{th} \text{ item if N is odd}$$

$$\text{where} = \frac{\text{Size of } \left(\frac{n}{2}\right)^{th} \text{ item + size off } \left(\frac{B}{2}+1\right)^{th} \text{ item}}{2} \text{ if N is even}$$

M = Median

N = Numbers of items

Illustration 14.

Determine the median of the following data

x	7	18	25	17	28	15	30	35	16

Solution :

igy ink dk vkjkgh Øe e djxA

Sr. no.	1	2	3	4	5	6	7	8	9
x	7	15	16	17	18	25	28	30	35

$$\text{Median} = \text{size of the } \left(\frac{n+1}{2}\right)^{th} \text{ items}$$

= Size of the 5the item

and it is 18

Illustration 15

Find the median of the following data

x	8	15	18	16	26	35	20

Solution :

igy ink dk vkjkgh Øe e fy[ku ij

Sr. no.	1	2	3	4	5	6	7	8
x	8	15	16	18	20	25	26	35

$$M = \text{size of the } \left(\frac{n+1}{2}\right) \text{item}$$

$$M = \text{size of } \left(\frac{8+1}{2}\right)^{th} \text{ item}$$

= Size of the 4.5 the item

= Size of $\frac{\text{4th + size of 5the item}}{2}$

$$= \frac{18+20}{2} = 19$$

2. [kf.Mr J.khµ [kf.Mr J.kh e efè;dk Kkr dju d fy; iFke] lp;h vkofÙk;k (Cumulative Frequences (c.f.) Kkr djr g rFkk fuEufyf[kr l=k dh lgk;rk l efè;dk Kkr djr gA

$$M = \text{size of } \left(\frac{n+1}{2}\right)^{th} \text{ item}$$

Illustration 16

Calculate median of the following data :

Values (X)	14	16	18	20	22	24	26	28
Frequenncy (f)	15	30	33	40	13	7	3	2

Values (X)	Frequency (f)	Cumulative Frequency (cf)
14	15	15
16	30	45
18	33	78
20	40	118
22	13	131
24	7	138
26	3	141
28	2	143

$$M = \text{size of the } \left(\frac{n+1}{2}\right)^{th} \text{ item}$$

$$M = \text{size of the } \left(\frac{143+1}{2}\right)^{th} \text{ item}$$

$$= \text{size of 72th item}$$

Which lies in the cumulative frequency 78, hence median = 18

[kf.Mr vFkok vfofPNUu J.kh

v[kf.Mr J.kh e efè;dk fudkyu d fy; iFke] lp;h vkofÙk Kkr djr gA efè;dk $\frac{n}{2}$ th in gkrk gA $\frac{n}{2}$ in ftu lp;h vkofÙk (c.f) e vkrk g mlI lEcfU/r oxkUrj efè;dk oxkUrj (median class interval) dgykrk gA vUr e fuEufyf[kr l=k dh enn l efè;dk Kkr djr gA

$$M = 1_1 + \frac{i}{f}(\text{in} - c)$$

where M = Median

I_1 = lower limit of medians class

i = Class interval of meadian class

f = $\frac{n}{2}$ frequency of median class

M =

C = Cumulative frequency of the class preceeding the median class.

;gk ij ;g è;ku nu dh ckr g fd Åij fyf[kr efè;dk dk l=k mioDr (exclusive) J.kh e ykx gkrk gA

Illustration 17 :

Find out the median from the following data :

Class	20–29	30–39	40–49	50–59	60–69	70–79
Frequency	8	15	40	28	14	4

Solution :

Given series is the inclusive continuous series. Inclusive series will be changed into exclusive series

Class	Frequency	Cf
19.5.29.5	8	8
29.5.39.5	15	23
39.5.49.5	40 (f)	63 (c).
49.5.59.5	28	91
59.5.69.5	14	105
69.5.79.5	4	109

$$\text{Median} = \text{Size of } \frac{n}{2} \text{ th item}$$

$$M = \text{size of } \left(\frac{109}{2}\right) \text{th item}$$

which lies in the cf 63, hence (39.5.49.5) is the median class.

$$M = 1_1 + \frac{i}{f}\,(m - c)$$

$$= 39.5 + \frac{10}{40}\,(54.5 - 23)$$

$$= 47.375$$

efè;dk d x.k

1. ljyµ efè;dk dk Kkr djuk ljy g rFkk ljyrk l le> e Hkh vk tkrk gA
2. xkiQ }kjk Kkr djukµ efè;dk xkiQ dh enn l Kkr fd;k tk ldrk gA
3. pje eY;k dk U;ure iHkkoµefè;dk ,d LFkkuµeè; g vkj pje&eY;k dk bl ij iHkko ugh iMrk gA
4. x.kkRed rF;k e mi;kxµx.kkRed rF;k tIk fd nfjærk] ckf¼d Lrj vkfn dk Kku ikIr dju e rFkk foopu e doy efè;dk gh mi;Dr ekè; gA
5. xf.krh; foopuµefè;dk fooj.k d iR;d in ij vk/kfjr ugh gkrh g rFkk bldk xf.krh; foopu dfBu gkrk gA fiQj Hkh ekè; fopyu (Mean Deviation) vkfn e efè;dk dk gh lokÙke i;kx djr gA
6. efè;dk l ink d fopyuk dk ;kx U;ureµnlj ekè;k dh vi{kk efè;dk l fy; x; fopyuk dk ;kx (½.k rFkk /u fpUgk dk NkMr g,) U;ure gkrk gA

efè;dk d nk"k

1. xf.krh; foopu dk vHkkoµefè;dk dk xf.krh; fØ;kvk e lkekU; r% i;kx ugh fd;k tk ldrkA
2. fuf'prrkµlekUrj ekè; dh vi{kk efè;dk dk eY; fuf'pr ugh gkrk gA
3. fun'ku dk vHkkoµfun'ku dk efè;dk ij lkekUrj ekè; dh vi{kk T;knk iHkko iMrk gA

foHkktu eY; (Partition Values)

efè;dk ledekyk dk nk leku Hkkxk dk ckVrk gA bl h idkj ;fn ledekyk dk pkj] ikp] vkB] nl ;k lk cjkcj Hkkxk e ckVk tk, rk foHkDr dju oky eY;k dk Øe'k% prFkd (Quartiles), iped (Quentiles), v"Ved (Octiles), n'ked (deciles) ;k 'kred (percentiles) dgr gA ;g foHkDr dju oky eY; (Partition Values) dgykr gA

foHkkt d eY;k dh x.kuk

prFkd] iped] v"Ved] n'ked rFkk 'kred dk Øe'k% Q, Qn, O, D rFkk P l inf'kr fd;k tk ldrk gA 0;fDrxr rFkk [kf.Mr J.kh e (N + 1) dk mI l[;k l Hkkx n nr g ftru Hkkxk e ledekyk dk foHkkftr djuk g rFkk efè;dk dh Hkkfr x.kuk dj yr gA mnkgj.kkFkμ

$$Q_1 = \text{size of } \left(\frac{N+1}{4}\right)^{th} \text{ item}$$

$$Q_1 = \text{size of } 3\left(\frac{N+1}{4}\right)^{th} \text{ item}$$

$$Qn_1 = \text{size of } \left(\frac{N+1}{4}\right)^{th} \text{ item}$$

$$Qn_3 = \text{size of } 3\left(\frac{N+1}{5}\right)^{th} \text{ item}$$

$$Q_3 = \text{size of } 3\left(\frac{N+1}{8}\right)^{th} \text{ item}$$

$$D_3 = \text{size of } 8\left(\frac{N+1}{10}\right)^{th} \text{ item}$$

$$P_{37} = \text{size of } 37\left(\frac{N+1}{100}\right)^{th} \text{ item}$$ vkfn

v[kf.Mr J.kh e (N + 1) d LFkku ij N dk i;kx djr gA

$$Q_3 = \text{size of } \frac{3N^{th}}{4} \text{ item}$$

$$D_7 = \text{size of } \frac{7N^{th}}{4} \text{ item}$$

$$D_{33} = \text{size of } \frac{33N^{th}}{100} \text{ item}$$ vkfn

efè;dk dh Hkkfr foHkktu eY; Kkr fd;k tkrk gA

$$Q_3 = 1_1 + \frac{i}{f}\left(\frac{3N}{4} - c\right)$$

$$D_7 = 1_1 + \frac{i}{f}\left(\frac{7N}{4} - c\right)$$

$$D_{33} = 1_1 + \frac{i}{f}\left(\frac{33N}{100} - c\right)\text{vkfn}$$

Illustration 18

Find out Q_1, Q_2, Qn_3, Q_5, D_7 P_{30} of the following data

44	17	22	35	40	18	23	27	34	33	80	72	65	55
31	9	38	54	62	92	6	11	21					

Solution

Put the given data in the ascending order

Sr. No.	1	2	3	4	5	6	7	8	9	10	11	12
Values x	6	9	11	17	18	21	22	23	27	31	33	34
Sr. No.	13	14	15	16	17	18	19	20	21	22	23	24
Values x	35	38	40	44	44	54	55	62	65	72	80	92

$$Q_1 = \text{Size of } \frac{N+1^{th}}{4} \text{ item}$$

$$= \text{Size of } \frac{24+1^{th}}{4}$$

= Size of 6¼th item

= Size if 6th item + ¼ (= Size of 7th item = size of 6th item)

= 21 + 1/4 (22 – 21) = 21.25

$$Q_3 = \text{size of } \frac{3(N+1)^{th}}{4} \text{ item}$$

$$= \text{size of } \frac{3(24+1)^{th}}{4} \text{ item}$$

$$= \text{item } 18\ \frac{3}{4}$$

= Size of 18th item + 3/4 (= Size of 19th item – = Size of 18th item)

= 54 + 3/4 (55 – 54) = 54.75

$$Qn_3 = \text{size of } 3\left(\frac{N+1}{5}\right)^{th} \text{ item}$$

$$= \text{size of } 3\left(\frac{24+1}{5}\right)^{th} \text{ item}$$

= size of 15th item = 40

$$= \text{size of } \frac{5(N+1)^{th}}{8} \text{ item}$$

$$= \text{size of } \frac{5(24+1)^{th}}{8} \text{ item}$$

$$= \text{size of } 15\frac{5}{8} \text{th item}$$

$$= 40 + \frac{5}{8}(44 - 40) = 42.5$$

$$= \text{size of } \frac{7(N+1)^{th}}{10} \text{ item}$$

$$= \text{size of } \frac{7(24+1)^{th}}{10} \text{ item}$$

$$= \text{size of } 17\frac{1}{2} \text{th item}$$

$$= 44 + \frac{1}{2}(54 - 44) = 49$$

$$P_{33} = \text{size of } \frac{33(N+1)^{th}}{100} \text{ item}$$

$$= \text{size of } \frac{33(24+1)^{th}}{100} \text{ item}$$

$$= \text{size of 8th item}$$

$$= 23 + \frac{1}{4}(27 - 23) = 24$$

Illustration 19

Determine the lower and upper Quartiles, 7th deciles and 57th deciles for the following data :

Value	10	13	16	19	22	25	28
Frequency	8	18	22	14	9	7	2

Solutioin :

Values x	**Frequency** f	***C.f.***
10	8	8
13	18	26
16	22	48
19	14	62
22	9	71
25	7	78
28	2	80

Lower Quartile (Q_1) = Size of $\frac{N+1}{4}$ th item

= Size of $\frac{80+1}{4}$ th item

= Size of 20.25 th item = 13

Upper Quartile (Q_3) = Size of 3 $\frac{(N+1)}{4}$ th item

Upper Quartile (Q_3) = Size of 3 $\frac{(80+1)}{4}$ th item

Size of 60.75 in item = 19

D_7 = size of $\frac{7(N+1)}{10} = \frac{7(80+1)}{10}$ 56.7 = item = 19

P_{37} = size of $\frac{57(N+1)}{100} = \frac{57(80+1)}{100}$ = 46.17 = item = 16

Illustration 20

Calculate the median, quartiles and 6th deciles for the following data :

Marks	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79
No of Students	5	8	7	12	28	20	10	10

Solution : Given series is an inclusive continuous series. To find out partition values we will convert it into exclusive continuous series.

Marks ***f***	No. of Students ***c.f.***	Cumulative Frequency
Below 9.5	5	5
9.5.19.5	8	13
19.5.29.5	7	20
29.5.39.5	12	32
39.5.49.5	28	60
49.5.59.5	20	80
59.5.69.5	10	90
69.5.79.5	10	100
	N = 100	

Median = Size of $\frac{N^{th}}{2}$ item = $\frac{100^{th}}{2}$ item = 50th item, which lies in the class (39.5.49.5) Now

$$M = 1 + \frac{i}{f}\ (c - c)$$

$$= 39.5 + \frac{10}{28}\ (50 - 32) = 45.93 \text{ marks}$$

$$Q_1 = \text{Size of } \frac{N^{th}}{4} \text{ item}$$

$$= \frac{100^{th}}{4} \text{ item, which lies in the class (29.5 - 39.5) Now.}$$

$$Q_1 = 1 = \frac{i}{f}\left(\frac{N}{4} - c\right)$$

$$= 29.5 + \frac{10}{12}\ (25 - 20)\ 33.67 \text{ marks}$$

$$Q_2 = \text{Size of } \frac{3N^{th}}{4} \text{ item}$$

$= \text{Size of } \frac{3 \times 100th}{4}$ item, which lies in the class (49.5 – 59.5). Now

$$Q_3 = 1_1 + \frac{i}{f}\left(\frac{3N}{4} - c\right)$$

$$= 49.5 + \frac{10}{20}\ (75 - 60) = 57 \text{ marks}$$

$D_6 = \text{Size of } \frac{6N^{th}}{10} \text{ item} = \text{Size of } \frac{6 \times 100th}{10} \text{ item} = 60 \text{ th item,}$

which lies in the class (39.5 – 49.5) Now.

$$D_6 = 1_1 + \frac{i}{f}\left(\frac{6N}{10} - c\right)$$

$$= 39.5 + \frac{10}{28}\ (60 - 32) = 49.5 \text{ marks}$$

Problems.

18. What do you understand by central tendency ? Under what condition in medies more suibable than other messures of cenral tendency ?

19. Defines median. What are the properties of median ? Discuss its merits and demerits

Twelve students obtained the following marks in three papers. Stab, with reason, in which paper the level of intelligence is the higher

A	36	56	41	46	54	59	55	51	52	44	37	59
B	58	54	21	51	59	46	65	31	68	41	70	36
C	65	55	26	40	30	74	45	29	85	32	80	30

[A : M = 51.5
B : M = 52.5 (Highest)
C : M = 42.5]

20. Find out the median of the following data

X	14	16	18	20	22	24	26	28	30
f	7	18	25	30	32	17	6	4	5

(M = 20)

21. Find the median and Quartiles from the following table

Size	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
Frequency	7	10	13	26	35	22	11	5

(M = 31.71. Q_1 = 25.93 Q_2 = 36.81)

22. Calculate median first quartile and eight-fifth percentile of the following data

Income (000 Rs.)	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40
No. of families	75	250	350	192	68	35	24	6

(M = 12.5, Q_1 = 8.5 P = 19.56)

x.kkÙkj ekè; (Geometric Mean)

fdlh led J.kh dk x.kkÙkj ekè; bld lHkh eY;k d x.kuiQy dk og ey (root) gkrk g] ml J.kh e bdkb;k g vFkkr

Geometric Mean (6) = Nx_1 x x x................x_a

tgk x_1, x_2,.....................x_3............N ink d eY; gA

bl l=k dk gy y?kx.kd (logarithm) dh lgk;rk l ljyrk l Kkr fd;k tk ldrk gA vFkkr

$$G = \text{Antilog}\left[\frac{\log x_1 = \log N_2 + \ldots\ldots\ldots\ldots + \log x_n}{N}\right]$$

G = Antilog ....(i)

l=k (i) 0;fDrxr J.kh e x.kkÙkj ekè; Kkr dju d fy, gA

Illustration 21

Find the Geometric Mean of the following Individual series—

536 52 8 0.6 0.03 0.034 0.006

Solution :

X	log X
536	2.7292
52	1.7160
8	0.9031
0.6	1.7782
0.03	2.4771
0.034	2.5318
0.006	3.7782
N = 7	$\Sigma \log x$ = T.9136

$$\text{G.M.} = \text{Antilog}\left(\frac{\Sigma \log x}{N}\right)$$

= Antilog T.9136

= Antilog 7 + 6.9136

= Antilog T. 98766

= 0.9721

[kf.Mr rFkk vfofPNUu J.kh

vfofPNUu J.kh e eè; fcUn Kkr djd J.kh dk [kf.Mr J.kh e ifjofrr dj yr gA [kf.Mr J.kh fuEufyf[kr l=k dh lgk;rk l x.kkÙkj ekè; Kkr dj yr gA

$$G = \text{Antilog}\left[\frac{f_1 \log X_1 + f_2 \log X_2 + fN \log XN}{\Sigma f}\right]$$

$$= \text{Antilog}\ \frac{\Sigma \log x}{N}$$

$$N = \Sigma f$$

Illustration 22

Find the Geometric Mean of the following series–

Class	3–7	7–11	11–15	15–19	19–23
Frequency	3	12	25	13	7

Solution :

Class	*f*	Mid Values	log X	*fx*log*x*
3–7	3	5	0.6990	2.0970
7–11	12	9	0.9542	11.4504
11–15	25	13	1.1139	27.8475
15–19	13	17	1.2304	15.9952
19–23	7	21	1.3222	9.2554
	Σf = N = 60		66.6455	

$$\text{Antilog}\left(\frac{\Sigma \log x}{N}\right)$$

$$\text{Antilog}\left(\frac{66.6455}{60}\right)$$

= Antilog 1.1108 = 1290

Hkkfjr x.kkÙkj ekè; (Weight Geometric Mean)

;fn fofHkUu eY;k dk lkif{kr egRo (relative importance) vyx&vyx gk rk Hkkfjr x.kkÙkj ekè; fudkyk tk ldrk gA blds fy, fuEufyf[kr l=k i;kx e yk;k tkrk gA

$$\text{Weighted G.M.} = \text{Antilog}\left(\frac{Wx \log x}{\Sigma W}\right)$$

tgk Ws Hkkjk dk inf'kr djr gA

Illustration 23

Use the following data and find weight Geometric Mean

Items	income Numbers	Weighted
Food	125	10
Cloth	135	6
Fule & light	140	2
Rent	180	3
Miscellaneous	170	4

Solution :

items	No.	Logx	Weight	W log x
Food	125	2.02969	10	20.9690
Cloth	35	2.1303	6	12.7818
Fule & Lights	140	2.1461	2	4.2922
Rent	180	2.2553	3	6.7652
Miscellaneous	170	2.2304	4	8.9216
			25	53.7309

$$\text{Weighted G.M.} = \text{Antilog}\left(\frac{\Sigma W \log x}{\Sigma W}\right)$$

$$= \text{Antilog}\left(\frac{53.7309}{25}\right)$$

$$= \text{Antilog } 2.1492 = 141$$

x.kkÙkj ekè; dk i;kxµ

x.kkÙkj ekè; dk i;kx fo'k"krkvk tul[;k&of¼ pØof¼ C;kt vkfn e gku oky vkl r ifr'kr ifjoruk dk Kkr dju e fd;k tkrk gA pØof¼ C;kt dju d fy; fuEufyf[kr l=k dk i;kx fd;k tkrk gA

$$P_N P_O (1 + r) \text{ or } r = \frac{n\sqrt{P_N} - 1}{P_O}$$

Where P_o = Present value

P_N = amont after n periods

r = rate of interest per unit per period

Illustration 24

The sale is inceased by 10% and 12% in the first and second years respectively. It decreased by 2% and 3% in the third and fourth years, respectively. Again it increased by 5% in the fifth year. Find the average change in the sale in last five years.

Years	Rate of Changed	Changed value taking 100 in the beginning (x)	log x
Ist	10% increase	100 + 10 = 110	2.0414
IInd	12%	100 + 12 = 112	2.0492
IIIrd	2% decrease	100 – 2 = 98	1.9912
IVth	3%	100 – 3 = 97	1.9868
Vth	5% increase	100 + 5 = 105	2.0212
N = 5			Σlogs = 10.0898

$$\text{G.M.} = \text{Antilog}\left(\frac{\Sigma f \log s}{N}\right)$$

$$= \text{Antilog}\left(\frac{10.0898}{5}\right)$$

Antilog 2.0180 = 104.2

Average increase in sales = 104.2 – 100 = 4.2%

x.kkÙkj ekè; d x.k

1. lHkh eY;k ij vk/kfjrµx.kkÙkj ekè; Kkr dju d fy; ledekyk d lHkh eY;k dk i;kx fd;k tkrk gA
2. chtxf.krh; foopukµchtxf.krh; foopu lHko gA
3. pje eY;k dk U;ure iHkkoµbl ij lhekUr eY;k dk de iHkko iMrk gA tgk ink dk eY; vlkeku gk x.kkÙkj ekè; lokf/d i;kx gkrk gA
4. vuikrk dk ekè;µifr'kr of¼ nj Kkr dju d fy; bl i;kx fd;k tkrk gA vkIr pØof¼ nj] tul[;k of¼ vkfn Kkr dju d fy; x.kkÙkj ekè; gh i;kx e yk;k gA

x.kkÙkj ekè; d nk"k

1. x.kuk tfVyµbldh x.kuk vfr tfVy gA y?kx.kd rFkk ifry?kx.kd (Antilor) dk i;kx gkrk gA
2. le>uk dfBuµbldk le>u e dfBukb gkrh gA
3. 'kU; vFkok ½.kkRed eY; gku ij ;fn fdlh Hkh in dk eY; 'kU; gk rk lHkh ink dk eY; x.kuiQy 'kU; gk tkrk gA rFkk x.kkÙkj ekè; Hkh 'kU; vk,xkA ;fn dkb in ½.kkRed g rk Hkh x.kuk dfBu g rFkk G.M. dkYifud vkrk gA

23. Define Geometric mean and discuss its merits and demerits. What are its main used?

24. Calculate G.M. frouth the following data

1 7 29 115 and 375

(G = 24.45)

25. Calculate Geometric mean for the following series

310 32 8 0.9 0.4 0.0035

(G = 1.468)

26. Find G.M. of the following

X	2	4	6	8	10
f	3	4	7	4	2

(G = 4.78)

27. Find G.M. in the following series

Marks obtained (below)	10	20	30	40	50
No. of candidates	12	27	72	92	100

(G = 21.4 marks)

28. As amount is doubled itself in 5 years. Find the average rate of interest.

29. A machine is depreciated 40% in value in the first year. 25% in second year and 10% per annum for the next three years, each percentage being calculated on the diminishing value. What is the average percentage of depreciation for the five year.

(20% App.)

gjkRed ekè; (Harmenic Mean)

gjkRed ekè; fdlh led ekyk e ink dh l[;k dk mud 0;Øek (reciprocals) d ;kx l Hkkx nu ij feyrk gA fuEufyf[kr l=kk dh lgk;rk l bl Kkr fd;k tk ldrk gA

1. 0;fDrxr J.kh

$$\text{(Harmonic Mean ((H.M.)} = \frac{N}{\frac{1}{X_1}+\frac{1}{X_2}+\ldots\frac{1}{X_N}}$$

$$= \frac{N}{\Sigma\frac{1}{x}}$$

$$= \text{Reciprocal } \frac{\text{Reciprocals}}{\text{N}}$$

Illustration 25

Find the Harmonic Mean from the following data

230 22 12.5 1.25 0.125 0.0034

Solution :

x	Reciprocals $\frac{1}{x}$
230	0.00435
22	0.04545
12.5	0.08000
1.25	0.80000
0.125	8.00000
0.025	40.00000
0.0034	294.11765
N = 7	$\Sigma\frac{1}{x}$ = 343.04745

$$\text{H.M. } \frac{N}{\Sigma\frac{1}{x}} = \frac{7}{343.0445} = 0.0204$$

[kf.Mr rF; v[kf.Mr J.kh

v[kf.Mr J.kh e eè; fcUn Kkr djd mI [kf.Mr J.kh e ifjofrr dj yr gA rRi'pkr fuEufyf[kr l=k dk i;kx fd;k tkrk gA

$$H = f_1 + f_2 + f_3 ......+ f_N$$

$$\frac{f_1}{x_1} + \frac{f_2}{x_2} + \frac{f_3}{x_3} + ... \frac{fN}{xN}$$

$$\frac{\Sigma f}{\Sigma\frac{f}{N}} = \text{Reciprocal}\left[\frac{f \times \text{Reciprocal}}{\Sigma f}\right]$$

Illustratin 26.

Find the H.M. in the following series :

Class	0–5	5–10	10–15	15–20
Frequency	15	30	32	14

Solution :

Class	f	mid value x	Receiprocals $\frac{1}{x}$	fx Reciprocals $\frac{r}{x}$
0–5	15	2.5	0.40000	6.0
5–10	30	7.5	0.13333	4.0
10–15	32	12.5	0.08000	2.56
15–20	14	17.5	0.05714	0.79996
91				$\Sigma \frac{f}{x} = 13.35996$

$$\text{H.M.} = \frac{\Sigma f}{\Sigma \frac{f}{\Sigma}} = \frac{91}{13.35996} = 68114$$

Hkkfjr gjkRed ekè;µ;fn fofHkUu eY;k dk lkif{kr egRo (Relative importance) vyx&vyx Hkkfjr gjkRed ekè; fudkyk tk ldrk gA bl d fy, fuEufyf[kr l=k i;kx e yk;k tkrk gA

$$\text{Weighted H.M.} = \frac{\Sigma W}{\Sigma \frac{w}{x}}$$

Illustration 27.

Complete the H.M. of 40, 45, 50, 55, 60 when their respective weights are 3, 7, 4, 3, 3

X	W	$\frac{1}{x}$	$\frac{w}{x}$
40	3	0.0250	0.0750
45	7	0.0222	0.1554
50	4	0.0200	0.0800
55	3	0.0182	0.0546
60	3	0.0167	0.0501
	20		0.4151

$$\text{Weighted H.M.} = \frac{\Sigma w}{\Sigma \frac{w}{x}}$$

$$= \frac{20}{0.4151} = 48.18$$

gjkRed ekè; dk mi;kxμ

oLrq dh ek=kk ifr #i;k ;k ox rFkk nlj h l lEcfU/r i'u gjkRed ekè; dh lgk;rk l gy fd;k tkrk gA ;fn ;gh i'u #i, ifr ek=kk ;k ^ox vkj le;* e fn;k gk rk leklUrj ekè; dh lgk;rk l gy fd, tk;xA

Illustration 28

A train goes at a speed of 20 k.m. per hour for first 16 km. at a speed of 40 k.m.j. for next 20 k.m. h. It covers last 10 k.m. at a speed of 15 k.m.h. Find its average speed.

Solution :

Here $x_1 = 20$ $w_1 = 16$

$x_2 = 40$ $w_2 = 20$

$x_3 = 15$ $w_3 = 10$

$$\therefore \text{H.M. (Average Speed)} = \frac{\Sigma W}{\frac{w^1}{x_1} + \frac{w^2}{x_2} + \frac{w^3}{x_3}}$$

$$= \frac{16 + 20 + 10}{\frac{16}{20} + \frac{20}{40} + \frac{10}{15}}$$

$$= \frac{46}{0.8 + 0.5 + 0.6667}$$

$$= \frac{46}{1.9667} \text{ 23.39 k.m. per hour}$$

Problems

30. Find H.M. of the following individual series

15 250 15.7 157 105.7 10.5 1.06 25.7 and 0.257

(H.M. = 1.737)

31. Calculate H.M. of the following data :

x	130	135	140	145	146	148	149	150	157
f	3	4	6	6	3	5	2	1	1

(H.M. = 142.36)

32. An aeroplane flies along the four sides of a square at a speed of 100, 200, 300 and 400 miles per hours respectively. What is the average speed of the aeroplane in its flight the square ?

4. lkjk'k (Summary)

lkf[;dh ekè; J.kh e 0;Dr ledk dk ifrfuf/Ro dju okyh ,d ,lh en g ftle vU; lHkh en ekè; d lehi gkrh gA ekè; dk dUæh; iofÙk dk eki blfy, dgr g D;kfd 0;fDrxr pj eY;k dk teko vf/drj mlh d vkl&ikl gkrk gA blfy, bldk LFkku lkekU;r% J.kh d eè; e gh gkrk gA lekUrj ekè; (Arithmetic Mean) bl ekè; e ,d vPN ekè; d lHkh x.k ik; tkr gA lekUrj ekè; og jkf'k g tk fdlh J.kh d ink d eY;k d ;kx dk mudh l[;k l Hkkx nu l ikIr gkrh gA Hkfe"Br ;k cgyd (Mode) cgyd led ekyk e lcl vfèkd ckj vku okyh en dk dgr gA efè;dk (Median) efè;dk J.kh dk vkjkgh vFkok vojkgh Øe e ifjofrr dju ij nk Hkkxk e bl idkj foHkkftr djrk g fd ftruh en mlI Åijh gkrh g mruh gh en mlI uhp gkrh gA

x.kkÙkj ekè; (Geometric Mean) x.kkÙkj ekè; ifr'kr of¼ vFkok ifr'kr deh fudkyu d fy; i;kx fd;k tkrk gA ;g led J.kh e lHkh enk d x.kuiQy dk led l[;k dk ey (root) gkrk gA

gjkRed ekè; (Harmonic Mean)— gjkRed ekè; fudkyu d fy, led ekyk dh dy l[;k dk enk d mYV (Reciprocals) d ;kx l Hkkx dju k iMrk gA

foHkktu eY; (Partition Values)— ledekyk dk pkj] ikp] vkB] nl ;k lk Hkkxk e foHkDr dju oky eY; ^foHkktu eY;* dgykr gA

5. iLrkfor iLrd (Recommended Books)

1. Introdution to Statistics by Dr. R. Hooda, Macmillan India Limited.
2. Statistical Methods by Dr. S.P. Gupta, Sultan Chand & Sons.
3. Business Statistics by S.C. Sharm, R.C. Jain, Arya Book Depot Delhi.
4. Business Statistics by M.L. Oswal, N.R. Aggarwal, H.L. Sharma Ramesh Book Depot, Jaipur
5. Business Statistics by T.R. Jain, V.K. India Enterprise, New Delhi

B.Com. Part-II

Paper - BC–204 : Business Statistics *Revised By :* **Dr. Heera Lal Sharma**

Lesson No. : 3

vifdj.k dk vFk] x.k ,o eki

Meaning, Characteristics and Measurement of Dispersion

lajpuk (Stracture)

1. ifjp; (Introduction)
2. mí's; (Objective))
3. fo"k; dk iLrrhdj.k
   - 3.1 vifdj.k dk vFk
   - 3.2 vifdj.k d mí's; ,o egRo
   - 3.3 vifdj.k dh x.kuk dju dh fof/;k
     - 3.3.1 lhek dh jhfr;k
       - 3.3.1.1 foLrkj
       - 3.3.1.2 vUrj prFkd foLrkj
       - 3.3.1.3 'kred foLrkj
     - 3.3.2 fopyu ekè; dh jhfr;k
       - 3.3.2.1 prFkd fopyu
       - 3.3.2.2 ekè; fopyu
       - 3.3.2.3 ieki fopyu
     - 3.3.3 fcUn j[kh; jhfr
       - 3.3.3.1 ykjt pØ
4. lkjk'k
5. iLrkfor iLrd
6. vH;kl d fy, ç'u

1. ifjp; (Introduction)

ledekyk dh cukoV dk i.k vè;;u doy dUæh; iofÙk dk eki fudkydj ugh fd;k tk ldrkA ledekyk dk iR;d in eY; dk ik;% ekè; d leku le>k tkrk gA lR; rk ;g g fd ekè; in eY; dk doy ifrfuf/Ro djrk g tSls fd ,d eu"; ftldh Åpkbz 5' 10" g ,d unh ftldh vkSlr xgjkbz 5'5" g ikj ugh dj ikrk gA gk ldrk g unh dgh ij 6 l vf/d xgjh gkA ekè; dk cukoV d vk/kj ij ge fuEufyf[kr led Jf.k;k dk vè;;u djxA

Sr. No.	Factory A Salary (Rs.)	Factory B Salary (Rs.)	Factory C
1.	500	480	100
2.	500	490	310
3.	500	500	500
4.	500	510	520
5.	500	520	1.070
Total	**2500**	**2500**	**2500**
$\bar{x}$	500	500	500
M	500	500	500

mi;Dr mnkgj.k l ;g Li"V g fd rhuk dkj[kkuk e dke dju oky etnjk dh vkSlr etnjh 500 #- gA ;g ekè; doy Factory A vkj Factory B e nh tku okyh etnfj;k dk ifrfuf/Ro rk djrk g yfdu Factory C tgk ,d etnj doy 100 #- ikIr djrk g vkSlr xjhc g] nljk 1,070 #- yrk g fd /uoku g] ekè; mfpr ifrfuf/Ro ugh djrk gA

bll ;g Li"V g fd ledekykvk dh ljpuk dk vè;;u dju d fy, doy ekè;k ij fuHkj gh fd;k tk ldrk gA bld fy, fuEufyf[kr pkj eki Kkr fd, tkr gA

(a) **dUæh; iofÙk dh eki (Measured of Cenral Technology)**—ble ,d vd Kkr fd;k tkrk g fd ifrfuf/ dk dk; djrk gA

(b) **vifdj.k dh eki (Measures of Dispersion)**—ble ekè;k d pkjk vkj eY;k dk foLrkj (iQyko) Kkr fd;k tkrk gA

(c) **fo"kerk dh eki (Measures of Skewness)**—ble in&eY;k dh fn'kk dk Kku djr gA vFkkr fc[kjko bl fn'kk e vf/d gA

(d) **iFk'kh"kRo dh eki (Measures of Kurtosis)**—ble vkofÙk fdj.k d udhyiu ;k pVi Viu dk Kku djr g vFkkr ink d teko dk vè;;u djr gA

bl vè;k; e ge vifdj.k d eki dk foLrr foopu djxA

2. mís'; (Objectives)—bl vè;k; dk i<u d ckn vki le> ldr gA

(i) vifdj.k dk vFk ,o ifjHkk"kkA

(ii) vifdj.k dk Kkr dju d fofHkUu ekiA

(iii) nk ;k nk l vf/d led Jf.k;k e ikb tku okyh vlekurkvk e vUrj dk ryukRed vè;;u djukA

(iv) led ekyk d ekè; l mld fofHkUu in eY;k dh vkl r njh Kkr djukA

(v) vkfFkd ,o lkekftd {k=k e vk; ,o lEifÙk d forj.k dh vlekurkvk dk eki ,o ryukRed vè;;u djukA

3. fo"k; dk iLrrhdj.k (Presentation of Contents)—

3.1 vifdj.k dk vFk ,o ifjHkk"kk (Meaning and Definition of Dispersion)—

ledekyk d lheklr ink d foLrkj dk vifdj.k dgr g vifdj.k ink d fopyuk dk eki gA ftl lhek rd O;fDrxr ink e fHkUurk gkrh g] mld eki dk vifdj.k dgr gA nlj er d vulkj vifdj.k ekè; d pkjk vkj d iQyko dk dgr gA

Lihey d vulkjµog lhek tgk rd led ,d ekè; eY; d nkuk vkj iQyu dh iofÙk j[kr g] og mu ledk dk fooj.k ;k vifdj.k dgykrk gA

cDl rFkk fMd d vulkj] ¶,d dUæh; eY; d nkuk vkj ik; tku oky pj&eY;k d fopyu ;k ilkj dh lhek gh vifdj.k gA¸

3.2 vifdj.k d mí'; ,o egRoµ

vifdj.k dh eki e[;r% fuEufyf[kr mí'; dh ifr djr gA

(i) ekè; dh fo'oluh;rk dh ijh{kk djukA

(ii) in eY;k dk foLrkj Kkr djukA

(iii) ledekyk d ekè; l mld fofHkUu in eY;k dh vkl r njh Kkr djukA

(iv) nk ;k nk l vf/d led Jf.k;k d chp ryuk djukA

(v) ledekyk dh ljpuk d fo"k; e tkudkjh ikIr djukA

vifdj.k d eki dk vè;;u iR;d {k=k e egRoi.k g pkg og vkfFkd gk ;k lkekftdA foKku d {k=k e rk ;g vkj Hkh egRoi.k gA vkt dh vlekurk dk Kku O;kikj e ekx ij ekle d dkj.k mrkj&p<ko dk Kku] ejht d Toj e mrkj&p<ko dk eki vkfn e vifdj.k dh egRoi.k Hkfedk n[kh tk ldrh gA

3.3 vifdj.k dh x.kuk dju dh fof/;kµ

vifdj.k Kkr dju dh e[;r% fuEufyf[kr fof/;k gA

(a) 3.3.1 lhek jhfr;k (Methods of Limits)

3.3.1.1 vUrj prFkd foLrkj (Later-Quartile Range)

3.3.1.2 'kred foLrkj

(b) 3.3.2 fopyu ekè; jhfr;k (Methods of Averaging Deviations)

3.3.2.1 prFkd fopyu (Quartile Deviation)

3.3.2.2 ekè; fopyu (Mean Deviation)

3.3.2.3 ieki fopyu (Standard Deviation)

(c) 3.3.3 fcUnq j[kh; jhfr (Graphic Method)

3.3.3.1 ykjt pØ (Lorens Curve)

3.3.1.1 foLrkjµfoLrkj led J.kh in lcls cM+s rFkk lcls NksVs eY;ks dk vUrj gksrk gS A v[kf.Mr J.kh dh voLFkk esa vioth gkuh pkfg,A

l=k = R = L – S

;gk¡ R = foLrkj

L = vf/dre eY;

S = U;wure eY;

foLrkj x.kkd (Coefficient of Range)—nks ;k nks ls vf/d Jf.k;ks esa rqyuk djus d fy, lki{k eki (Relative measure) dh vko';drk iMrh gS A bl dk; d fy, foLrkj x.kkd fuEu l=k ls fudkyk tk,xkA

Coefficient of range = $\frac{L - S}{L + S}$

Illustration - 1

Calculate range and its coefficient

Values :	1–10	11–20	21–30	31–30
Frequency	10	15	7	4

Solution :

igys bl J.kh dks mioth J.kh esa cnyuk gksxkA

0.05–10.5	10.5–20.5	20.5–30.5	30.5–40.5
10	15	7	4

L = 40.5, N = 0.5

Range = L – S = 10.5, 0.5 = 40

Coefficient of Range = $\frac{L - S}{L + S} = \frac{40.5 - 0.5}{40.5 + 0.5} = \frac{40}{41} = 0.976$

Problems

Calculate range and its coefficient of the date given below

4.24, 26.38, 17.25, 35.45, 44.54, 13.50, 18.22, 16

(50 : 0.862)

Find range and its coefficient

Class	5–9	10–14	15–19	20–24	25–29	30–34
Frequency	6	12	18	17	9	3

(30 : 0.769)

foLrkj d x.kµ

(1) **ljy ekiµ** bldh x.kuk rFkk le>uk cgr ljy gA bldk i;kx lk/kj.k ckypky dh Hkk"kk e vf/dre gkrk gA mnkgj.k d fy, egkokj fcØh (vFkkr ,d eghu dh vf/dre rFkk U;ure fcØh dk foLrkj) fofue; njk vkfn e bldk i;kx gkrk gA

(2) **lhekvk dk Li"Vhdj.kµ** J.kh dk iR;d eY; fn, x, foLrkj d ekè; e gkrk gA

foLrkj d nk"kµ

(1) **pj.k eY;k e iHkkforµ** foLrkj d nk pj.k eY;k vf/dre vkj U;ure ij vkèkkfjr gA J.kh d vU; eY;k dk bl ij dkb iHkko ugh iMrk gA

(2) **J.kh dh ljpuk dk Kku u gkukµ** bl J.kh dh ljpuk dk Kku ugh gkrk gA J.kh d fc[kjko e eè; eY;k dk cgr egRo g yfdu foLrkj ij eè; dk dkb iHkko ugh iMrk gA

(3) **vLFkkb ekiµ** ;fn J.kh dk pje eY; ifjoru gkrk g rk foLrkj cny tk,xk pkg vU; eY;k e dkb ifjoru ugh gvk gkA

(4) **J.kh d lHkh eY;k dk i;kx ughµ** foLrkj eki dju e mlh eY;k dk i;kx ugh gkrk gA blh dkj.ko'k bldk xf.krh; i;kx ugh gkrk gA

3.3.1.2 vUrj prFkd foLrkj (Later-Quartile Range)

ledekyk d rrh; prFkd (Q_3) e l iFke prFkd (Q_1) ?kVku ij vUrjiod (LR) ikIr gkrk gA

l=kµ Inter Quartile Range = $Q_3 - Q_1$

x.kµ

1. **ljyµ** bldh x.kuk vfr ljy gA
2. pje eY;k l iHkkfor ughµ ;g foLrkj dh Hkkfr pje&eY;k ij vk/kfjr ugh gA iFke prFkd d igy d 25% rFkk rrh; prFkd d ckn d 25% in&eY;k dk i;kx ugh gkrk gA blfy, ;g fore[kh J.kh (Open end Series) e vifdj.k dju e vf/d mi;kxh gA

vox.k

1. **lHkh eY;k ij vk/kfjr ughµ** foLrkj dh Hkkfr ;g Hkh ledekyk d lHkh eY;k ij vkèkkfjr ugh gA
2. **chtxf.krh; i;kx lHko ughµ** foLrkj dh Hkkfr lHkh eY;k ij vk/kfjr gku d dkj.ko'k ;gk ij Hkh chtxf.krh; foopu lHko ugh gA

3.3.1.3 'kre; foLrkjµ

'kred foLrkj 90ok 'kred (P_{99}) e l 10ok 'kred (P_{pp}) ?kVku ij vkrk gA

Perceatile Range = P90 – 910

$$\text{Coefficient of Range} = \frac{P_{99} - P_{10}}{P_{90} + P_{10}}$$

;g vUrj prFkd foLrkj l vf/d mfpr g bl e P_{10} d igy 10% rFkk P_{10} d ckn d 10% eY; dk NkM tkr g rFkk chp d 80% eY; i;kx e yk, tkr g tcfd vUrj&prFkd foLrkj e doy chp d 50% eY; gh i;kx e yk, tkr gA ckdh x.k nk"k ogh g tk fd vUrj&prFkd foLrkj d gA

3.3.2.1 prFkd fopyu %

vUrj prFkd foLrkj d eY; dk nk l Hkkx nu ij prFkd fopyu Kkr fd;k tkrk gA bl v¼&vUrj prFkd foLrkj (Semi inter Quartile Range) Hkh dgr gA

$$\text{Quartile Deviation (Q. D.)} = \frac{Q_3 - Q_1}{2}$$

prFkd&fopyu dk x.kkd µprFkd fopyu vifdj.k Kkr dju dk fuji{k (Absolute) eki gA nk ;k nk l vf/d Jf.k;k dh ryuk dju d fy, prFkd&fopyu x.kkd Kkr djr g tk fd lki{k eki gA

$$\text{Coefficient of Quartile Deviation Range} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

x.k nk"kµbld x.k&nk"k ogh g tk fd vUrj prFkd foLrkj d gA

Illustration 2 :

Calculate Inter-Quartile Range, Percentile Range, Quartile Deviation and its coefficient for the following data :

Size (x)	4	8	12	16	20	24	28
Frequency (f)	6	10	18	22	11	5	3

Solution :

Size (x)	**Frequency (f)**	**Cumulative Frequency (c.f)**
4	6	16
8	10	16
12	18	34
16	22	56
20	11	67
24	5	72
28	3	75
	75	

$$Q_1 \quad = \text{Size of } \frac{N+1^{th}}{4}$$

$$= \text{Size of } \frac{75-1}{4} = \frac{76}{4} = \text{19th item} = 132$$

$$Q_3 = \text{Size of } \frac{3\,(N+1)^{th}}{4} \text{ item}$$

$$= \text{Size of } \frac{3 \times 76}{4} = 57^{th} \text{ item} = 20$$

$$P_{10} = \text{Size of } \frac{10\,(N+1)^{th}}{100} \text{ item}$$

$$= \text{Size of } \frac{10\,(75+1)}{100} = \frac{10 \times 76}{100} = 76^{th} \text{ item} = 8$$

$$P_{90} = \text{Size of } \frac{90\,(N+1)^{th}}{100} = \frac{90 \times 76}{100} = 68.1^{th} \text{ item 'w' } 24$$

(i) Inter Quartile Range = $Q_3 - Q_1 = 20 - 12 = 8$

(ii) Percentile Range = $P_{90} - P_{10} = 24 - 8 = 16$

(iii) Quartile Deviation = $\frac{Q_3 - Q_1}{2} = \frac{20-12}{2} = 4$

(iv) Coefficient of Quartile Deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{20-12}{20+12} = \frac{3}{32} = 0.25$

Problems :

3. Find out Inter-Quartile Range of the following :

   20, 8, 14, 10, 22, 12, 16

4. Calculate Percentile Range

x	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
f	10	16	14	24	56	40	20	20

(R.R. = 53.75)

5. Calculate the Q.D. and its Coefficient the following data

Marks less than	10	20	30	40	50	60	70	80	90
No. of Students	5	15	98	242	367	405	425	438	439

(Q. D. = 8; 10, C.Q.D. = 0.22)

3.3.2.2. ekè; fopyu (Mean Deviation)

ledekyk d fdlh lkf[;dh; ekè; (lekUrj ekè; eè;dk ;k cgyd) l fofHkUu eY;k d fy, x, fopyuk d fun'k eku dk lekUrj ekè; ml J.kh dk ekè; fopyu gkrk gA fopyuk dk ;kx Kkr djr le; chtxf.krh; fpUg ?ku (+) vkj ½.k (–) dk NkM fn;k tkrk gA

When δ = Mean Deviation $\delta_1 = \frac{\sum ldl_s}{N}$ [δ is pronounced as Delta]

(d) = Deviation from any statistical average (mean, median or mode) ignorig + and – signs.

N = number of items.

fofHkUu lkf[;dh; ekè; l fy; x, ekè; fopyu d l=k fuEu idkj l fy[k tk ldr gA

I - 0;fDrxr J.kh e ekè; dk fopyu %

Mean Deviation from Mean $\delta_1 = \frac{\sum ldl}{N}$

Mean Deviation from Mean $\delta_m = \frac{\sum ld_m l}{N}$

Mean Deviation from Mean $\delta_m = \frac{\sum ld_m l}{N}$

[kf.Mr rFkk v[kf.Mr J.kh e ekè; fopyu

$$\delta_m = \frac{\sum fld{-}_m 1}{N}$$

$$\delta_m = \frac{\sum fld{-}_m 1}{N}$$

$$\delta_m = \frac{\sum nd_x 1}{N}$$

ekè; fopyu x.kkdμ ekè; fopyu ,d fuji{k (Absolute) eki gA nk ;k nk l vf/kd Jf.k;k e ryuk dju d fy, lki{k (Relative) eki dh vko';drk iMrh gA ekè; fopyu x.kkd lki{k eku g tk fd ekè; fopyu e mlh ekè; l Hkkx n djd Kkr fd;k tk ldrk g ftl ekè; d fopyu fy, x, gA

Coefficient of Mean Deviation from = $\frac{6 -_n}{x}$

Coefficient of Mean Deviation from median = $\frac{\delta m}{m}$

ukVµ ;fn i'u e ;g u fn;k gk fd ekè; fopyu fdl ekè; dk vk/kj eku dj Kkr fd;k tk;] tk lekUrj ekè;] l ekè; fopyu Kkr djuk pkfg, D;kfd lekUrj ekè; l fy, x, fopyuk dk ;kx U;ure gkrk gA

(a) **0;fDrxr J.khµ**

(i) **ekfè;dk l ekè; fopyuµ**y?k jhfr l fuEufyf[kr l=k }kjk Kkr fd;k tk ldrk gA

$$\delta_m = \frac{\Sigma X_A \Sigma X_v}{N}$$

When δ_m = Mean Deviation from medium

ΣX_A Sum of values of items above the medium

ΣX_μ Sum of values of item below the identian

ukVµ ;fn ekè; in J.kh e mifLFkr gk rk ml in dk NkM fn;k tkrk gA

(ii) lekUrj ekè; l ekè; fopyu fuEu l=k l Kkr djr gA

$$\delta_x = \frac{\Sigma X_A \Sigma X_v - (N_A N_p)\bar{x}}{N}$$

Where δ_m = Mean Deviation from mean

ΣX_A = Sum of values of items above the mean

ΣX_μ = Sum of Values of item below the mean

N_A = Number of items above the mean

N_p = Number of items below the mean

[kf.Mr rFkk v[kf.Mr J.khµv[kf.Mr J.kh e oxkUrjk d eè;&fcUn Kkr djd ml [kf.Mr J.kh e ifjofrr dj yr gA bu eè;&fcUnvk dk (X) ekuk tkrk gA rRi'pkr ekè; fopyu Kkr dju dh fØ;k [kf.Mr rFkk v[kf.Mr J.kh e leku gkrh gA

$$\delta_x = \frac{\Sigma fX_A \Sigma fX_m - (N_A N_p \Sigma X)}{N}$$

and

$$\delta_m = \frac{\Sigma fX_A \Sigma fX_m - (N_A - N_p)m}{N}$$

Where ΣfX_A = Sum of the values above the average (X or M)

ΣfX_μ = Sum of Values below the average

N_A = Sum of frequencies above the average

N_μ = Sum of frequencies below the average

Illustration 3.

The following are weight of 10 students

Weight (Kg.) :	49	50	62	73	47	80	59	58	48	74

Calculate Mean Deviation from mean and its coefficient

Solution :

Sr.	Weight (x)	Deviation from 60 (+ and – signs ignored)
1	49	11
2	50	10
3	62	2
4	73	13
5	47	13
6	80	20
7	59	1
8	58	2
9	48	12
10	74	14
N = 0	600	98

$$\bar{x} = \frac{\sum X}{N} = \frac{600}{10} = 60 \text{ kgs}$$

Mean Deviation from Mean = $\delta_m = \frac{\sum dx}{N}$

$$= \frac{98}{10} = 9.8 \text{ kgs}$$

Short Cut Method

$$\Sigma X_A = 62 + 73 + 80 + 74 = 289$$

$$\Sigma X_\mu = 49 + 50 + 47 + 59 + 58 + 48 = 311$$

$$\delta_{\bar{x}} = \frac{\sum X_A - \sum X\mu(N_A - N\mu)\bar{x}}{N}$$

$$= \frac{289 - 311 - (4 - 6)60}{10}$$

$$= \frac{-22 + 120}{10} = 9.8 \text{ kgs}$$

$$\text{Coefficient Mean Deviatin} = \frac{\delta_{\bar{x}}}{X}$$

$$= \frac{9.8}{60} = 0.163$$

Illustration 4.

Determine the Mean Deviation from Mean and Meadian in the following distribution

Value	0–10	10–20	20–30	30–40	40–50
Frequency	2	4	5	6	3

Solution :

(i) Mean Deviation from Mean

(a) Direct Method

Value	Mid-value (x)	Frequency (f)	f.x.	Deviation from Mean |dx|	f |d$\bar{x}$|
0–10	5	2	10	22	44
10–20	5	4	60	12	48
20–30	25	5	125	2	10
30–40	35	6	210	8	48
40–50	45	3	135	18	54
		20	540		204

$$\bar{x} = \frac{\sum fx}{N} = \frac{540}{20} = 27$$

$$\delta_m = \frac{\sum fd\,|\bar{x}|}{N} = \frac{204}{20}\ 10.2$$

Short Cut Method

Where $\Sigma fX_A = 210 + 135 = 345$

$\Sigma fX_B = 10 + 60 + 125 = 195$

$N_A = 6 + 3 = 9$

$N_\mu = 2 + 4 + 5 = 11$

$$\delta- = \frac{\sum fx_A - \sum fx_\mu (N_A - N_B)\bar{x}}{N}$$

$$= \frac{345 - 195 - (9 - 11)27}{20}$$

$$= \frac{150 - (-2)27}{20} = \frac{150 + 54}{20} = \frac{204}{20} = 10.2$$

(ii) Mean Deviation from Median

(a) Direct Method

Value	Frequency (f)	c.f.	Mid Value (x)	|dm|	$f\|d_m\|$	for shortcut Method (fx)
0–10	2	2	5	23	46	10
10–20	4	6	15	13	52	60
20–30	5	11	25	3	15	125
30–40	6	17	35	7	42	42
40–50	3	20	45	17	51	51
	20				206	

$$m = \text{Size of } \frac{N}{2} \text{th item}$$

$$m = \text{Size of } \frac{20}{2} \text{th item}$$

which lies in the class (20 – 30)

$$M = 1 + \frac{1}{f}(m - C)$$

$$= 20 + \frac{10}{5}(10 - 6) = 28$$

$$\delta_m = \frac{\Sigma dml}{N}$$

$$= \frac{206}{20} = 10.3$$

Short Cut Method

$$\Sigma fx_A = 210 + 135 = 345$$

$$\Sigma fx = 10 + 60 + 125 = 195$$

$$N_A = 6 + 3 = 9$$

$$N = 2 + 4 + 5 = 11$$

$$\delta_m = \frac{\Sigma fx_A - \Sigma fx_P (N_A - N_{30})_m}{N}$$

$$= \frac{345 - 195 - (9 - 11)28}{20}$$

$$= \frac{150-(-2)28}{20}$$

$$= \frac{150+56}{20} = \frac{206}{20} = 10.3$$

ekè; fopyu d x.kµ

1. ljyµbldh x.kuk ljy g vkj vklkuh l le> e Hkh vk tkrh gA
2. leLr eY;k ij vk/kfjrµekè; fopyu J.kh d leLr eY;k ij vk/kfjr g vr% J.kh dh ljpuk ij i;klr idk'k Mkyrk gA
3. pje eY;k ij iHkkoµbl ij vfr lhekUr in eY;k dk iHkko de iMrk gA
4. fdlh Hkh ekè; l lHkoµbldk Kkr dju d fy, dkb Hkh ekè; fy;k tk ldrk gA

nk"k

1. **chtxf.krh; i;kxµ**ekè; fopyu Kkr dju e /u (+) rFkk ½.k (–) fpUgk dk NkM fn;k tkrk gA ;g ,d xf.krh; v'kf¼ g vkj mPp Lrjh; xf.krh; i;kx lHko ugh gA
2. **vfuf'prµ**ekè; fopyu dh x.kuk fdlh Hkh ekè; l dh tk ldrh g vkj vyx&vyx ekè; dk vk/kj ekudj fudky x; ekè; fopyu vyx&vyx vkr gA bl vfuf'prk d dkj.k nk ;k nk l vf/d Jf.k;k dh ryuk e dfBukb jgrh gA

Problems

6. Determine Mean Deviation from mean and median and then Coefficients from the data given below

46,	48,	62,	50,	54,	40

(δx = 5.32; Coeff. = 0.106; δm = 5.33; coeff. = 0.109)

7. Calculation Mean Deviation from median and mean their Coefficient from the following

Wages (Rs.)	5	10	15	20	25
No. of Workers	4	6	8	10	12

(δx = 5.5; Coeff. = 0.275; δx = 5.75; coeff. = 0.329)

8. Calculation Median, Mean Deviation and its Coefficient from the following data

Scores :	140–150	150–160	160–170	170–180	180–190	190–200
Frequency	4	6	10	18	9	3

(M = 173; δm = 10.24; coeff. = 0.06)

9. The following table gives the figures of wages earned by workers of two factories. Calculate the Mean deviation and state which factory has greater variation in wages

Weekly Wages (Rs.)	No. of Workers	
	Factory A	Factory B
0–5	28	15
5–10	18	20
10–15	30	35
15–20	25	30
20–25	20	18
25–30	15	17

(Factory A : δm = 6.68, Coeff. = 0.47 : Factory B δm = 6.26; Coeff = 0.43)

3.3.2.3 iekf.kd fopyu (Standard Deviation)

iekf.kd fopyu ftls ekud fopyu Hkh dgrs gSa ,d oKkfud ,o vkn'kZ vifdj.k Kkr djus dh ;g fof/k gSA ;g fof/k Js.kh ds lekUrj eè;d ij vk/kfjr gS rFkk blesa /ku (+) rFkk ½.k (–) fpUgksa dks NksMus dh vko';drk ugha iMrh gSA blds i;ksx lo0;kih gSA

Js.kh ds lekUrj eè; ls fy, x, fopyuksa ds oxksZa ds lekUrj eè; ds oxZey dks iekf.kd fopyu dgrs gSA vFkkZr

$$\sigma = \sqrt{\frac{\Sigma d^2}{N}}$$

;gka σ = iekf.kd fopyu (σ is knows as small sigma)

d = lkekUrj eè; ls in dk fopyu (d = x – x)

N = inksa dh la[;k

iekf.kd fopyu fof/k dk i;ksx loiFkke dkyZ fi;lZu (Karl Pearson) us fd;k FkkA blds ekè;&fo"k; (mean error), eè;&oxZ fo"k; (mean sqaure error) rFkk eè;; ls Kkr fopyu oxZ eè; eY; (poor mean square deviation from mean) Hkh dgrs gSA

eki fopyu vk/kfjr dy 0;;

1. iekf.kd fopyu dk i.kkd (Coefficient pf Standard Deviation) nks ;k nks ls vf/kd Jf.k;ksa ds vifdj.k dh rqyuk djus ds fy, bldk i;ksx djrs gSA ;g ,d lkisf{kd eki gSA

Coefficient of Standard Deviation = $\frac{\sigma}{N}$

2. fopj.k x.kkd (Coefficient of Variation)—iekf.kd fopyu ds x.kkd dks 100 ls x.kk djds blls Kkr djrs gSa vFkkZr vf/kd ek Kkr djuk dgrs gSA

Coefficient of Variation = $\frac{\sigma}{\bar{x}} \times 100$

fopj.k x.kkd dk loiFkke dkyZ fi;lZu us fd;k FkkA bldk i;ksx nks ;k nks ls vf/kd led Jf.k;ksa dh vfLFkjrk (Variability) fLFkjrk dk lxfr (Stability of Consistency) dh rqyuk djrs gSA ftl Js.kh dk fopyu x.kkd vf/kd gksxk og Js.kh vfLFkj (Variable) gksxhA

3. ilj.k (Variance) iek.k fopyu (σ) d ox dk ilj.k (Variance) dgr gA

Variance = V = σ^2

y?k fof/;k& iek.k fopyu Kkr dju dh fuEufyf[kr nk y?k fof/;k gA

(a) iR;d eY; dk ox fudkyk djA

$$\sigma = \sqrt{\frac{\Sigma X^2}{N} - (X)^2}$$

ble in eY;k (x) dk ox fudkyuk iMrk gA

(b) dfYir ekè; l fopyu Kkr djdA

$$\sigma = \sqrt{\frac{\Sigma d^2 x}{N}\left(\frac{\Sigma dx^2}{N}\right)} \qquad ...(1)$$

$$= \frac{1}{N}\sqrt{N\Sigma d^2 x – (\Sigma dx)^2} \qquad ...(2)$$

$$= \sqrt{\frac{N\Sigma d^2 x}{N}}(\bar{X} = A)^2 \qquad ...(3)$$

tgk A = dfYir ekè;

dx = X – A

Illustration : The following are the marks secured by 10 students

40, 42, 48, 49, 50, 51, 53, 54, 56, 57

Find the Standard Deviation and its coefficient

Sr. No.	Marks X	Deviation from Mean ($\bar{x}$ = 50) d = ($\bar{x}$ – x)	d2	for Short cut Methods: deviation from Assumed mean (A = 49) dx = X – A	d2x	x2
1.	40	–10	100	–9	81	1,600
2.	42	–8	64	–7	49	1,764
3.	48	–2	4	–1	1	2,304
4.	49	–1	1	0	0	2,401
5.	50	0	0	1	1	2,500
6.	51	1	1	2	4	2,601
7.	53	3	9	4	16	2,809
8.	54	4	16	5	25	2,916
9.	56	6	36	7	49	3,136
10	57	7	49	8	64	3,249
	500	0	280	10	290	25,280

(a) Direct Method

$$\bar{x} = \frac{\Sigma X}{N} = \frac{500}{10} = 50 \text{ Marks}$$

Standard Deviation $$\sigma = \sqrt{\frac{\Sigma d^2 x}{N}} = \sqrt{\frac{280}{10}} = \sqrt{28} = 529 \text{ Marks}$$

(b) Short Cut Methods :

First Formula : $$\sigma = \sqrt{\frac{\Sigma d^2 x}{\text{N}} - \left(\frac{\Sigma dx}{N}\right)^2}$$

$$= \sqrt{\frac{290}{10} - \left(\frac{10}{10}\right)^2}$$

$$= \sqrt{29 - 1} = \sqrt{28} = 5.29 \text{ Marks}$$

Second Formula $$\sigma = \frac{1}{N}\sqrt{N\Sigma d^2 x - (\Sigma dx)^2}$$

$$= \frac{1}{10}\sqrt{10 \times 290 - (10)^2}$$

$$= \frac{1}{10}\sqrt{2900 - 100}$$

$$= \frac{1}{10}\sqrt{2800}$$

$$= \frac{52.9}{10} = 5.29 \text{ Marks}$$

Third Formula $$\sigma = \sqrt{\frac{\Sigma d^2 x}{N} - (x - 4)^2}$$

$$= \sqrt{\frac{290}{10}} - (50 - 49)^2$$

$$= \sqrt{29 - 1} = \sqrt{28} = 5.29 \text{ Marks}$$

Fourth Formula $\sigma = \sqrt{\frac{\Sigma x^2}{N} - (\bar{x})^2}$

$$= \sqrt{\frac{25280}{10}} - (50)2$$

$$= \sqrt{2528 - 2500}$$

$$= \sqrt{28 = 5.29} \text{ Marks}$$

[kf.Mr rFkk v[kf.Mr J.kh e ieki fopyu Kkr djuk %

v[kf.Mr J.kh e eè; fcUn (x) fudkyu d i'pkr [kf.Mr rFkk v[kf.Mr Jf.k;k d fy, fuEufyf[kr lw=kk dk i;kx djd ieki fopyu Kkr djr gA

(a) iR;{k fof/

$$\sqrt{\frac{\Sigma fx^2}{N}}$$

Where d = X – $\bar{x}$

N = Σf

(b) y?k fof/ ;kμ ;fn leUrj ekè; n'keyo e vk tk, rk ieki fopyu dh x.kuk tfVy gk tkrh g , lh fLFkfr e dfYir ekè;e l fopyu fudky dj ieki fopyu fudkyk tk ldrk gA bld fy, fuEufyf[kr lw=kk e fdlh ,d dk i;kx djr gA

$$\sigma = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx^2}{N}\right)^2} \qquad ...(1)$$

$$\sigma = \frac{1}{N}\sqrt{N\Sigma d^2 x - (\Sigma fxk)^2} \qquad ...(2)$$

$$= \sqrt{\frac{\Sigma fd^2 x}{N}}(\bar{X}\sigma = A)^2 \qquad ...(3)$$

tgk dx = X – A (A = Assumed mean)

N = Σf

Illustration 6

In the following series determine Standard Devision

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70
No of Students	10	15	25	25	10	10	5

Solution :

(a) Direct Method

Marks	Mid Value X	No. of Student f	fx	Deviation fees $\overline{X} = 31$ $d = x - \overline{X}$	(fxd) fd	fdxd fd^2
0–10	5	10	50	–26	–260	6,760
10–20	15	15	225	–16	240	3,840
20–30	25	25	625	–6	150	900
30–40	35	25	875	4	100	400
40–50	45	10	450	14	140	1,960
50–60	55	10	550	24	240	5,760
60–70	65	5	325	34	170	5,780
Total	100	3,100				25,400

$$\text{Arithmetic Mean} = \overline{X} = \frac{\Sigma fx}{N} = \frac{3100}{100} = 31 \text{ Marks}$$

$$\text{Arithmetic Mean} = \sigma = \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{25,400}{100}} \quad \sqrt{254} = 1594 \text{ Marks}$$

(b) Short Cut Method

Marks	Mid Value x	No. of Student	Deviation fees Assumed Mean (A = 35) fdx = X – A	(fxd) fd	fdxxdx fd^2
0–10	5	10	–30	–300	9,000
10–20	15	15	–20	–300	6,000
20–30	25	25	–10	–250	2,500
30–40	35	25	0	0	0
40–50	45	10	10	100	1,000
50–60	55	10	20	200	4,000
60–70	65	5	30	150	4,500
Total	100			–400	27,000

$$\overline{X} = A + \frac{\Sigma fdx}{N} = 35 + \frac{-400}{100} = 35 - 4 = 31 \text{ Marks}$$

$$\sigma = \sqrt{\frac{\Sigma fd^2 x}{N}\left(\frac{\Sigma fdx^2}{N}\right)} = \sqrt{\frac{27000}{100} - \frac{(-400)^2}{100}} = \sqrt{254} = 1594 \text{ Marks}$$

(a) in fopyu fof/ (Step Deviation)— ;fn ox foLrkj leku gk rk dfYir eè; l fopyu Kkr dju d i'pkr bu fopyuk dk leku ?kVd (Common Factor) tk fd lkekU;% ox foLrkj (i) d cjkcj gkr g] l Hkkx ndj fuEufyf[kr l=k dk i;kx djr gA

$$\sigma = \frac{\sqrt{\Sigma fd^2 s}}{N} - \left(\frac{\Sigma fds}{N}\right)^2 \times 2$$

$$\text{Where, } ds = \frac{dx}{i}$$

Isslutration 7.

Solve illustration 6 by step deiviation method.

Solution :

Marks	Mid Value x	No. of Student f	Deviation fees Assumed Mean (A = 35) dx = X – A	(i = 10) $dx = \frac{dx}{i}$	fds	fds^2
0–10	5	10	–30	–3	–30	90
10–20	15	15	–20	–2	–30	60
20–30	25	25	–10	–1	–25	25
30–40	35	25	0	0	0	0
40–50	45	10	10	1	10	10
50–60	55	10	20	2	20	40
60–70	65	5	30	3	15	15
Total	100				–40	270

$$\bar{X} = A - \frac{\Sigma fds}{N} \times i \qquad \sigma = \sqrt{\frac{\Sigma fd's}{N}\left(\frac{\Sigma fds}{N}\right)} \times i$$

$$35 + \frac{-40}{100} \times 10 \qquad = \sqrt{\frac{270}{100} - \frac{(-40)^2}{100} \times 10} = \sqrt{2.7 - (4)^2 \times 10}$$

$$35 - 4 \qquad = \sqrt{2.7 - 0.16 \times 10}$$

$$= 31 \text{ Marks} \qquad = \sqrt{2.54 \times 10}$$

$$= 15.94 \times 10$$

$$= 15.94 \text{ Marks}$$

Illustration 8.

Following is the table giving weight of students of two classes. Calculate the coefficient variation of the two distributions. Which series is more variable ?

Weight in kgn	**Class A**	**Class B**
20–30	7	6
30–40	10	9
40–50	20	21
50–60	18	15
60–70	7	6
Total	62	56

Solution :

Weight in kgs	mid value	(A = 45) d	Class A			Class B		
	x	(= x – A)	f	fdx	fd^2x	f	fdx	fd^2x
20–30	25	–20	7	–140	2,800	5	–100	2,000
30–40	35	–10	10	–100	1,000	9	–90	900
40–50	45	0	20	0	0	21	0	0
50–60	55	10	18	180	1,800	15	150	1,500
60–70	65	20	7	140	2,800	6	120	2,400
Total			62	80	8,400	56	80	6,800

Class A

$$\text{Mean} = A + \frac{\Sigma fdx}{N}$$

$$= 45 + \frac{80}{62}$$

$$= 45 + 1.29$$

$$= 46.29 \text{ kgs}$$

$$\sigma = \sqrt{\frac{\Sigma fd^2x}{N}\left(\frac{\Sigma fdx}{N}\right)^2}$$

Class B

$$\text{Meran} = A + \frac{\Sigma fdx}{N}$$

$$= 45 + \frac{80}{56}$$

$$= 45 + 1.43$$

$$= 46.43 \text{ kgs}$$

$$\sigma = \sqrt{\frac{\Sigma fd^2x}{N}\left(\frac{\Sigma fdx}{N}\right)^2}$$

$= \sqrt{\frac{8,400}{62} - \frac{80^2}{62}}$ $\qquad$ $= \sqrt{\frac{6800}{56} - \left(\frac{80}{56}\right)^2}$

$= \sqrt{133.48 - 1.66}$ $\qquad$ $= \sqrt{121.43 - 2.04}$

$= = \sqrt{133.82 - 1.66}$ $\qquad$ $= \sqrt{119.39}$

= 11.568 kgs. $\qquad$ = 10.925 kgs

Coefficeint of Variation = $\frac{\sigma}{x} \times 100$ $\qquad$ C.V. = $\frac{\sigma}{x} \times 100$

$= \frac{11568}{46.29} \times 100$ $\qquad$ $= \frac{10.925}{46.43} \times 100$

= 24.98% $\qquad$ = 23.53%

Since coefficient of variation of class A is more than B, Hence weights of class A in more variable then class B.

Problem

10. Ten students of B. Com. class of a college have obtained following marks in statistics out of 100 marks. Calculate the Standard Deviation of marks obtained.

Sr. No.	1	2	3	4	5	6	7	8	9	10
Marks	5	10	20	25	40	42	45	48	70	80

How is standard Deviation affected of every marks is (a) increased, (b) decreased, (c) multiplied, (d) divided by 2

$\left[\begin{array}{l} \sigma = 23.06 \\ \text{(a), (b) unaffected} \\ \text{(c) } 2 \times \sigma \text{(d) } \frac{\sigma}{2} \end{array}\right]$

11. From the following figures find the standard deviation and coefficient of variation

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of stu.	5	10	20	40	30	20	10	4

($\bar{x}$) = 39.5, = σ = 15.6, C.V. = 39.2%)

12. The following table gives goal scored by two terms A and B in a football season. Find the term which is more consistant in its performance.

No. of Goals scored in a match	No. of Matches	
	Team A	Team B
0	27	17
1	9	9
2	8	6
3	5	5
4	4	3

(Team A : $\bar{x}$ = 1,0566, σ = 1.3081, C.V. = 123.9%)

(Team B : $\bar{x}$ = 1.2, σ = 1.3081, C.V. = 108.97%)

B is more consistant

13. The average monthly prices (in Rs.) of two shares are given below -

Share X :	12	14	18	20	13	15	18	23	19	18
Share Y :	135	154	163	109	100	122	140	131	128	118

Show which share has more stabiliven prices

(C.V. X = 19.1%. Y = 14.06%, Y is greater stability)

14. Find out which of the following two series has more variation ?

Agegroup	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Papulation City A	19	16	15	12	10	5	2	1
Papulation City B	10	12	24	32	29	11	3	1

(C.V., A = 67.3%, B = 44.4%)

lkefgd iek.k fopyu (Combined Standard Deviation)

;fn vyx&vyx legk d ieki fopyu] ekF; rFkk in l[;k, Kkr gk rk lkefgd ieki fopyu dh miek ljyrk l dh tk ldrh g bld fy, fuEufyf[kr fØ;k, dh tkrh gA

(i) igy] lHkh legk dk lkefgd okD; (Combined mean) Kkr djr gA

$$\frac{N1x_1 + N_2\bar{x}_2 + N3\bar{x}_3 + ...}{N_1 + N_2 + N_3 + ...}$$

(2) lkefgd ekè; (x) dh iR;d leg d ekè; l ?kVkdj fuEufyf[kr vUrj Kkr djr gA

$D_1 = (\bar{X} - \bar{X}), D_2 = (\bar{X}_2 - \bar{X})$ etc.

tgk $\bar{X}, \bar{X}_2$ vkfn vyx&vyx legk d ekè; gA

(3) fuEu l=k dk i;kx djd lkefgd iekk fopyu (σ) Kkr fd;k tkrk gA

$$\sigma = \sqrt{\frac{N_1\left(\sigma_1^2 + D_1^2\right) + N_2\left(\sigma_2^2 + D_2^2\right) + N_3\left(\sigma_3^2 + D_3^2\right)}{N_1 + N_2 + N_3}}$$

σ = Combined Standard Deviation

σ_1, σ_2 etc. = Standard deviation of different groups

$D_1 = (\overline{X} - \overline{X})$; D2 = $(\overline{X}_2 - \overline{X})$ etc.

N_1, N_2, N_3 etc. Number o items in different groups

Illustration 9.

The means of two samples of sizes 50 and 100 respectively are 54.1 and 50.3 and the standard deviations are 8 and 7. Find the mean and standard deviation of the sample of size 150 obtained by combining the two samples.

Combined Mean $\overline{X} = \frac{N_1\overline{X} + N_2\overline{X}}{N_1 + N_2}$

$$= \frac{50 \times 54.1 + 100 \times 50.3}{50 + 100} = 51.57$$

Differences $D_1 = (\overline{X} - \overline{X}) = 54.1 - 51.57 = 2.53$

$D_2 = (\overline{X}_2 - \overline{X}) = 50.3 - 51.57 = 1.27$

Combined Standard Deviation

$$\sigma = \sqrt{\frac{N_1\left(\sigma_1^2 + D_1^2\right) + N_2\left(\sigma_2^2 + D_2^2\right)}{N_1 + N_2}}$$

$$= \sqrt{\frac{50(8^2 + 253^2) + 100[7^2(-1.27)^2]}{50 + 100}}$$

$$= \sqrt{\frac{50(64 + 6.4009) + 100(49 + 1.6129)}{150}}$$

$$= \sqrt{\frac{50 \times 70.4009 + 100 \times 50.6129}{150}}$$

$$= \sqrt{\frac{8581.335}{150}}$$

$= \sqrt{57.2089} = 7.56$

Problems :

15. The following are some of the particulars of the distribution of weights of boys and girls in a class

Number	Boys	Girls
Mean Weight (K.g.)	100	50
Standard Deviations	60	45
Calculation combine Deviation	3	4

($\sigma = 7.57$)

16. From the following table, compute the missing values

Subgroup	Number	Arithmetic Mean	Variance
I	?	25	9
II	259	?	16
III	300	15	?
Combined	750	16	51.733

($N_1 = 200$ $\overline{X}_2 = 10, \sigma\frac{2}{3} = 15$)

17. An analysis of the monthly wages paidd to workers in two firms A and B belonging to the same industry, gives the following result

Firms	Average monthly wages (Rs.)	Standard Deviation (Rs.)	Numbers of wage earners
A	52.5	10	586
B	47.5	11	643

(a) Which firm A or B pays out larger amounts as monthly wage ?

(b) What are measures of

(i) average monthly wage and

(ii) the standard deviation in two firms – A and B taken together

(a) A = Rs. 30.765 (b) (i) = 49.9

B = Rs. 30.780 (ii) = 10.8

3.3.3 mÙkj t oØ (Lareas Curve)

vifdj.k Kkr dju dh ,d fcUn j[kh; jhfr dk i;kx fd;k tkrk g ftl dk i;kx loiFke Mk- eDl vk- Ikjt u /u o vk; d fooj.k dk vè;;u dju d fy, fd;k FkkA bld }kjk ,d ox leg e /u o vk; d chp fdruh fo"kerk g] Kkr dh tk ldrh gA ykjt oØ ,d ljy] vkd"kd o iHkkoh fof/ gksu ij Hkh bl oKkfud ugh ekuk tkrk gA D;kfd bll vifdj.k dh ek=kk dh vdjkRed eki lHko ugh g ykjt oØ leku forj.k dh j[kk l ftruk nj gkxk] vifdj.k e vlekurk mruh gh vf/d gkxh rFkk og ftruk fudV gkxk vifdj.k dh ek=kk mruh de gkxhA

4. lkjka'k (Summary)

ledekyk d lheklr ink d foLrkj dk vifdj.k dgr gA vifdj.k ink d fopyuk dk eki gA vifdj.k dh fofHkUu eki fdlh Hkh {k=k e fopyu ;k fo[kjko lEcU/h leL;kvk dk vè;;u dju d fy, cgr egRoi.k gA vkfFkd ,o lkekftd {k=k e vk; ,o lifÙk d forj.k dh vlekukvk dk eki ,o ryukRed vè;;u dju e vifdj.k d eki cgr mi;kxh gkr gA vifdj.k Kkr dju dh fofHkUu jhfr;k e l foLrkj (Range), prFkd fopyu (Quartile Deviation) ekè; fopyu (Mean Deviation) o iekf.kd fopyu (Standard Deviation) ie[k gA

foLrkjµfdlh led J.kh e lcl vf/d eY; (H) vkj lcl NkV eY; (I) d vUrj dk foLrkj dgr gA

prFkd fopyuµ;g J.kh d rrh; o iFke prFkd d vUrj dk vk/k gkrk gA

ekè; fopyuµledekyk d fdlh lkf[;dh ekè; l fofHkUu eY;k d fy, x, fopyuk d fun'k eku dk lekUrj ekè; ml J.kh dk ekè; fopyu gkrk gA

iekf.kd fopyuµJ.kh d lekUrj ekè; l fy, x, fopyuk d oxk d lekUrj ekè; d oxey dk iekf.kd fopyu dgr gA

tlk fd vki tku pd gkx fd iR;d {k=k e led J.kh d ekè; l ikl;r vi.k ,o vkf'kd lpuk dh lifr vifdj.k dh eki }kjk dh tkrh gA

5. iLrkfor iLrd (Recommended Books)

(i) Introduction to Statistics – Dr. R, P. Hooda.

(ii) Statistical Methods – S.P. Gupta.

(iii) Business Statistics – Oswal Aggarwal, Sharma.

(iv) Business Statistics – T.R. Jain.

(v) Business Statistics – S. C. Sharma, R.C. Jain.

6. vH;kl d fy, i'u (Self Assement Questions)

(i) vifdj.k dk ifjHkkf"kr dhft,A bld fofHkUu ekik dh O;k[;k dhft,A budh mi;kfxrk Hkh crkb,A

(ii) iekf.kd fopyu D;k g\ bldh fo'k"krk,i crk,A bld x.k o nk"k Hkh crkb,A

(iii) Find Range and its Coefficient from the following Data

X	10	11	12	13	14	15
F	5	8	12	20	15	10

(iv) The Mean Deviation of a Normal Distribution is 20. Find the value of its Q.D. and S. Deviations.

(v) If the Standard Deviation of a N.D. is 24, Find the value of its Q.D. and M.D.

B.Com. Part-II

Paper BC–204 : Business Statistics ***Revised By :*** **Dr. Heera Lal Sharma**

Lesson No. : 4

Skewness, Moments, Kurtosis

l jpuk (Structure)

1. ifjp; (Introduction)

vkdMk d fo'y"k.k dh fØ;k e ,d lk[;d (Statistician) led J.kh dh iofÙk dk tkuu d fy, vudk lkf[;dh jhfr;k dk i;kx djrk gA bue fo"kerk dk eki Hkh ,d egRoi.k fof/ gA dUæh; iofÙk d eki (Measures of Cenral Tendency) o vifdj.k d eki (Measures of Dispession) led J.kh d fo'y"k.k d fy, vR;Ur vko';d lpuk, inku djr g] ijUr bul ;g tkudkjh ikIr ugh gkrh fd led J.kh dk vkdkj dlk g vFkkr dUæh; iofÙk l eY;k dk fc[kjko ;k i lkj leferh; g ;k vleferh; gA blfy, J.kh d okLrfod vkdkj d vè;;u d fy, ge fo"kerk d ekik dh enn yuh iMrh gA

2. mí'; (Objectives)

bl vè;k; dk vè;;u dju d mí'; fuEufyf[kr gµ

1. fo"kerk dk vFk ,o ifjHkk"kk dk vè;;u djukA
2. led J.kh d okLrfod Lo:i dk le>ukA
3. vkofÙk;k d ?kuRo dh ek=kk rFkk iÑfr dk Kkr djukA
4. vkofÙk forj.k d oØ dh tkudkjh ikIr djukA
5. led J.kh e dUæh; iofÙk ,o fo"kerk dk Kku djuk rkfd ml J.kh dh cukoV dk vè;;u fd;k tk ldA
6. iFk'kh"kd (Kurtosis) }kjk oØ 'kh"kd dh iÑfr (Peak of a curve) ij idk'ku MkyukA

3. fo"k; dk iLrrhdj.k (Presentation of Countents)

3.1 fo"kerk (Skewness)—

fo"kerk J.kh d Lo:i dk fnXn'ku djrh g] fo"kerk d eki J.kh dk lfEer ;k vlfEer vkèkkj Li"V djr g tcfd vifdj.k d eki J.kh d in eY;k d fc[kjko ;k iQyko dk vè;;u djr gA

3.1.1 fo"kerk dk vFk ,o ifjHkk"kk (Meaning and Definition of Skewness)

fo"kerk dk 'kkfCnd vFk g lerk dk u gkuk vFkok vlerk dk ik;k tkukA ;fn fdlh ledekyk dk vkofÙk forj.k ml d eè; e nkuk vkj lekt u gk rk ,lh fLFkfr dk fo"kerk (Skeweness) dh fLFkfr dk gkuk ekuk tkrk gA nlj 'kCnk e fo"kerk dk eki g ,d ,lk l[;kRed eki tk fdlh J.kh dh vlfefr (Asymmetry) dk idV djrk gA fofHkUu fo}kuk u fo"kerk dk fofHkUu idkj l ifjHkkf"kr fd;k gA bue l dN fo}kuk dh ifjHkk"kk, bl idkj gA

(i) iMu rFkk fyUMfDoLV (Paden and Liandquist) d vulkj ¶,d forj.k dk fo"ke dgk tkrk g tcfd mlu lefefr (Symmetry) dk vHkko gk] vFkkr ekik d foLrkj d ,d vkj ;k nljh vkj gh eY; dfUær gk tkr gA¸

(ii) th flilu o dkiQdk (G. Simpson and Kafka) d vulkj] ¶fo"kerk dk eki fn;k o fo"kerk dh ek=kk crkr gA ,d lefefr forj.k e ekè;] efè;dk o Hkf"Bd leku gkr gA ekè; Hkf;"Bd l ftruk vf/d njh ij gkxk] fo"kerk dh ek=kk mruh gh vf/d gkxhA¸

(iii) ,p- vfdu o dkYVu d vuljk] ¶fo"kerk l vfHkik; fdlh Hkh vkofÙk forj.k l lfEer l nj gVu dh iofÙk gA¸

(iv) ekfjl geox d vuljk] ¶fo"kerk ,d vkofÙk forj.k l vlferrk vFkok leferrk d vHkko dk vkdkj d :i e crykrk gA ;g y{k.k dUæh; iofÙk d dy ekik d ifrfuf/ dk fu.k; gr fo'k"k egRo dk gA¸

3.1.2 vkofÙk forj.k d vuljk (Types of Frequency Distribution)

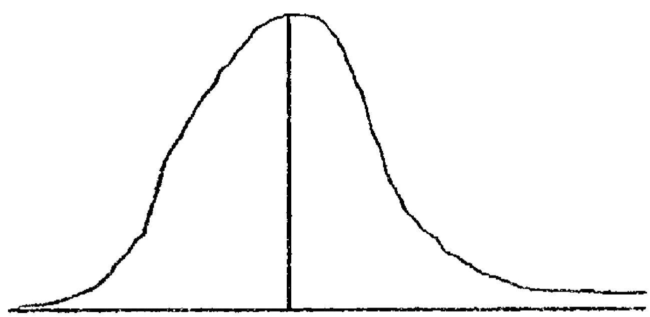
lkekU; vkofÙk forj.k fuEu idkj d gk ldr gA

(i) lkekU; forj.k (Normal Distribution)—

bl idkj d forj.k e vkofÙk;k ,d fuf'pr Øe l c<rh g fiQj ,d fuf'pr fcUn ij vfèkdre gku d ckn mlh Øe l ?kVrh gA ;fn bu vkofÙk;k dk fcUnj[kh; i=k ij inf'kr fd;k tk; rk ?kUVh d vkdkj dk oØ (Bell Shaped Curve) cuxkA bl n'kk e lekUrj ekè; ($\overline{X}$) eè;dk rFkk cgyd (Z) dk eY; ,d leku gkrk gA (In a symmetrical distribution $\overline{X}$ = M = Z) fp=k l[;k 1 l ;g Li"V gA

Fig. 1

Normal Distribation Curve



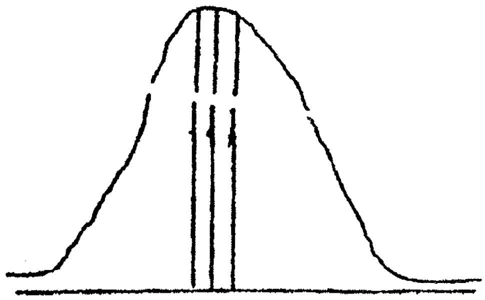
$\overline{X}$ = M = Z

$Q_3 - M = M - Q_1$

(i) fo"k; forj.k (Asymmetrical Distribution)—bl idkj d forj.k e vkofÙk;k Hkfe"Bd d ,d vkj vf/d rFkk nljh vkj de gkrh gA ;gk lekUrj ekè; ($\overline{X}$) eè;dk (M) rFkk cgyd (Z) lHkh ,d fcUn ij ugh gkrA ,l forj.k tk leferh; u gk] fo"kerk okyk forj.k ;k vlfer vkofÙk forj.k dgykrk gA ;g fo"kerk Hkh nk idkj dh gk ldrh gA

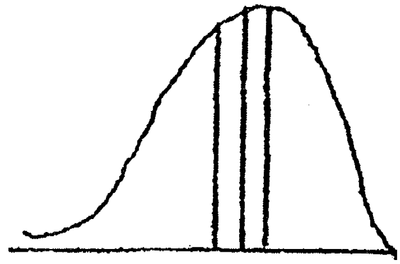
(A) ?kukRed fo"kerk (Positive Skewness)—;fn lekUrj ekè; dk eY; eè;dk l vfèkd g vkj eè;dk dk cgyd l vf/d g rk fo"kerk ?kukRed gkxhA nlj 'kCnk e] ;fn oØ nkfguh vkj vf/d >dk g rk fo"kerk ?kukRed gkxhA (fp=k l[;k 2)

(B) ½.kkRed fo"kerk (Negative Skevness)—;fn lekUrj ekè; dk eY; eè;dk l de g vkj eè;dk cgyd l de gkrh g rk fo"kerk ½.kkRed gkxhA ,lh n'kk e oØ ck;h vkj vf/d >dk gkrk gA fp=k l[;k (3)



$\overline{X} > M > Z$

Fig : 2 Positive Skewness



$\overline{X} < M < Z$

Fig : 3 Negative Skewness

3.1.3 fo"kerk d mís'; (Objectives of Skewness)

fdlh ledekyk e fo"kerk Kkr dju d ie[k mís'; fuEufyf[kr gkr gA

(i) fo"kerk vkofÙk;k d ?kuRo dh ek=kk rFkk iÑfr dk tkuu e lgk;rk inku djrh g] fi0j pkg ?kuRo de eY;k e ;k vf/d eY;k eA

(ii) lekUrj ekè; ($\overline{X}$) eè;dk (M) rFkk cgyd (Z) d chp vkuikfrd lEcU/ vkf'kd :i e fo"k;e forj.k ij gh vk/kfjr gA fo"kerk d eki gh ;g inf'kr djr g fd ;g vkuikfrd lc/ fdl lhek rd lgh gA

(iii) fo"kerk ,d vkofÙk forj.k dh lekU;rk Kkr dju e Hkh lgk;d gkrk gA lkekU; forj.k dh ekU;rk ij lkf[;dh d vud eki fuHkj djr gA

3.1.4 vifdj.k rFkk fo"kerk e vUrj
(Difference between Skeweness and Dispersion)—

vifdj.k rFkk fo"kerk e e[; vUrj fuEufyf[kr gµ

(i) vifdj.k fdlh J.kh e fopj.k dh ek=kk d ckj e crkrh gA tcfd fo"kerk fopj.k dh fLFkfr vFkok lefefr l fopyuk dh tkudkjh nrh gA

(ii) vifdj.k e fofHkUu ink d leg e fLFkfr ij fopkj fd;k tkrk g tcfd fo"kerk e mudh iofÙk ijA

(iii) vifdj.k forj.k dh cukoV d ckj e tkudkjh nrk g tcfd fo"kerk forj.k dh vkÑfr ;k Lo:i d ckj e tkudkjh nrk gA

(iv) vifdj.k d eki f}?kkrh; ekik ij vk/kfjr g tcfd fo"kerk d eki ie[k :i l iFke Hkkjrh; ekik ij vk/kfjr gkr gA

(v) vifdj.k d lHkh eki ik;% ?kukRed gkr g tcfd fo"kerk x.kkd ?kukRed o ½.kkRed nkuk gh idkj d gk ldr gA

(vi) vifdj.k dk fp=kk }kjk inf'kr ugh fd;k tk ldrk tcfd fo"kerk dh tkudkjh fp=ke; in'ku }kjk ljyrk l dh tk ldrh gA

3.1.5 fo"kerk dk ijh{k.k (Test of Skewness)

bl ckr dh tkp djus d fy, fd fdlh led J.kh e fo"kerk dh fo|ekurk g ;k ugh] bld fy, db tkp fof/;k dk i;kx fd;k tkrk g tk fuEufyf[kr g %&

(i) ekè;k dk lEcU/—;fn ,d forj.k e lekUrj ekè; ($\overline{X}$), eè;dk (M) rFkk cgyd (Z) vki l e cjkcj g vFkkr ($\overline{X}$ = M = Z) rk mle fo"kerk dk vHkko gkrk g vkj ;fn forj.k e lekUrj ($\overline{X}$) eè;dk (M) rFkk cgyd (Z) cjkcj u gk vFkkr ($\overline{X}$ = M = Z) rk ml forj.k e fo"kerk ikb tkrh gA

(ii) fopyuk dk ;kx—fdlh led ekyk d lekUrj ekè; ($\overline{X}$) eè;dk (M) rFkk cgyd (Z) l fy, x; ?kukRed (+ve) fopyuk dk ;kx ½.kkRed (–ve) fopyuk d ;kx d cjkcj g rk J.kh e fo"kerk ugh gkrh gA

(iii) cgyd d nkuk vkj vkofÙk;k—;fn cgyd d nkuk vkj dh vkofÙk;k dk ;kx cjkcj gkrk g rk fo"kerk ugh gkrh gA

(iv) eè;dk l prFkd eY;k dh njh—fo"kerk dh n'kk e iFke prFkd (Q_1) o rrh; prFkd (Q_3) efè;dk l leku njh ij ugh ikr] vFkkr ;fn $Q_3 - M = M - Q_1$ rk forj.k e fo"kerk dk vHkko gkrk g vkj ;fn $Q_1 - M \neq M - Q_1$ rk forj.k e fo"kerk fo|eku jgrh gA

(v) forj.k dk pØ—vkofÙk forj.k dk fcUnj[kh; i=k ij vfdr djus ij lkekU; oØ (Normal Curve) u cu rk fo"kerk vo'; gh fo|eku gkrh gA

3.1.6. fo"kerk d eki (Measures of Skewness)

fo"kerk d pkj eki gkr g rFkk bue l iR;d eki dk nk :i k e fn[kk;k tk ldrk g ftUg fuji{k eki (Absolute Measure) rFkk lki{k eki (Relative Measure) dgr gA fo"kerk d fuji{k eki }kjk J.kh dh fo"kerk dh dy ek=kk (Degree) rFkk ?kukRed (+ve) o ½.kkRed (–ve) iÑfr ek=k gh Kkr gk ikrh gA bl idkj dk eki ryukRed vè;;u gr mi;Dr ugh gkrkA vr% fo"kerk dk lki{k eki fudkyk tkrk gA bl eki dk vuikr e 0;Dr djr gA ; lki{k eki fo"kerk dk x.kkd dgykrk gA

fo"kerk d fuEufyf[kr pkj eki g ftudk vè;;u ge bl vè;k; e djxA

(i) dky ih;jlu dk eki (Karl Pearson's Measure)

(ii) ckmy dk eki (Bowle's Measure)

(iii) dyh dk eki (Kelly's Measure)

(iv) vifdj.k dh ?kkr dk eki (Moments Measure)

(i) dky fi;jlu dk eki (Karl Pearson's Measure)

;g eki ledekyk d ekè;k dh fLFkfr ij fuHkj djrk gA ,d fo"k; vkofÙk forj.k e $\overline{X} \neq M \neq Z$ vFkkr vlleku eY; gkr gA bu ekè;kdk d eè; vUrj ftruk vf/d gkxk] forj.k mruk gh vf/d fo"ke gkxkA ;g ½.kkRed o ?kukRed gk ldrk gA dky fi;jlu e fo"kerk dk eki dju d fy, fuEu l=k fn, gA

(A) leKUrj ekè; ,o cgyd d vUrj ij vk/kfjr eki

(i) Skewness (SK) = Mean ($\overline{X}$) – Mode (Z)

(ii) Coefficient of Skewness (i) = $\dfrac{\text{Mean } (\overline{X}) - \text{Mode (Z)}}{\text{S.D. } (\sigma)}$

(v) leKUrj ekè; rFkk efè;dk d vUrj ij vk/kfjr eki

(i) Skewness (Sk)3 = (Mean – Median)

(ii) Coefficient of Skewness (i) = $\dfrac{3 \text{ (Mean – Median)}}{(\sigma)}$

dky fi;lu fo"kerk x.kkd dk eY; lkekU;k—1 rFkk +1 d chp ik;k tkrk gA ;fn cgyd (z) vfu/kj.kh; (ill defined) gk] rc fo"kerk x.kkd dk eY; –3 rFkk +3 d chp ik;k tkrk gA

Example 1.

The following information relative to two distribution state which distribution is more

	Disbtibution I	Distribution II
Mean	100	90
Median	95	95
Standard Deviation	10	10

Solution : The values of Mode is not given in this problems. As such it will be solved by 2nd formula given by karl Pearson

Distribution I— j = $\dfrac{3(x-m)}{\alpha} = \dfrac{3(100-95)}{10} = 1.5$

Distribution II— j = $\dfrac{3(x-m)}{\alpha} = \dfrac{3(90-95)}{10} = 1.5$

Thus both the distributions reveal the same amount of skewness. But it is position in I distribution while negative in II distribution.

Example 2.

Calculate Karl Pearson's Coefficient of skewness by finding out Arithmetic Mean, mode and standard Deviation

Measurement	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	10	12	18	25	16	14	5

Solution : Calculation of Karl Pearson's Coefficient of Skewness

Measurement	M.V. (x)	Frequency (f)	Deviation from A = 35 i = 10 dx	Deviations multiplied with Freq- (fdx')	Squares of deviations
0–10	5	10	–3	–30	90
10–20	15	12	–2	–24	48
20–30	25	18	–1	–18	18
30–40	35	25	0	0	0
40–50	45	16	+1	16	16
50–60	55	14	+2	28	56
60–70	65	5	+3	15	45
Total	–	100		– 72 + 59 = – 13	273

$$\overline{X} = A + \frac{\Sigma dx}{N} \times i \;\; 35 + \frac{-13}{100} \times 10 = 35 - 13 = 33.7$$

Mode lies in group 30–40 having highest frequency

Formula $$Z = 1_1 + \frac{\nabla_1}{\nabla_1 + \nabla_2} \times i = 30 + \frac{7}{7+9} \times 10$$

$$= 30 + \frac{70}{16} = 30 + 4.38 = 34.38$$

$$\alpha = \frac{i}{n} \sqrt{\Sigma d^2 x^1 N - \Sigma(fdx)^2} = \frac{10}{100} \sqrt{273 \times 100 - (-13)^2}$$

$$= \frac{1}{10} \sqrt{27300 - 169} \text{ or } \frac{1}{10} \sqrt{27131} = 16.47$$

$$j = \frac{\overline{X} - z}{\alpha} = \frac{33.70 - 34.38}{16.47} = \frac{-0.68}{16.47} \text{ or} - 0.04$$

The Skewness is negative in small degree

Example 3.

Calculate Coefficient of Variation and Karl Pearson's Coefficient of Skewness from the data given below

Age under (in years)	10	20	30	40	50	60	70	80
No. of Persons	5	15	30	50	80	100	120	125

Solution : Problem is given in commulative frequency. It is to be converted into ordinary frequency before it is.

Solved

Calculation of C.V. and Coefficient of Skewness

Age in years	M.V.	No. of Persons f	A = 35 i = 10 (dx')	fdx'	fd^2x'
0–10	5	5	–3	15	45
10–20	15	10	–2	–20	40
20–30	25	15	–1	–15	15
30–40	35	20	0	0	0
40–50	45	30	+1	+30	30
50–60	55	20	+2	+40	80
60–70	65	20	+3	+60	180
70–80	75	5	+4	+20	80
Total		125		+100	470

$$\overline{X} = A = \frac{\Sigma dx'}{N} \text{ xi} = 35 + \frac{100}{125} \times 10 = 35 + 8 = 43 \text{ years}$$

Mode lies in group 40–50 (by inspection), formula is

$$Z = 1_1 + \frac{\nabla^1}{\nabla^1 + \nabla^2} \times i = 40 + \frac{10}{10+10} \times 10 \text{ or } 45 \text{ years}$$

$$\alpha = \frac{i}{n} \sqrt{\Sigma fd^2 x^1 N - (2\, fdx')^2} = \frac{10}{125} \sqrt{470 \times 125 - (100)^2}$$

$$= \frac{10}{125} \sqrt{58750 - 10000} \text{ or } \frac{10}{125} \times 220.79 \text{ or } 17.66 \text{ years}$$

$$\text{C.V.} = \frac{\alpha}{\overline{x}} \times 100 = \frac{17.66}{43} \times 100 = 41.07\%$$

$$j = \frac{\overline{x} - z}{\alpha} = \frac{43 - 45}{17.66} = -0.113$$

Thus C.V. = 41.07% and Karl Pearsons j_z = – 0.113

(2) ckmy dk eki (Bowley's Measure)

ik- ckmy u fo"kerk dk ekiu dh ,d vU; fof/ nh gA ;g fof/ eè;dk (M), iFke prFkd (Q_1) rFkk rhlj prFkd (Q_3) ij vk/kfjr gA bl fof/ dk fo"kerk ekiu dh prFkd (Quartile Measure of Skewness) Hkh dgk tkrk gA ckmy dh fo"kerk dk eki dk i;kx ,lh fLFkfr e fd;k tk ldrk g 0;; ,d forj.k d cgyd fuf'pr u gkA bl ekiu dk i;kx [ky 'kh"kd ox gku dh fLFkfr e Hkh fd;k tk ldrk gA ckmy u fo"kerk dk eki dju d fy, fuEufyf[kr l=k fn, gµ

(i) fo"kerk dk prFkd eki

Skewness (Sk) = $(Q_1 - M) - (M - Q_1)$

or SK = $Q_3 + Q_1 - 2M$

(ii) fo"kerk dk prFkd x.kkdµ

$$\text{Coefficient of Skewness (i)} = \frac{(Q_3 - M) - (M - Q_1)}{(Q_3 - M) + (N - Q_1)}$$

$$\text{or j} = \frac{Q_3 + Q_1 - 2}{Q_3 - Q_1}$$

Example 4

From the following data calculate the coefficient of skewness based upon median the quartile and comment on your result.

Weekly Wages (Rs.)	**No. of Workers**
12.5-17.5	12
17.5-22.5	16
22.5-27.5	25
27.5-32.5	14
32.5-37.5	13
37.5-42.5	10
42.5-47.5	6
47.5.52.5	3
52.5-57.5	1
Total	100

Solution : Calculation of Bowley's Coefficient of Skewness

Weekly Wages (Rs.)	No. of Workers	Cumulative Frequency
12.5-17.5	12	12
17.5-22.5	16	28
22.5.27.5	25	53
27.5.32.5	14	67
32.5.37.5	13	80
37.5.42.5	10	90
42.5.47.5	6	96
47.5.52.5	3	99
52.5.57.5	1	100

m = Size of $\frac{N}{2}$ th item = Size of $\frac{100}{2}$ th item of 50th item which lies in 53 c.f. or 22.5 – 27.5 group

M = 1 + $\frac{i}{f}$ (m – C) = 22.5 + $\frac{5}{25}$ (50 – 28) or 22.5 + 4.4 = Rs. 26.9

q_1 = size of $\frac{N}{4}$ th item = Size of $\frac{100}{4}$ or 25th item, lies in 17.5.22.5 group

Q_3 = 1 + $\frac{i}{f}$ (Q1 - c) = 17.5 + $\frac{5}{16}$ (25 – 12) or 17.5 + 4.06 = Rs. 21.56

Q_3 = size of $\frac{3N}{4}$ or $\frac{3(100)}{4}$ or 75th item, lies in 32.5.37.5 group

Q_3 = 1 + $\frac{i}{f}$ (Q_1 – c) = 32.5 + $\frac{5}{13}$ (75 – 67) or 32.5 + 3.08 = Rs. 35.58

JQ = $\frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1} = \frac{35.58 + 21.56 - (2 \times 26.9)}{35.58 - 21.56}$ =0.2382

So Bowley's Coefficient of Skewness = + 0.2382

Example 5.

Find the appropriate measure of skewness and dispersion from the following data.

Age (Years)	No. of Workers
Below 20	13
20–25	29
25–30	46
30–35	60
35–40	112
40–45	94
45–50	45
50 & above	21

Solution : In open end distribution, Dr. Bowley's measure of skewness is suitable one.

Age (Years)	No. of Employees	Cumulative Frequency
Below 20	13	13
20–25	29	42
25–30	46	88
30–35	60	148
35–40	112	260
40–45	94	354
45–50	45	399
50 & above	21	420

$$q_1 = \text{Size of } \frac{420}{4} \text{ th or 105th item, m = Size of } \frac{420}{2} \text{ th or 210th item}$$

$$q_3 = \text{Size of } \frac{3(420)}{4} \text{ th or 315 th item}$$

So $$Q_1 = 30 + \frac{5}{60}(105 - 88) = 30 + \frac{5 \times 17}{60} \text{ or 31.42 years}$$

$$M = 35 + \frac{5}{112}(210 - 148) = 35 + \frac{5 \times 62}{1120} \text{ or 37.77 years}$$

$$Q_3 = 40 + \frac{5}{94}(315 - 260) = 40 + \frac{5 \times 55}{44} \text{ or 42.92 years}$$

Bowley's Coefficient of Skewness

$$j_q = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1} = \frac{42.92 - 31.42 - (2 \times 37.77)}{42.92 - 31.42}$$

$$= \frac{37.34 - 75.54}{11.50} = \frac{-1.20}{11.50} = -0.104$$

Coeff. of Quartile Dispersion

$$= \frac{Q_3 - Q_1}{Q_3 - Q_1} = \frac{42.92 - 31.42}{42.92 + 31.42} = \frac{1.5}{34} \text{ or } 0.154$$

So j_q = 0.104 and C or Q. D = 0.154

(i) dyh dk eki (Kelly's Measure)

dyh dk eki mi;Dr ekih dk eè; ekx gA dky fi;jlu dk eki ,d forj.k dh leLr enk ij vk/kfjr gkrk g tcfd ckmy dk eki eè; dh 50% enk ij vk/kfjr gkrk g bl fof/ e eè; dh 80% enk ij è;ku fn;k tkrk g rFkk 10% nkuk vkj d pje eY;k dk NkM fn;k tkrk g dyh fo"kerk n'ked o 'kred ij vk/kfjr g ;g fo"kerk fuEu nk l=kk l fudkyh tk ldrh gA

(i) Kelly's Skewness (Sk)= $P_{10} + P_{90} - 2$ Median

Coefficient of Skewness (i) = $\dfrac{P_{10} + P_{10} - 2M}{P_{90} - P_{10}}$

(ii) Kelly's Skewness (Sk) = $D_7 + D_1 - 2M$

Coefficient of Skewness (j) = $\dfrac{D_9 + D_1 - 2M}{D_9 - D_1}$

Example 6 : From the following data given for a frequency Distribution. Calculate kelly's Coefficient of Skewness.

P_{90} = 101, P_{10} = 5812, Median = 79.06

Solution : Coefficient of Skewness (j) = $\dfrac{P_9 + P_{10} - 2M}{P_{90} - P_{10}}$

$$= \frac{101 + 5812 - 2(79.06)}{101 - 5812}$$

$$= \frac{15912 - 15812}{4288} = +0.023$$

Example 7.

Calculate Coefficient of Skewness in the following two distributions and tell which distribution is more Skewed ?

Mark	55	58	61	64	67–70	Total
Group A	12	17	23	18	11	81
Group B	20	22	25	13	07	87

Solution : This problem can be solved by any measure of skewness, but Karl Pearson's masure is more reliable and more appropriate measure, as such the same measure of Skewness has been used.

Calculation of Karl Pearsons' Coefficient of Skewness

Marks	$A = \frac{62.5}{3}$	Group A			Group B		
	dx'	Frequency	fdx'	fd^2x'	Frequency	fdx'	fd^2x
55–58	–2	12	–24	48	20	–40	80
58–61	–1	17	–17	17	22	–22	22
61–64	0	23	0	0	25	0	0
64–67	1	18	18	18	13	13	13
67–70	2	11	22	44	7	14	28
Total	–	81	–1	127	87	–35	143

Group A : $\overline{X} = A + \frac{fdx'}{N} \times I = 62.5 \times \frac{-1}{81} \times 3 = 62.463$ marks

$$\alpha = \frac{1}{N}\sqrt{\Sigma fd^2 x' N - (\Sigma fdx')^2} = \frac{3}{81}\sqrt{127 \times 81 - (-1)^2}$$

$$= \frac{3}{81}\sqrt{10281 - 1} = \frac{3}{81} \times 101.39 \text{ or } 3.76 \text{ marks}$$

Mode by inspection lies in (61 – 64) group Formula is

$$Z = I_n + \frac{\nabla_1}{\nabla_1 + \nabla_2} \times i \text{ or } 61 + \frac{6}{6+5} \times 3 = 61 + \frac{18}{11} \text{ or } 62.64 \text{ marks}$$

$$J = \frac{\overline{x} - Z}{\sigma} = \frac{62.463 - 62.64}{3.76} = 0.047$$

Group B : $\overline{X} = A + \frac{fdx'}{N} \times I = 62.5 \times \frac{-35}{87} \times 3 \text{ or } 62.5 - \frac{105}{87}$

= 62.5 – 1.207 or 61.293 Marks

$$\alpha = \frac{1}{N}\sqrt{\Sigma fd^2 x' N - (\Sigma fdx')^2} = \frac{3}{87}\sqrt{143 \times 87 - (-35)^2}$$

$$= \frac{3}{87}\sqrt{12441 - 1225} \text{ or } \frac{3}{87} \times 105.906 \text{ or } 3.65 \text{ marks}$$

Z by inspection lies in (61 – 64) group. Fomula is

$$Z = L_1 + \frac{\nabla_1}{\nabla_1 + \nabla_2} \times i$$

$$61 + \frac{3}{3+12} \times 3 \text{ or } 61 + 0.6 \text{ or } 61.6 \text{ marks } J = \frac{\overline{x} - z}{\alpha} = \frac{61293 - 616}{365} = -0.06$$

Hence Group B is more skewed than Group A.

Example 8.

Bellow is the frequency distribution of Ages at which drinking is begun for a sample or 200 drinkers

Age in years	15.18.9	19-22.9	23.26.9	27.30.9	31 and over
Frequency	55	80	30	25	10

From the above data, find out the Age which lies midway between the first and third quartiles.

Solution : First of all the class groups would be

Age in years	14.95-18.95	18.95-22.95	22.95-26.95	26.95-30.95	30.95-34.95
Frequency	55	80	30	25	10
Cum. Frequency	55	135	165	190	200

$Q_1 = \frac{200}{4} = 50\text{th item}$ $\qquad$ $M = \frac{200}{4}\text{th} = 100\text{th item}$

It lies in (14.95-18.95 group $\qquad$ It lies in (18.95-22.95) group

$$Q_1 = 1_1 + \frac{i}{f}(q1 - c) = 14.95 + \frac{4}{55}(50 - 0)$$

$$M = li = \frac{i}{f}\ (m - C)\ M = 1895 + \frac{4}{80}(100 - 55)$$

= 18.59 years $\qquad$ M = 18.95 + 2.25 = 21.20 years

$$Q_3 = \frac{200 \times 3}{4} = 150\text{th item or } Q_3 \text{ group is } 22.95.\ 26.95$$

$$Q_3 = l1 + \frac{1}{f}(q_3 - C) = 22.95 + \frac{4}{30}(150 - 135) = 24.95 \text{ yrs.}$$

$$\text{Q.D. } \frac{Q_3 - Q_1}{2} = \frac{24.95 - 18.95}{2} = 318 \text{ years}$$

Age is midway between Q_1 and Q_3 that is Q_1 + Q. D = 18.59 + 3.18 = 21.77 years. Since the value of Q_1 + Q. D. is more than median distribution has positive skewness.

3.2 ifj?kkr (Moments)

3.2.1 ifj?kkr dk vFk ,o ifjHkk"kk

fdlh Hkh lkf[;dh vuql/ku e lkf[;dh fo'y"k.k o vkofÙk forj.k dh fo'k"krkvk dk Lo:i dh leh{kk vfr vko';d o egRoi.k ifØ;k gA lkf[;dh fo'y"k.k o vkofÙk forj.k dh ljy o Li"V 0;k[;k dju e ifj?kkr (Moments) fof'k"V Hkfedk fuHkkr gA lkf[;dh e ifj?kkr 'kCn dk vk'k; Hkkfrdh ,o lkf[;dh foKku l vyx g ;fn ifj?kkr 'kCn dk vFk mlh HkkokFk e fy;k tk; rk ge lkf[;dh e fofHkUu fcUnvk ij iMu okyh Hkkj dh ,ot e ox vkofÙk;k yx rFkk ey fcUnvk dh njh d LFkku ij leUrkj ekè; dfYir ekè; vFkok 'kU; l fofHkUu ij eY;k d fopyuk l yrh gA

ifj?kkr dk ifjHkkf"kr djr g, A.E. Waugh u fy[kk g fd ifj?kkr fdlh ledekyk l leUrj ekè; l mld fofHkUu eY;k rd d fopyuk d fofHkUu ekè;k (iFke] f}rh;] rrh;] prFk] ipe bR;kfn) d leUrj ekè; gA

fu"d"k d :i e dg ldr g fd ifj?kkr og lkf[;dh eki g ftudk i;kx fdlh vkofÙk forj.k dh fo'k"krkvk dh 0;k[;k rFkk fo'y"k.k dju d fy, fd;k tkrk gA

3.2.2 ifj?kkrk d mís'; (Object of moments)

ifj?kkrk dh Hkkfex.kuk dju e ie[k mís'; fuEufyf[kr g

(i) led J.kh d ckn eY;k d vifdj.k vFkok fo[kjko dk vè;;u lekUrj ekè; d lEcU/ e djukA

(ii) lekUrj ekè; d lnHk e led J.kh dh ljpuk dk vè;;u djukA

(iii) vkofÙk forj.k d oØ dh tkudkjh iklr djukA oØ i lkekU; (Normal) udhy 'kh"k okyk vFkok piV 'kh"k okyk gk ldrk gA

3.2.3 ifj?kkrk dh x.kuk (Calculation of moments)

ifj?kkrk dh x.kuk dju dh fuEufyf[kr jhfr;k gµ

(A) iR;{k jhfr (Direct Method)

(B) y?k jhfr (Short Cut Method)

(C) infopyu jhfr (Step Deviation Method)

(A) iR;{k jhfr (Direct Method)—;fn lekUrj ekè; i.kkd e vk tk; rk iR;{k jhfr ljy jgrh gA blh jhfr d vuljkj fuEufyf[kr fØ;k viukb tkrh gA

(i) loiFke led J.kh d lekUrj ekè; dk fu/kj.k fd;k tkrk gA ($\bar{x}$)

(ii) iR;d ineY; dk lekUrj ekè; ($\bar{x}$) l fopyu Kkr fd;k tkrk gA (d = x – $\bar{x}$)

(iii) fopyuk d ox (d^2) ?ku (d^3) rFkk prFk ?kkr (d^4) Kkr dj mUg tkM nr gA vFkkr Σd^2 Σd^3 o Σd^4 iklr fd, tkr gA

(iv) [kf.Mr ;k lrr J.kh e fopyu ?kkrk dk lEcfU/r vkofÙk;k l x.kk djd mud tkM Kkr fd, tkr gA vFkkr Σfd, Σfd^3 Σfd^3 o Σfd^4 iklr fd, tkr gA

(v) vUr e ink dh l[;k ;k vkofÙk;k d ;kx (N) l Hkkx ndj iFke pkj dUæh; ifj?kkrk dh x.kuk dh tkrh gA

ifj?kkrk d l=kk dk fuEu idkj j[kk tk ldrk gµ

Moments	Individual Series	Discrete/Continous Series
1. First Moment about the Mean M_1	$\frac{\Sigma(x-\bar{x})}{N} = \frac{\Sigma d}{N}$	$= \frac{\Sigma f(x-\bar{x})}{N} = \frac{\Sigma fd}{N}$
(ii) Second Moment about the Mean M_2	$\frac{\Sigma(x-\bar{x})^2}{N} = \frac{\Sigma d^2}{N}$	$= \frac{\Sigma f(x-\bar{x})^2}{N} = \frac{\Sigma fd^2}{N}$
(iii) Third Moment about the Mean M_3	$\frac{\Sigma(x-\bar{x})^3}{N} = \frac{\Sigma d^3}{N}$	$= \frac{\Sigma f(x-\bar{x})^3}{N} = \frac{\Sigma fd^3}{N}$
(iv) Fourth Moment about the Mean M_4	$\frac{\Sigma(x-\bar{x})^4}{N} = \frac{\Sigma d^4}{N}$	$= \frac{\Sigma f(x-\bar{x})^4}{N} = \frac{\Sigma fd^4}{N}$

Compute first four moments about mean of the fllowing data.

Student	A	B	C	D	E	F
Marks Obtianed	14	16	18	20	25	27

Solution : Calculation of first moment about the mean

Student	Marks Obtained	(xd – k)	d^2	d^3	d^4
A	14	–6	36	–216	1,296
B	16	–4	16	–64	256
C	18	–2	4	–8	16
D	20	0	0	0	0
E	25	+5	25	+125	625
F	27	+7	49	+343	2,401
Total	120	0	130	+180	4,594

$$\bar{x} = \frac{\Sigma x}{N} = \frac{120}{6} = 20$$

$$\mu_1 = \frac{\Sigma d}{N} = \frac{0}{6} = 0 \qquad \mu_3 = \frac{\Sigma d^3}{N} = \frac{180}{6} = 30$$

$$\mu_2 = \frac{\Sigma d^2}{N} = \frac{130}{6} = 2167 \qquad \mu_4 = \frac{\Sigma d^4}{N} = \frac{4995}{6} = 7656$$

(B) y?k jhfr **(Short-cut Method)**

;fn lekUrj ekè; i.kkdk e ugh gkrk g rk y?k jhfr dk i;kx ljy gkrk gA bl fof/ dh ifØ;k fuEu idkj gA

(i) loiFke fdlh lfo/ktud eY; dk dfYir ekè; (A) eku yr gA ;g eY; J.kh l ckgj dk Hkh gk ldrk gA

(ii) bl dfYir ekè; l J.kh d fofHkUu eY;k d fopyu (dx) Kkr djd mud ox (dx²) ?ku (dx³) o prFk ?kkr (dx⁴) Kkr fd, tkr gA

(iii) [kf.Mr vkj lrr J.kh dh n'kk e vkofÙk;k dk x.kk dju d ckn ;kx yxk;k tkrk g vkj Σfdx, Σfdx² o Σfdx³ rFkk Σfdx⁴ ikIr djr gA

(iv) bld ckn fudky x; ;kxk dk N l Hkkx ndj fuEu l=kk d vk/kj ij ^dfYir ey fcUn l ifj?kkr (Moment about orbitary origin) dh x.ku dh tkrh gA

Moments about orbitary orgin	Individual Series	Directe Constlement
V_1	$\frac{\Sigma(x-A)}{N} = \frac{\Sigma dx}{N}$	$\frac{\Sigma f(x-A)}{N} = \frac{\Sigma fdx}{N}$

V_2	$\frac{\Sigma(x-A)^2}{N} = \frac{\Sigma d^2 x}{N}$	$\frac{\Sigma f(x-A)^2}{N} = \frac{\Sigma f d^2 x}{N}$
V_3	$\frac{\Sigma(x-A)^3}{N} = \frac{\Sigma d^3 x}{N}$	$\frac{\Sigma f(x-A)^3}{N} = \frac{\Sigma f d^3 x}{N}$
V_4	$\frac{\Sigma(x-A)^4}{N} = \frac{\Sigma d^4 x}{N}$	$\frac{\Sigma f(x-A)^4}{N} = \frac{\Sigma f d^4 x}{N}$

(v) lekUrj ekè; $(\bar{x})$ l ifj?kkrk dk Kkr dju d fy, dfYir ekè; l fudky x; ifj?kkrk e lek;ktu dju d fy, fuEu l=kk dk i;kx fd;k tkrk gA

(dfYir ekè; l fudky x; ifj?kkrk l dUæh; ifj?kkrk dk fu/kj.k)

$M_1 = V_1 - V_1 = 0$

$M_2 = V_2 - V_1^2 = \sigma^2$

$M_3 = V_3 - 3V_2 V_1 = 2 V_1^3$

$M_4 = V_4 - 4V_3 V_1 + 6V_2 V_1^2 - V_1^4$

Calculate first four moments about the mean by Short-Cut method from the following data

Length (in inches)	1.0	2.0	3.0	4.0	5.0	6.0	7.0
Frequency	5	38	65	92	70	40	10

Solution : Calculation of first four meoments

(Short-Cut Method)

Length in inches	frequency	dx A=4.0	fdx^1	fdx^2	fdx^3	fdx^4
1.0	5	–3	15	45	–135	405
2.0	38	–2	–76	152	–304	608
3.0	65	–1	–65	65	–65	65
4.0	92	0	0	0	0	0
5.0	70	+1	+70	70	+70	70
6.0	40	+2	+80	160	+320	640
7.0	10	+3	+30	90	+270	810
Total	320		+24	+582	+156	+2598

Moments about an arbitrary origin

$V_1 = \frac{\Sigma f dx}{N} = \frac{24}{320} = +0.075$ $\qquad$ $V_3 = \frac{\Sigma f d^3 x}{N} = \frac{156}{320} = +0.488$

$V_2 = \frac{\Sigma fd^2 x}{N} = \frac{582}{320} = +1.819$ $\qquad V_4 = \frac{\Sigma fd^4 x}{N} = \frac{2598}{320} = +8.119$

Moments about the Mean

$\mu_1 = V - V$ or $.075 - .075 = 0$

$\mu_2 = V_2 - V_1^2$ or $1.819 - (-075)^2 = 1.813$

$\mu_3 = V_3\ 3V . V_1 + 2V_1^3$

$= 0.488 - 3(1.819 \times 0.075) + 2\ (.075)^3$

$= 0.488 - 0.409 + 0.00084 = .0798$

$\mu_3 = V_4 - 4V_3\ V_1 + 6V_2^3\ V_1^2 - 3V_1^4$

$= 8.119 - 4\ (.488 \times .075) + 6\ (1.819)\ (.075)^2 - 3\ (.075)^4$

$= 8.119 - 0.146 + 0.0614 - 0.00060$ or 8.034

Computation of Moments about orbirary origin and form Central Moment

On the basis of Central moments the moment about any orbirary origin can biocomputed by using following steps :

(i) Find the different between arithmetic mean (x) and assumed mean (A) :

$[(\bar{d}\ x = \bar{x} - A)]$

(ii) Thereafter the following formulas are used

$V_1 = (\mu + \bar{d}\ x)^1 = \mu_1 + \bar{d}\ x\ (\therefore \mu_1 = 0)$ so $V_1 = \bar{d}\ x$

$V_2 = (\mu + \bar{d}\ x)^2 = \mu_2 + \bar{d}\ x + \bar{d}\ x^2 = \mu_2 + \bar{d}\ x^2\ (\therefore \mu_1 = 0)$

$V_3 = (\mu + \bar{d}\ x)^3 = \mu_3 + 3\mu_2\ \bar{d}\ x^2 + 3\ \mu_1\ \bar{d}\ x^2 + \bar{d}\ x^3$

or $\mu_3 + 3\ \mu_2\ \bar{d}\ x + \bar{d}\ x_3$

$V_4 = (\mu + \bar{d}\ x)^4 = \mu_4 + 4\mu_3\ \bar{d}\ x + 6\mu_2\ \bar{d}\ x^2 + 4\mu_1\ dx^3 + dx^4$

or $(\mu_3 + 4\mu_3\ \bar{d}\ x + 6\mu_2\ \bar{d}\ x^2 + \bar{d}\ x^4\ (\therefore \mu_1 = 0)$

Note : $(\bar{x} - A)$ has been denoted here as $\bar{d}\ x$. it may be denoted by (Greek sign Δ (delta) also.

From the following first four moments of a distribution about the arbitrary orgin 4, find out the mean of the distribution and Calculate moment about mean and also about the arbitrary mean zero.

$V_1 = 1_1\ V_2 = 4_1\ V_1 = 10$ and $V_4 = 45$

Solution : $\bar{x} - A + \frac{\Sigma dx}{N}$

and $V_1 = \frac{\Sigma dx}{N}$ so $\bar{x} = A + V_1$

in the problem A = 4 and $V_1 = 1$ Hence $\bar{x} = 4 + 1 = 5$

(i) First four Moment about the mean ($\bar{x} = 5$)

$\mu_1 = V_1 - V_1 = 1 - 1 = 0$

$\mu_3 = V_2 - V_1^2 = 4 - (1)^2 = 4 - 1 = 3$

$\mu_4 = V_4 - 4V_3 V_1 + 6V_2 V_1^2 \; 3V_1^4$

$= 45 - 4 (10 \times 1) + 6 (4) (1)^2 - 3 (1)^4$

$= 45 - 40 + 24 - 3 = 26$

So $\bar{x} = 5, \mu_1 = 0, \mu_2 = 3, \mu_3 = 0, \mu_4 = 26$

(ii) First four Moment about Arbitrary origin zero (0)

Since $\bar{x} = 5$, A = 0

So $\bar{d}x = (\bar{x} - A) = (5 - 0) = 5$

$V_1 = \mu_1 + \bar{d}x = 0 + 5 = 5$

$V_2 = \mu_2 + \bar{d}x^2 = 3 + (5)^2 = 28$

$V_3 = \mu_3 + 3\mu_2 \bar{d}x + \bar{d}x^3 = 0 + 3 (3) (5) + (5)^2$ or $0 + 45 + 125 = 170$

$V_4 = \mu_4 + 4\mu_3 \bar{d}x + 6\mu_2 \bar{d}x^2 + dx^4 = 26 + 4 (0) (5) + 6 (3) (5)^2 (5)^4$

$= 26 + 0 + 450 + 625 = 110$

So, $V_1 = 5$, $V_2 = 28$, $V_3 = 170$ and $V_4 = 1101$

(C) in fopyu jhfr (Step Deviation Method)

lekU oxkZUrj okyh J.kh e x.kuk fØ;k dk vkIku cuku d fy, in fopyu jhfr dk i;kx fd;k tk ldrk gA ;g jhfr y?k jhfr dh rjg gh gA V_1, V_2, V_3 vkj V_4 dk eku fudkyu d ckn lek;ktu djr le; fuEu l=kk dk i;kx fd;k tkrk gA

$\mu_1 = [V_1 - V_1] \times 1$

$\mu_2 = [V_2 - V_1^2] \times i^2$

$\mu_3 = [V_3 - 3V_2 V_1 + 2V_1^3] \times 1^1$

$\mu_4 = [V_4 - 4V_3 V_1 + 6V_1^2] - 3 V_1^4] \times 1^4$

3.2.4 'kh"kd dk l'kks/u (Sheppard's Correction) :

lrr vFkok v[kf.Mr J.kh e ifj?kkrk dk Kkr djr le; ;g eku fy;k tkrk g fd fofHkUu oxkUrjk dh vkofÙk;k mud eè; fcUn ij dUæhr gA ox d eè; fcUn ij leLr vkofÙk;k dk dUæh; gkuk eku tku d dkj.k ifj?kkrk (Moments) e dN foHk; (Errors) gk tkrk gA bl nj dju d fy, ifl¼ W.F. Sheppard u dN l'kk/u fd, g] ftUg 'kiM l'kk/u (Shepapard Correction) d uke l tkuk tkrk gA dUæh; ifj?kkrk d fy, 'kiM l'kk/u fuEu idkj g%&

l'kkf/r (Corrected) $\mu_1 = \mu_1$ (l'kk/u vko';d ugh)

l'kkf/r (Corrected) $\mu_2 = \mu_2 \frac{i^2}{12}$ (i = o.k foLrkj)

l'kkf/r (Corrected) $\mu_3 = \mu_3$ (l'kk/u vko';d ugh)

l'kkf/r (Corrected) $\mu_4 = \mu_4 \quad \frac{x^2 \times x^2}{2} + \frac{7}{240} 1^4$

3.2.5 ikfy;j dk 'k¼rk ijh{k.k (Charlier's Check of accuracy)

ifj?kkrk dh x.kuk dh 'k¼rk dk ijh{k.k dju d fy, ikfy;j d 'k¼e ijh{k.k l=kk dk i;kx fd;k tk ldrk g ; l=k fuEu idkj g %&

iFke ifj?kkr $\Sigma f(dx + 1) = \Sigma fdx + N$

f}rh; ifj?kkr $\Sigma f(dx + 1)^2 = \Sigma fdx^2 + 2\Sigma\, fdx + N$

rrh; ifj?kkr $\Sigma f(dx + 1)^3 = \Sigma fdx^3 + 3\Sigma fd^2\, x + 3\Sigma fdx + N$

prFk ifj?kkr $\Sigma f(dx + 1)^4 = \Sigma fd^4 x + 4\Sigma fd^3 x + = 6\Sigma fd^2 x + 4\Sigma fdx + N$

3.3 iFk'kh"kRo (Kurtosis)

3.3.1 iFk'kh"kRo dk vFk ,o ifjHkk"kk (Meaning and Defination of Kurtosis)

fdlh Hkh vkofÙk forj.k e vklr] vifdj.k o fo"kerk ekiu d ckn forj.k d 'kh"k dh iÑfr vFkkr 'kh"kd (Peakedness) dk Hkh ekik tkrk gA fdlh vkofÙk forj.k d eè; Hkkx e vkofÙk;k d leku vkofÙk pØ d cgyd d {k=k e piViu dk udhyiu dh ek=kk l gA

flEilu ,o dkiQdk d vulkj] ¶,d forj.k u iFk'kh"kd dh ek=kk dk eki lkekU; oØ d cukoV e dh tkrh gA

dky ,o 'kdkn (Clark and Shalade) d vulkj ¶iFk'kh"kRo ,d forj.k dh og fo'k"krk gA mld lkif{kr dk 0;Dr djrh gA

dky fi;jlu u 1905 e fuEu rhu 'kCnk dk i;kx fd;k FkkA

(i) leptokurtic udhy 'kh"k okyk oØ (Peaked Curve)

(ii) Platykurtic piV 'kh"k okyk oØ (Flat Topped Curve)

(iii) Mesokurtic lkekU; oØ (Normal Curve)

3.4.2 iFk'kh"kd dk eki (Measurement Kurtosis)

iFk'kh"kRo (Kurtosis) dk lki{k eki β_2 f}rh; rFkk prFk ifj?kkrk ij vk/kfjr g rFkk ftl fiDjlu dk iFk'kh"kRo x.kkd (Pearson's Coefficient of Skewness) dgk tkrk gA dky fi;jlu d l=kkulkj

$$\beta^2 = \text{(Bita two)} \ \frac{\mu_1 \text{(fourth moment)}}{\mu_2 \text{(Second moment)}}$$

lkekU; forj.k e β^2 dk eku 3 d cjkcj gkrk gA β^2 dh ekU;rk dju d i'pkr fu"d"k fuEu idkj fudky tk ldr gA

(i) $\beta_2 = 3$ oØ lkekU; gA (Mesokurtic)

(ii) $\beta_2 > 3$ oØ udhyk gA (Leptokurtic)

(iii) $\beta_2 < 3$ oØ piVk gA (Platykurtic)

iFk'kh"kRo d eki gr μ_2 (xkek) dk Hkh i;kx fd;k tk ldrk gA bld vulkj ;fn]

(i) γ_2 or $B - 3 = 0$ oØ lkekU; gA

(ii) γ_2 ?kukRed g] oØ udhyk gA

(iii) γ_2 ½.kkRed g] oØ piVk gA

4. lkjk'k (Summary)

dUæh; iofÙk l eY;k dk fc[kjko ;k ilkj lfEefyr g vFkok ugh g bld vè;;u d fy, ge fo"kerk d egÙk dk lgkjk yuk iMrk gA fo"kerk dk eki ,d ,lk l[;kRed eki g tk fdlh J.kh dh vlfefr dk idV djrk gA fo"kerk ?kukRed vFkok ½.kkRed gkrk gA fo"kerk de ;k vf/d gk ldrh gA ;fn oØ eku iQyk gvk gk rk fo"kerk lk/kj.krk de vkj oØ d vf/d iQyk gku dh n'kk e fo"kerk vf/d gkrh gA ;g vkofÙk;k e ?kuRo dh ek=kk rFkk iÑfr Kkr dju e lgk;rk inku djrk g] pkg ?kuRo U;u eY;k e gk vFkok vf/d eY;k eA fo"kerk d fuji{k eki }kjk fo"kerk dh dy ek=kk rFkk ?kukRed (+ve) o ½.kkRed (–ve) iÑfr ek=k gh Kkr gk ikrh g vr% nk ;k nk l vf/d forj.kk d ryukRed vè;;u gr fo"kerk dk lki{k eki egRoi.k gkrk gA ; lki{k eku fo"kerk dk x.kkd dgykrk gA ifj?kkr fdlh led J.kh d lekUrj ekè; dk dfYir ekè; vFkok 'kU; d vk/kj ij fy, x, fopyuk d ?kkrk dk lekurj ekè; gA iFk'kh"kd ,o lkf[;dh eki g tk oØ d 'kh"k dh iÑfr ij idk'k Mkyrk g vFkkr cgyd d {k=k e piViu ;k udhyiu dk fn[kkrk gA

5. iLrkfod iLrd (Recommended Books)

(i) Business Statistics — Prof. M.L. Oswal, N.P. Aggarwal, Dr. H.L. Sharma, Parveen Khurana

(ii) Business Statistics — T.R. Jain

(iii) Business Statistics — S.C. Sharma, R.C. Jain

(iv) Business Statistics — Shukha & Sahai

vkReck/ d fy, i'u (Self Assessment Question)

1. fo"kerk l D;k vk'k; gA fo"kerk d fofHkUu ekik dk o.ku dhft,A lk/kj.k% dku&lk eki dke e fy;k tkrk g rFkk D;k \
2. ,d vkofÙk forj.k e fo"kerk dh tkp fdl idkj dh tk;xh \ mnkgj.k lfgr le>kb; rFkk vifdj.k o fo"kerk e vUrj dhft,A
3. lkf[;dh e ifj?kkr dh ifjHkk"kk nhft,A bUg Kkr dju dh fofHkUu jhfr;k dk o.ku dhft,A
4. iFk'kh"kRo D;k g\ bll fdl mí'; dh ifr gkrh g\ D;k vkfFkd ,o lkekftd foKkuk e iFk'kh"kRo dk vè;;u mi;kxh g\ ;fn ugh] rk D;k \
5. ifj?kkr ,o iFk'kh"kRo e vUrj dhft,A
6. Find the Quartile Measure of Skewness and its Coefficients from the following

X	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40
F	2	5	7	18	21	16	8	3

7. Calculato Bowley's Coefficient of Skewness from the following data

Size of Collar (cm)	0	2	3	4–5	6–9	10–14	15–25
No.of Shirts	1	3	2	4	7	3	1

B.Com. Part-II

Paper - BC–204 : Business Statistics ***Revised By :*** **Dr. Heera Lal Sharma**

Lesson No. : 5

lg&lEcU/

Co-rrelation

ljpuk (Structure)

1. ifjp; (Introduction)
2. mís'; (Objective)
3. fo"k; dk iLrqrhdj.k
   - 3.1. lg&lEcU/ dh ifjHkk"kk
   - 3.2. lg&lEcU/ d idkj
   - 3.3. lg&lEcU/ dk egRo
   - 3.4. lg&lEcU/ Kkr dju dh jhfr;k
     - 3.4.1. fcUn j[kh; jhfr;k
       - 3.4.1.1. fo{ki fp=k jhfr
       - 3.4.1.2. lk/kj.k fcUn j[kh; jhfr
     - 3.4.2. xf.krh; jhfr;k
       - 3.4.2.1. dkyfi;lu dk lg&lEcU/ x.kkd
       - 3.4.2.2. fLi;jeu dk vufLFkfr lg&lEcU/ x.kkd
       - 3.4.2.3. lxkuh fopyu x.kkd
   - 3.5. fu/kj.k x.kkd
   - 3.6. lg&lEcU/ ,o dk;&dkj.k lEcU/
4. lkjk'k
5. iLrkfor iLroQ
6. vH;kl oQ fy; i'u

lg&lEcU/

(Co-rrelation)

lg&lEcU/ dk ifjp; (Introduction to Correlation) :

(Co-rrelation-I)

,lk n[kus e vkrk g fd tc fdlh ,d pj vFkok rÙo e ifjoru gkrk g rk nlj pj vFkok rÙo e Hkh ifjoru gkrk gA mnkgj.kkFk] tc fdlh oLr dh ekx c<rh g rk ml oLr oQ eY; e of¼ gkrh gA bl h idkj vPNh cjlkr gku ij mit Hkh vPNh gkrh gA cPpk dh vk; oQ lkFk&lkFk mudh yEckb Hkh c<rh gA lg&lEcU/ fo'y"k.k }kjk ge bl idkj d ikjLifjd lEcU/k dk vè;;u djr gA vr% tc nk jkf'k;k ;k pj bl idkj ifjofrr gkr g fd ,d pj e ifjoru gku l nlj pj e Hkh ifjoru gkrk gk rk ;g dgk tk,xk fd o pj lg&lEcfU/kr gA ;g ifjoru ,d gh fn'kk e gk ldrk g ,o foijhr fn'kk e HkhA ,d gh fn'kk e gku oky ifjoru dk /kukRed lg&lEcU/ o foijhr fn'kk e gku oky ifjoru dk ½.kkRed lg&lEcU/k dgr gA

2. mís'; (Objectives) :

bl lg&lEcU/ d vè;;u dju d e[; mís'; bl idkj g—

(i) lg&lEcU/ fo'y"k.k }kjk nk ;k vf/d pjk d eè; lEcU/k dk vè;;u djukA

(ii) ;g tkuuk fd O;kolkf;d fu.k;k e lg&lEcU/] vkUrjx.ku] ckã x.ku ,o iokuekuk e oQl lgk;d gkrk gA

(iii) lg&lEcU/ dh fn'kk rFkk ek=kk dk vuekku yxkukA

(iv) lg&lEcU/ dh x.kuk dju dh fofHkUu jhfr;k d ckj e tkuukA

(v) fu/kj.k x.kkd dh x.kuk djukA

3. fo"k; dk iLrqrhdj.k (Presentation of Contents) :

3.1. lg&lEcU/ dh ifjHkk"kk (Definition of Correlation) :

lg&lEcU/ dh oQN e[; ifjHkk"kk, fuEufyf[kr g—

1. foQx oQ 'kCnk e] ^^lg&lEcU/ dk vFk g fd ledekyhvk vFkok rF; legk oQ dkj.k vFkok ifj.kke dk lEcU/ ik;k tkrk gA**

   ("Correlation means that between two series or groups of data then exists some casual connection." W.I. King.)

2. ØkDlVu ,o dkmMu d vulkj] ^^tc lEcU/ vkfdd iÑfr dk gkrk g rk mlu [kktu ,o ekiu rFkk l{e l=k e O;Dr dju dh mfpr lkf[;dh fof/ dk lg&lEcU/ dgr gA**

   ("When the relationship is of quantitative nature, the appropriate statistical tool for discovering and measuring the relationship and expression it in brief formula is known as correlation." Croxton and Cowden)

3. ;k&yu&pkÅ d 'kCnk e] ^^lg lEcU/] fo'y"k.k fofHkUu pjk d lEcU/ dh ek=kk dk eki dgr gA**

(Correlation analysis attempts to determine the degree of relationship between variables.'' Ya-Lun-Chow)

bl h idkj dkuj d 'kCnk e] ^^tc nk ;k vf/d jkf'k;k lgkuHkfr e bl idkj fopj.k djrh g fd ftll ,d e gku okyh ifjoru oQ iQyLo:i nlj h jkf'k e Hkh ifjoru gku dh iofÙk ikb tkrh g] ; jkf'k;k lg&lEcfU/r dgykrh gA**

(If two or more quantities vary in sympathy, so that movements in one tend to the accompanied by the corresponding movement in the others, then they are said to the correlated.''—Conner)

bl idkj mijkDr ifjHkk"kkvk l Li"V g fd fdUgh nk led Jf.k;k e lkFk&lkFk ifjoru gku dh iofÙk dk gh lg&lEcU/ dgr gA

3.2. lg&lEcU/ d idkj (Types of Corrleation) :

lg&lEcU/ e[;r% fuEufyf[kr idkj dk gk ldrk g—

(i) /ukRed vFkok ½.kkRed lg&lEcU/

(ii) ljy] vkf'kd rFkk cgx.kh lg&lEcU/

(iii) j[kh; vFkok vj[kh; lg&lEcU/

(i) /ukRed vFkok ½.kkRed lg&lEcU/ (Positive or Negative Correlation)—

;fn ,d pj dk eY; c<u ij nlj pj dk eY; Hkh c< ;k ,d pj d eY; ?kVu ij nlj pj dk eY; Hkh ?kV rk bl idkj d lg&lEcU/ dk /ukRed lg&lEcU/ dgr gA mnkgj.k—

X	*Y*
10	20
14	22
16	24
20	30
25	32

X	*Y*
15	25
12	20
9	16
7	14
4	11

tc fdlh ,d pj d eY; c<u ij nlj pj dk eY; de gkrk gk rk ml ½.kkRed lg&lEcU/ Hkh dgr gA mnkgj.k—

X	10	20	30	40	50
Y	50	40	30	20	10

fdlh oLr d eY; ,o ekx e ½.kkRed lg&lEcU/ ik;k tkrk gA

(ii) ljy] vkf'kd vFkok cgx.kh lg&lEcU/ (Simple, Partial or Multiple Correlation)—

nk pj eY;k e ;fn lg&lEcU/ Kkr fd;k tk, rk ml ljy lg&lEcU/ dgk tkrk gA vkf'kd lg&lEcU/ e Hkh nk pj eY;k dk lg&lEcU/ Kkr fd;k tkrk g] ijUr ,d vU; Lor=k dj eY; dk leko'k djoQ ,o mldk iHkko fLFkj j[kk tkrk gA nk l vf/d pj eY;k d eè; lglEcU/ cgx.kh lglEcU/ dgykrk gA (ukV % vkioQ ikB;Øe l ek=k ljy lglEcU/ Kkr djuk gA)

(iii) j[kh; vFkok v&j[kh; lglEcU/ (Linear and Non-Linear Correlation)—

;fn nks pjksa d s eè; ifjorZu dk vuqikr fLFkj jgrk rks mls j[kh; lglEcU/ dgrs gSa bls ;fn xzkfQ isij ij vfdr fd;k tk, rks ;g ,d lh/h js[kk dk :i ikIr djxkA ;fn nks pjksa eY;ksa e s ifjorZu dk vuqikr leku jgrk gS bl izdkj d s lglEcU/ dks v&j[kh; vFkok oØj[kh; lglEcU/ dgrs gSA

mnkgj.k %

(i) j[kh; lglEcU/ %

X	3	6	9	12	17	23
Y	4	8	12	16	6	9

(ii) v&j[kh; lglEcU/ %

X	3	5	8	12	23
Y	2	3	4	7	69

3.3. lg&lEcU/ dk egÙo (Significance Correlation) :

O;kogkfjd thou e s o lkf[;dh e s lg&lEcU/ fl¼kUr (Theory of Correlation) dk cgqr egÙo gSA lg&lEcU/ }kjk vkfFkZd rFkk O;kikfjd leL;kvksa dk fo'ys"k.k oKkfud :i ls lEHko gks tkrk gSA blls ;fn ge fdlh ,d pj dk eY; Kkr gks rks nwljs dk vuqikr yxk;k tk ldrk gSA mnkgj.k d s fy, ;fn vkids fuEu led fn, tk,¡ ,o ;s dgk tk, fd ;fn eY; 50 ifr fdyk gks rks ekx Kkr djks rks vki dks

Price per kg	Demand in kg
10	100
20	50
30	35
40	25

mÙkj yxHkx 20 kg d s vkl&ikl gksxk ;g vkiu fdl izdkj Kkr fd;k \ lg&lEcU/ }kjk vr% vkfFkZd o O;kikfjd {k=k e s bldk cgqr vf/d egÙo gSA lkf[;dh e s Hkh izrhixeu (Regression) ,o forj.k&vuqikr (Ratio of Variation) Hkh lg&fl¼kUr ij vk/kfjr gSA blds vfrfjDr thou d s iR;sd {k=k e s bldk egÙo gSA MkDVj Hkh jksx d s ckjs e s tkudkjh ikIr djus d s fy, lg&lEcU/ dk i;ksx djrk gSA

uhLorj (Neiswanger) d s 'kCnksa e s ckr dks vkSj vf/d Li"V djrs gSA mud s vuqlkj] ^^lg&lEcU/ fo'ys"k.k vkfFkZd O;ogkj dks le>us e s ;ksxnku nsrk g] fo'ks"k egÙoi.k pjksa] ftu ij vU; pj fuHkZj djrs g] dh [kkst e s lgk;rk nsrk g] vFkZ'kkL=kh dks mu lEcU/ksa dks Li"V djus g] ftuls xMcMh iSnk gksrh g rFkk ml mu mik;ksa dk l>ko nsrk g ftuds }kjk fLFkjrk ykus okyh] 'kfDr;k iHkkoh gks ldrh gA**

3.4. lglEcU/ Kkr djus dh jhfr;k (Methods of Determining Correlation) :

lglEcU/ Kkr djus dh ie[k jhfr;k fuEufyf[kr g—

3.4.1. fcUnq js[kh; jhfr;k¡ (Graphic Methods) :

3.4.1.1. fo{ksi fp=k ;k fc[kjko fp=k ;k fcUnq fp=k (Scatter Diagram or Scattergram or Dot Diagram)

3.4.1.2. lk/kkj.k fcUnq js[kh; jhfr (Simple Graphic Method)

3.4.2. xf.krh; jhfr;k¡ (Mathematical Methods) :

3.4.2.1. dkyZ fi;lZu dk lglEcU/k x.kkad (Karl Pearson's Coefficient of Correlation)

3.4.2.2. fLi;jeSu dk vuqfLFkfr lglEcU/k x.kkad (Spearman's Rank Coefficient of Correlation)

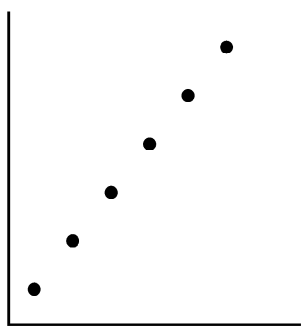
3.4.2.3. laxkeh fopyu x.kkad (Coefficient of Concurrent Deviations)

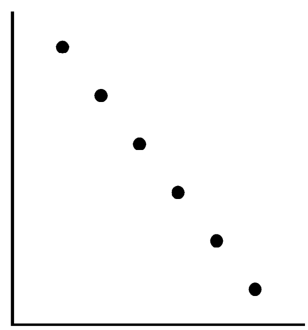
3.4.1. fcUnq js[kh; jhfr;k¡ (Graphic Methods) :

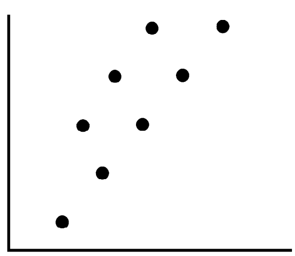
tSlk fd uke ls Li"V gS fd bl fof/k esa lglEcU/k xzkQ }kjk Kkr fd;k tkrk gSA blds oxZ esa eq[;r% nks fof/k;k¡ gSa ftudk o.kZu bl izdkj gS—

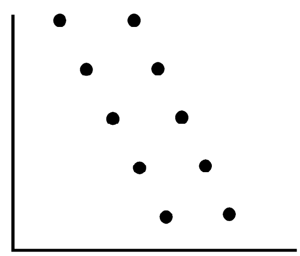
3.4.1.1. fo'k"k fp=k ;k fc[kjko fp=k ;k fcUnq fp=k (Scatter Diagram or Scattergram or Dot Diagram) :

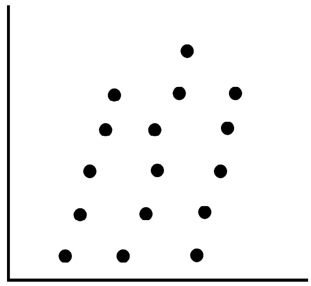
;g ,d ljy ,oa vkd"kZd rjhdk gSA blds fo{ksi fp=k cukus ds fy;s xzkQ isij ij Lora=k pj (independent variable) dks *X*-v{k (*X*-axis) ij rFkk vkfJr pj dks *Y*-v{k (*Y*-axis) ij fn[kk;k tkrk gSA bl izdkj ftrus in&;qXe (Paris of values) gksaxs mrus gh fcUnq vafdr fd;s tk;saxsA ;s fcUnq dkSu dkSu vkdkj cuk;saxs ,oa blh vk/kkj ij gh lg&lEcU/k Kkr fd;k tk,xkA bl fof/k ls ek=k lg&lEcU/k dh fn'kk dk (Direction of Correlation) Kkr fd;k tk ldrk gSA blls lg&lEcU/k dk vuqeku fuEu <x ls yxk;k tkrk gS—


r = + 1

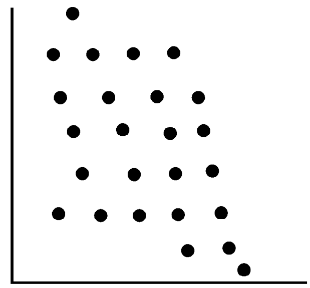

r = – 1


Positive Correlation
(High Degree)


Negative Correlation
(High Degree)

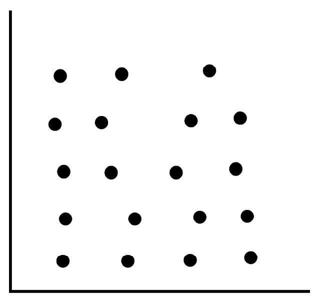


Positive Correlation
(Low Degree)



Negative Correlation
(Low Degree)

bl fof/ dk e[; nk"k ;g g fd ;g lg&lEcU/ dh ek=kk (Degree of Correlation) dk vPNh rjg Kkr ugh dj ldrhA



Absence of Correlation

3.4.1.2. lk/kj.k fcUnq js[kh; jhfr (Simple Graphic Method) :

bl jhfr l Hkh led (data) dk xkiQ ij O;Dr fd;k tkrk gA ij vc *X*-v{k (*X*-axis) ij Øe l[;k] de [kp@le;] LFkku vkfn dk fy;k tk;xkA nkuk pjk (Variables) dk *Y*-v{k (*Y*-axis) ij fy;k tk;xkA bl idkj nk oØk dk fuek.k gkxkA vc bu oØk d vk/kj ij gh ; n[kk tk;xk fd fdl idkj dk lglEcU/ nkuk Jf.k;k e ik;k tkrk gA bu lEcU/ e fuEufyf[kr oQN fu;e g—

(a) ;fn nkuk oØ ,d gh fn'kk e c<r ;k ?kVr gk rk /ukRed lglEcU/ gkxkA (Positive Correlation)

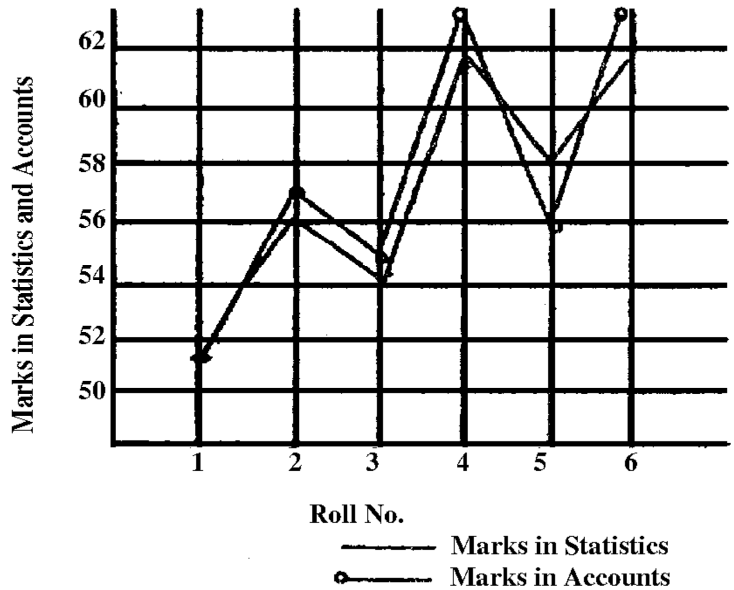
(b) ;fn nkuk Jf.k;k oQ oØ foijhr fn'kkvk e pyr gk rk ½.kkRed lglEcU/ ik;k tk,xkA (Negative Correlation)

(c) ;fn nkuk oØk d ckj e Li"V #i l oQN ugh dgk tk ldrk fd ,d gh fn'kk e tk jg g ;k foijhr fn'kk e rk o ekuk tk;xk fd dkb lg lEcU/ ugh ik;k tkrkA (Absence of Correlation)A

Example : From the following data find our correlation using graphic method.

Roll No.	1	2	3	4	5	6
Marks in Statistics	52	56	54	60	58	62
Marks in Accounts	51	57	55	62	56	62

Solution. igyk xkiQ cuk;k tk;xk



bl xkiQ ls Li"V gS fd nksuks oØ ,d lkFk ,d gh fn'kk dh vksj py jgs gSA vr% (Positive Correlation) ik;k x;k gSA

3.4.2. xf.krh; jhfr;k (Mathematical Methods) :

fcUnq js[kh; jhfr;ksa ls ge ;g rks Kkr gks tkrk gS fd lglEcU/k /kukRed gS vFkok ½.kkRed ijUrq ;g Kkr djuk fd ;g fdruk /kukRed vFkok ½.kkRed gS] dfBu gSA xf.krh; jhfr;ksa }kjk ;g Kkr fd;k tk ldrk gSA

(Thus mathematical methods are able to tell us both the degree and direction of correlation)

fuEufyf[kr jhfr;k xf.krh; jhfr;ksa gSa ,oa buesa ls lcls igyh dkyZfi;lZu jhfr dk o.kZu fd;k x;k gSA 'ks"k jhfr;ksa dk o.kZu vxys ikB esa fd;k x;k gS ftlls correlation-II dk uke fn;k x;k gSA

(i) Karl Pearson's Coefficient of Correlation

(ii) Spearman's Rank Coefficient of Correlation

(iii) Coefficient of Concurrent Deviations

3.4.2.1. dkyZ fi;lZu dk lglEcU/k x.kkad (Karl Pearson's Coefficient of Correlation) :

dkyZ fi;lZu fof/k }kjk lglEcU/k dh x.kuk bl izdkj dh tk,xh—

(a) O;fDrxr J.kh (Individual Series)

(b) iR;{k fof/k (Direct Method)A

bl fof/k esa nksuks pjksa dk lglEcU/k lekUrj ekè; (Arithmetic Mean) ls fy, x, fopyuksa (deviations) dk iz;ksx djoQ Kkr fd;k tkrk gSA blesa fuEufyf[kr esa ls fdlh Hkh l=k dk iz;ksx fd;k tk ldrk gS—

$$r = \frac{\text{Co-variance of } X \text{ and } Y}{\sigma_x \sigma_y}$$

$$\left(\text{Co-variance of } X \text{ and } Y = \frac{\Sigma xy}{N} \right)$$

rk

$$r = \frac{\Sigma xy}{N\sigma_x \sigma_y}$$

(x and y dk formula fy[ku ij)

$$r = \frac{\Sigma xy}{N\sqrt{\frac{\Sigma x^2}{N}}\sqrt{\frac{\Sigma y^2}{N}}}$$

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 - \Sigma y^2}}$$

vr% bue lfdlh Hkh l=k dk i;kx fd;k tk ldrk gA

Example : Co-variance between *X* and *Y* is 6 and Standard Deviation of *X* is 3 and of *Y* is 2. Find Correlation.

Solution.

$$r = \frac{\text{Co-variance of } X \text{ and}}{\sigma_x \sigma_y}$$

$$r = \frac{6}{3 \times 2} = 1$$

$r = 1.$ **Ans.**

Example : Given : Number of pairs of observations of *X* and *Y* series = 15

$$\bar{x} = 25, \sigma_x = 3.01$$

$$\bar{y} = 18, \sigma_y = 3.03$$

Summation of products of corresponding deviations of *X* and *Y* series = + 122. Calculate the Co-efficient of correlation between *X* and *Y* series.

$$r = \frac{\Sigma dxdy}{N\sigma_x \sigma_y}$$

$$= \frac{122}{15 \times 301 \times 303}$$

$$= \frac{122}{136.81}$$

$= +.89.$ **Ans.**

Example : From the following data and Correlation.

Solution.

X	2	4	6	8	10	12	14
Y	3	6	9	12	15	18	21

$X - \bar{X}$ $\quad$ $Y - \bar{Y}$

X	x	x^2	Y	y	y^2	xy
4	– 4	16	6	– 6	36	24
6	– 2	4	9	– 3	9	6
8	0	0	12	0	0	0
10	+ 2	4	15	+ 3	9	6
12	+ 4	16	18	+ 6	36	24
14	+ 6	36	21	+ 9	81	54
$Ex = 56$	0	112	84	0	252	168
$N = 7$		Σx^2	Σy		Σy^2	Σxy

lcl igy X and Y Kkr fd;k tk,xkA

$$X - \bar{X} = \frac{\Sigma X}{N}$$

$$\Sigma X = 56$$

$$N = 7$$

$$\bar{X} = \frac{56}{7} = 8$$

$$\bar{Y} = \frac{\Sigma Y}{N}, \ \Sigma Y = 84, N = 7$$

$$= \frac{84}{7} = 12$$

blds ckn X ,o Y ls fopyu (deviations) Kkr fd;k tk,xk ftlls x ,o y dk eY; Kkr gks tk,xkA blds ckn Σx^2, Σy^2 vkj Σxy dk eku Kkr fd;k tk;xkA

blds ckn fuEu l=k dk i;kx djo lglEcU/ Kkr fd;k tk;xkA

$$r = \frac{\Sigma xy}{\sqrt{(\Sigma x^2 \cdot \Sigma y^2)}}$$

$$= \frac{168}{\sqrt{112 \cdot 252}}$$

$$= \frac{168}{168}$$

$$r = +1. \quad \textbf{Ans.}$$

Example : Find Karl Pearson's Coefficient of Correlation. Take deviation from the actual means 52 and44 respectively.

X	44	46	46	48	52	54	54	56	60	60
Y	36	40	42	40	?	44	46	48	50	52

Solution. bl i'u e lcl igy *Y* J.kh dk ,d vKkr eY; Kkr djuk gkxkA

We know $\overline{Y} = \frac{\Sigma Y}{N}$

$$\Sigma y = 36 + 40 + 42 + 40 + x + 44 + 46 + 48 + 50 + 52 = 398 + x$$

$$\overline{Y} = 44 \text{ (given)}$$

$$N = 10$$

$$\overline{Y} = \frac{\Sigma Y}{N}$$

$$44 = \frac{398 + x}{N}$$

$$440 = 398 + x$$

$$440 - 398 = x$$

$$42 = x$$

vc 'k"k i'u igy tlk fd;k tk,xkA

X	$X - \overline{X}$	X^2	Y	$Y - \overline{Y}$	Y^2	xy
44	– 8	64	36	8	64	+ 64
46	– 6	36	40	– 4	16	+ 24
46	– 6	36	42	– 2	4	+ 12
48	– 4	16	40	– 4	16	+ 16
52	0	0	42	– 2	4	0
54	+ 2	4	44	0	0	0
54	+ 2	4	46	+ 2	4	+ 4
56	+ 4	16	48	+ 4	16	+ 16
60	+ 8	64	50	+ 6	36	+ 48
60	+ 8	64	52	+ 8	64	+ 64
	0	304		0	224	248
	Σx	ΣX^2		Σy	ΣY^2	Σxy

$$-r = \frac{\Sigma xy}{\sqrt{(\Sigma x^2 \cdot \Sigma y^2)}}$$

$$= \frac{248}{\sqrt{304 \times 224}}$$

$$= .95$$ **Ans.**

y?k fof/ (Short-Cut Method)

tc okLrfod lekUrj ekè; (Actual Arithmetic Mean) fHkUu (fraction) vk, rk iR;{k fof/ dk i;kx djus e dfBukb vkrh gA bl dfBukb l cpu d fy, y?k fof/ dk i;kx fd;k tkrk g ftle fopyu dfYir ekè; dk i;kx djoQ Kkr fd;k tkrk gA bl fof/ l fuEu l=k dk i;kx fd;k tkrk gA

$$r = \frac{N\Sigma dxdy - \Sigma dx \cdot \Sigma dy}{\sqrt{N\Sigma dx^2 - (\Sigma dx)^2}\sqrt{N\Sigma y^2 - (\Sigma dy)^2}}$$

or
$$r = \frac{\Sigma dx \cdot dy - \dfrac{\Sigma dx - \Sigma dy}{N}}{\sqrt{\Sigma dx^2 - \dfrac{(\Sigma dx)^2}{N}}\sqrt{\Sigma dy^2 - \dfrac{(\Sigma dy)^2}{N}}}$$

bu l=kk e $\Sigma dxdy$ = Sum of the products of deviations of X and Y series when deviations are taken from assumed mean

$\Sigma dx/\Sigma dy$ = Sum of the deviations of X/Y Series from its assumed mean

$\Sigma dx^2/\Sigma dy^2$ = Sum of the squares of the deviations of X/Y series from its assumed mean

N = Number of Pairs

Example : Calculate the coefficient of Correlation between the values of X and Y given below.

X	78	89	97	69	59	79	68	61
Y	125	137	156	112	107	136	123	108

J.kh X			J.kh Y			fopyu
X	$dx/78$	dx^2	Y	$dy/123$	dy^2	$dxdy$
78	0	0	125	+2	4	0
89	+11	121	137	+14	196	154
97	+19	361	156	+33	1089	627
69	–9	81	112	–11	121	99
59	–19	361	107	–16	256	301
79	+1	1	136	+13	169	13
68	–10	100	123	0	0	0
61	–17	289	103	–15	225	225
$N = 8$	–24	1314	8	20	2060	1452
	Σdx	Σdx^2	N	Σdy	Σdy^2	$\Sigma dxdy$

ukV % fopyu acrual mean d vkl&ikl yu l x.kuk de gkxhA

$$r = \frac{N\Sigma dxdy - \Sigma dx.\Sigma dy}{\sqrt{N\Sigma dx^2 - (\Sigma dx)^2}\sqrt{Nd\Sigma y^2 - (\Sigma dy)^2}}$$

$$= \frac{8\times1452 - (-\ 24).(20)}{\sqrt{8\times1314 - (-\ 24)^2}\sqrt{8\times2060 - (20)^2}}$$

$$= \frac{11616 + 480}{\sqrt{10512 - 576}\sqrt{16480 - 400}}$$

$$= \frac{12096}{\sqrt{9936}\sqrt{16080}}$$

$r = +\ 0.956.$ **Ans.**

Example : From the following data calculate. Coefficient of Correlation bewteen age and playing habit.

Age group	No. of Employees	No. of Regular Players
20 – 30	25	10
30 – 40	60	30
40 – 50	40	12
50 – 60	20	2
60 – 70	20	1

Solution. ukV % bl i'u e ge vk; ,o depkfj;k dh [kyu dh vknr dk lglEcUèk Kkr djuk gA vk; dk X ekuk tk,xk ,o vk; dk vkl r (mid-value) fy[kk tk,xkA bl idkj X dk eY; Øe'k% 25, 35, 45, 55 rFkk 65 vk;xkA [kyu dh vknr dk Y ekuk tk,xk ,o bl Kkr djuk gkxkA bl fdlh lkekU; l[;k d vk/kj ij Kkr fd;k tk,xkA bl i'u e ge ifr 100 ekuxA ;bl idkj Kkr fd;k tk,xkA

No. of Employees	No. of Regular Players	% of Regular Players
25	10	10/25 × 100 = 40%
60	30	30/60 × 100 = 50%
40	12	12/40 × 100 = 30%
20	2	2/20 × 100 = 10%
20	1	1/20 × 100 = 5%

vr% fuEu dk lglEcU/ Kkr fd;k tk,A

Hint :

X	Y	x esa 45 , oa y esa 30 dks
25	40	assumed mean ekuus i j
35	50	$\Sigma dx = 0$
45	30	$\Sigma dx = -15$
55	10	$\Sigma dx^2 = 1000$
65	5	$\Sigma dy^2 = 1525$
		$\Sigma dxdy = -1100$
		$N = 5$
		Ans. 904

'k"k viu vki djA

'k"k vxy ikB e crk;k x;k gA

O;fDrxr J.kh dh rjg oxhÑr J.kh e Hkh lglEcU/ Kkr fd;k tk ldrk gA bl idkj d i'u e iR;d ox dh vkofÙk (frequency) dk lEcU/ nkuk pjk (variables) l gkrk gA

bl idkj d i'uk e fuEufyf[kr l=k dk i;kx fd;k tkrk g—

$$r = \frac{N\Sigma fdxdy - \Sigma fdx \cdot \Sigma fdy}{\sqrt{N\Sigma fdx^2 - (\Sigma fdx)^2}\ \sqrt{N\Sigma fdy^2 - (\Sigma fdy)^2}}$$

$$= \frac{N\Sigma fdxdy - \dfrac{\Sigma fdx \cdot \Sigma fdy}{N}}{\sqrt{\Sigma fdx^2 - \dfrac{(\Sigma fdx)^2}{N}}\ \sqrt{\Sigma fdy^2 - \dfrac{(\Sigma fdy)^2}{N}}}$$

oxhÑr J.kh e rkfydk cuku dh fof/ fuEufyf[kr g—

(i) X o Y Jf.k;k d eè; fcUn Kkr djk o dfYir ekè; ydj fopyu Kkr djk ftUg dx o dy }kjk fn[kkvkA

(ii) dx o dy l Fdx o Fdy Kkr djk fiQj tkMdj ΣFdx o ΣFdy Kkr djkA

(iii) blh idkj ΣFdx^2 o ΣFdy^2 Kkr djkA

(iv) fiQj iR;d dk"B e ftle F dh value nh gk $Fdxdy$ Kkr djkA budk ;kx dju l $\Sigma Fdxdy$ Kkr djkA

(v) bloQ ckn l=k dk i;kx djoQ lg&lEcU/ Kkr fd;k tk ldrk gA

fuEu i'u e ;g fof/ Li"V gkrh g—

Example : Calculate the coefficient of Correlation between the ages of 100 mothers and daughters.

Age of Daughters (*X*)

Age of Mothers	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	Total
15 – 25	6	3	—	—	—	9
25 – 35	3	16	10	—	—	29
35 – 45	—	10	15	7	—	32
45 – 55	—	—	7	10	4	21
56 – 65	—	—	—	4	5	9
Total	9	29	32	21	9	100

Solution.

		x	5 – 10	10 – 15	15 – 20	20 – 25	25 – 20			
		mv	7.5	12.5	17.5	22.5	27.5			
Y	*Mv*	*dy* / *dx*	– 2	– 1	0	+ 1	+ 2	*fdy*	*fdy*^{2}	*fdxfdy*
15 – 25	20	– 2	24 6	6 3	—	—	—	18	36	30
25 – 25	30	– 1	6 3	16 16	0 10	—	—	– 29	29	22
35 – 45	40	+ 0	—	0 10	0 15	0 7	—	0	0	0
45 – 55	50	+ 1	—	—	0 7	10 10	8 4	21	21	18
55 – 65	60	+ 2	—	—	—	3 4	20 5	18	36	28
Total			9	29	32	21	9	8	122	98

		fdx	– 18	– 29	0	– 21	+ 18	Σ*fy* Σ*dx*	Σ*dy*^{2}	Σ*dxdy*
		fdx *fdxdy*	36 30	29 22	0 0	21 18	36 28	Σ*dx*^{2}		

$$r = \frac{N.\Sigma fdxdy - \Sigma fdx.\Sigma fdy}{\sqrt{N.\Sigma fdx^2 - (\Sigma fdx)^2}\ \sqrt{N\Sigma fdy^2 - (\Sigma fdy)^2}}$$

$$= \frac{100 \times 93 - (-8)\ (-8)}{\sqrt{100 \times 122 - (-8)}\ \sqrt{100 \times 122 - (-8)}}$$

$$= \frac{9800 - 64}{\sqrt{12200 - 64\sqrt{12200 - 64}}}$$

$$= \frac{9736}{\sqrt{12136 - \sqrt{12136}}}$$

$$r = \frac{9736}{12136} = 0.302$$

$r = +0.302$ **Ans.**

fLi;jeu dk vuifLFkfr lglEcU/ x.kkd (Spearman's Rank Coefficient of Correlation) : bl jhfr dk ifriknu ik- fLi;jeu u fd;kA bl fof/ dk i;kx ml le; vfèkd mi;Dr ekuk tkrk gA tc rF;k dk l[;kRed (Quantitative) eki lHko u gk ,o mUg Øe (rank) d vuljk gh j[kk tk ldrk gkA mnkgj.k d fy, lUnjrk dk vdk d LFkku ij Øe nuk vf/d mi;Dr jgxkA blh idkj cf¼erk dk Hkh fuf'pr Øe tl iFke] f}rh; bR;kfn nuk vfèkd mi;Dr djxkA

bl fof/ l lg&lEcU/ Kkr dju d fy, fuEu fØ;k dh tk,xh—

(i) lcl igy Øe (rank) Kkr fd, tk;xA (ukV % ;fn i'u l igy l gh Øe fn;k x;k gk fiQj Kkr dju dh vko';drk ugh gkxhA) nkuk Jf.k;k e lcl vf/d vkdkj oky eY; dk 1 mll de oky dk 2 ,o mll Hkh de oky eY; dk 3 Øe fn;k tk,xkA ;g dk; blh idkj fd;k tk;xkA

mnkgj.k % fuEu ink dk Øe inku djkA

X	10	7	6	12	8	16	3

gy % bll lcl cMk vd 16 gA vr% bl Øe 1 fn;k tk,xkA mud ckn vd 12 vkrk gA bl Øe 2 fn;k tk,xk blh dk; dk ckj&ckj fd;k tk,xkA

X	Rank
10	3
7	5
6	8
12	2
8	4
16	1
3	7

;fn J.kh e dkb in gk ;k mll vf/d ckj vk tk, rk mudk Øe nu dh fof/ e oQN ifjoru djuk iMrk gA leku vkdkj d ink dk Øe Kkr dju d fy, mudh Øe'k% feyu oky Øek dk vkIr Kkr djuk gkxk tk fd fuEu mnkgj.k l Li"V gA

mnkgj.k % fuEu ink dk Øe inku djkA

X	3	10	7	3	2	9	7	6	1

gy % ble iFke LFkku 10 vd dk ikIr djxk] D;kfd og lcl vf/d gA Øe l[;k 2 blo Q ckn 7 dk ikIr gkxkA bld ckn 7 nk ckj vk jgk gA vr% bldk Øe l[;k 3 o 4 dk vkIr fn;k tk,xkA

$$\frac{3+4}{2}=3.5$$

bl idkj bu nkuk dk 3, 5 Øe ikIr gkxkA bld ckn 6 dk Øe l[;k 5 nh tk,xh u fd 4 D;kfd 6 l vf/d 4 vd (10, 9 o 2 ckj 7) gA blh dk ckj&ckj fd;k tk,xk o Øe bl idkj gkxkA

X	3	10	7	3	2	9	7	6	1
Rank	6.5	1	3.5	6.5	8	2	3.5	5	9

mnkgj.k % fuEu ink dk Øe inku djkA

X	10	6	7	6	5	6	4	3	4	12

gy %

X	Rank
10	2
6*	5
7	3
6*	5
5**	7
6*	5
4***	8.5
3	10
4***	8.5
12	1

$$* \frac{4+5+6}{3} = 5$$

**bloQ ckn 5 dk Øe l[;k 7 nh tk,xh] D;kfd 5 l vf/d 6 vd gA

$$** \frac{8+9}{2} = 8.5$$

(ii) X J.kh ,o Y J.kh oQ Øek dk ?kVkdj ØekUrjk d vUrj (Rank Differences) Kkr fd, tkr g ,o D }kjk 0;Dr fd;k tkrk gA

ukV % Σd lno 'kU; gkxk vU;Fkk vc rd d dk; e dkb xyrh gA

(iii) D dk ox dj D^2 Kkr dj ,o tkM n bll ΣD^2 Kkr gk tk,xkA

(iv) fiQj fuEu l=k (formula) dk i;kx fd;k tk,xk ,o lglEcU/ Kkr fd;k tk,xkA

$$rk = 1 - \frac{6\Sigma d^2}{N^3 - N} \quad ;k \quad rk = 1 - \frac{6\Sigma d^2}{N(N^2-1)}$$

rk = Rank Correlation Coefficient

ΣD^2 = Total of squares of Rank Differences

N = Number of pairs of items

vc ,d ljy i'u fd;k tk,xk ftl e eY; lc/h dkb dfBukb ugh gA

Example : Calculate the Coefficient of Correlation from the following data using the method of rank differences.

X	75	88	96	70	60	80	81	49
Y	120	134	160	115	110	140	142	90

Solution.

X	Rank of X (Rx)	Y	Rank of Y (Ry)	Rank difference $Rx - Ry(d)$	Squares of difference (d^2)
75	5	120	5	0	0
88	2	134	4	– 2	4
96	1	160	1	0	0
70	6	115	6	0	0
60	7	110	7	0	0
80	4	140	3	1	1
81	3	142	2	1	1
49	5	90	3	0	0
				0	$\Sigma d^2 = 6$

$$rk = 1 - \frac{6\Sigma d^2}{N^3 - N}$$

$$rk = 1 - \frac{6 \times 6}{6^3 - 6}$$

$$rk = 1 - \frac{36}{504}$$

$$rk = 1 - 0.07$$

$$\boxed{rk = .93 \quad \textbf{Ans.}}$$

lekU eY; lc/h dfBukb% tc Hkh nk ;k nk l vf/d eY; lekU vkr gk rk Øe (rank) Kkr dju e rk dfBukb vkrh gh g] ijUr lkFk&lkFk lg&lEcU/ Kkr dju d l=k e Hkh l'kk/u djuk iMrk g tkfd fuEufyf[kr g—

$$rk = \frac{6\left[6\Sigma D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + \ldots\right]}{N^3 - N}$$

og m mu in eY;k dh l[;k g ftud Øe lekU g ,o $1/12\,(m^3 - m)$ mruh ckj vk,xk ftruh ckj ij eY;k dh l[;k cjkcj jghA

; fuEu mnkgj.k l Li"V g—

Example : Following are the marks obtained by the studetns in Accountancy and Statistics.

Calculate Coefficient of Correlation between them by Rank Method.

Accountancy	45	56	39	54	45	40	56	70	30	36
Statistics	40	36	30	44	36	32	75	42	20	36

Solution.

Marks in Accountancy X	Rank Rx	Marks in Statistics Y	Rank Ry	D	D^2
45	5.5	40	4	+ 1.5	2.25
56	2.5	36	6	3.5	12.25
39	8	30	9	– 1	1.00
54	4	44	– 2	+ 2	4.00
45	5.5	36	6	– 0.5	50.25
40	7	32	8	– 1	1.00
56	2.5	75	1	+ 1.5	.225
70	1	42	3	– 2	4.00
30	10	20	10	0	0.00
36	9		6	+ 3	9.00
				$\Sigma D = 0$	$\Sigma D^2 = 36.00$

nkuk Jf.k;k e oQN ,d leku ink dh iujkofÙk gb gA vr% l=k e $1/12(m^3 - m)$ dk lek;ktu fd;k tk,xk nkuk Jf.k;k e l iujkofÙk 3 ckj (2 ckj X e ,o ,d ckj Y e) gb g rk ; lek;ktu 3 dc fd;k tk,xkA

$$rk = 1 - \frac{6\left[\Sigma D + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m)\right]}{N^3 - N}$$

$$rk = 1 - \frac{6\left[36 + \frac{1}{12}(2^3 - 2) + \frac{1}{12}(2^3 - 2) + \frac{1}{12}(3^3 - 3)\right]}{10^3 - 10}$$

$$= 1 - \frac{6\left[36 + \frac{1}{2} + \frac{1}{2} + 2\right]}{1000 - 10}$$

$$= 1 - \frac{6 \times 39}{990}$$

$$= 1 - \frac{234}{990}$$

$= 1 - 234/990$

$= 1 - 0.236$

$= 0./764.$ **Ans.**

3.4.2.3. lxkeh fopyu jhfr (Concurrent Deviation Method) :

bl jhfr }kjk lglEcU/ dh fn'kk ,o ek=k nkuk dk Kku gkrk g] ijUr bldk i;kx vfèkdrj mlh le; fd;k tkrk g tc ek=kk dh vud fn'kk dk vf/d egRoi.k ekuk x;k gkA bl jhfr e fuEu x.ku fd;k i;kx e ykb tk,xhA

(i) iR;d J.kh e vxy in&lph dh mlls fcYoqy io (Previous) d in eY; ls ryuk dh tk,xhA ;fn eY; de g rk (–) vf/d g rk (+) ,o ;fn cjkcj gk rk (= ;k 0) dk i ;kx fd;k tk,xkA

bl fØ;k e ,d ckr dk è;ku jg fd fopyuk (Deviations) dh l[;k oQy in&;Xek e ,d de gkxhA

(ii) X ,o Y Jf.k;k d fopyu fpUgk d vk/kj ij lxkeh iofÙk Kkr dh tkrh gA ftu in ;Xek e ,d lkFk of¼] deh ;k cjkcjh jgh g] mld lkeu (+) dk fpUg yxk;k tkrk gA

(iii) oQy (+) d fpUgk dk ;kx fd;k tkrk g ,o ml c }kjk 0;Dr fd;k tkrk gA

(iv) lglEcU/ Kkr dju d fy, fuEu l=k dk i;kx fd;k tkrk g—

$$\gamma_c^+ = \pm\sqrt{\frac{\pm(2C-N)}{N}}$$

γ_c = Coefficient of Concurrent Deviations

C = Number of Concurrent Deviations

N = Number of Pairs of Deviations

Example : From the following data calculate Coefficient of Correlation using Concurrent Deviation.

X	90	96	80	85	15	72	80	90	105
Y	60	65	66	72	72	72	80	30	70

Solution.

X	Direction of change in *X* (*Dx*)	*Y*	Direction of Change in *Y* (*Dy*)	*Dxdy*
90		60		
96	+	65	+	+
80	–	66	+	–
85	+	72	+	+
85	=	72	=	+
72	–	72	=	–
80	+	80	+	+
90	+	30	–	–
105	+	70	+	+

$$yc = \pm\sqrt{\frac{\pm(2c-n)}{y}}$$

$$= \pm\sqrt{\frac{\pm(2\times 5\times 8)}{3}}$$

$$= \pm\sqrt{\frac{\pm 2}{8}}$$

$$= \pm\sqrt{\pm 25}$$

$= + 5.$ **Ans.**

3.5. fu/kj.k x.kkd (Coefficients of Determination) :

lglEcU/ x.kkd dk oxZ fu/kj.k x.kkd dgykrk gA blls fopyu x.kkd Hkh dgrs gA bl idkj Coefficient of Determination = r^2.

fu/kj.k x.kkd ls ;g Kkr fd;k tkrk g fd fdlh ,d pj (variable) ds eY; ls gksus okys ifjorZu e fdruk ;kxnku nlj pj ds ifjorZu n jg gA mnkgj.k d fy, mit (*Y*) ,o ;gk (*X*) e lglEcU/ dh ek=kk + 0.8 ikbZ xbZ rk fu/kj.k x.kkd 0.64 gkxkA ($r^2 = (0.8)^2 - 0.60$)A bldk vFk ;g gvk fd mit e 64 ;k 64% ifjorZu o"kkZ d dkj.k o 'k"k 36% ifjorZu vU; dkj.kk l g,A

vr% ge ;g i< ldr g fd fdlh Hkh pj e gksu okys ifjorZu dk nks Hkkxk e ckVk tk ldrk g—

(i) Li"Vhdj.k ilj.k **(Explained Variance)**— ;s ,lk ifjorZu g tks fd nljk pj Li"V dj jgk g] tlS fd mnkgj.k e crk;k x;k g fd 14% o"kkZ Li"V dj jgh gA

(ii) vLi"Vhdj.k ilj.k **(Unexplained Variance)**—; ok ifjorZu g tks nljk pj (Variable) Li"V ugh dj jgkA tlk fd mnkgj.k l Li"V g fd ifjorZu mit e vU; dkjdk l g,A

Li"Vhdj.k ilkj.k dk eku gk fu/kj.k x.kkd l fd;k tkrk gA

3.6. lglEcU/ ,o dk;&dkj.k lEc/ (Correlation and Causation)

lglEcU/ ;k dk;&dkj.k lEcU/ e oQN crkus ls igys nkuk 'kCnk dk vFk vkuk vko';d lg&lEcU/ d ckj e crk;k tk pdk g] dk;&dkj.k dk vFk g fd ,d pj dkj.k gk ,o nljk pj mnkgj.k d fy, vPNh cjlkr gkuk dkj.k g ,o vPNh mit gkuk iHkkoA

lg&lEcU/ rk ek=k nk ;k vf/d pjk d eè; ik, tku oky lEcU/ dh ek=kk dk crkrk g dk;&dkj.k dh vkj dkbZ lo™r ugh djrkA ;fn lglEcU/ dk eY; vf/d vk tk, rk dHkh ugh ekud fd mue dk;&dkj.k lEcU/ Hkh vf/d gkxk] og rk vf/d gk ldrk gA mnkgj.k d fuEu leadk dk n[kk—

Table : (Not the actual data)

Year	1982	1982	1983	1984	1985	1986
No. of Automobiles	70	100	140	200	400	650
No. of Accidents in Japan	35	49	72	301	251	330

ledk dk n[kdj dgk tk ldrk g fd Hkkjr e Automobiles dh l[;k crk jgh g ,o ;g Accidents dh l[;k Hkh c< jgh g ,o ;fn bldk lglEcU/ Kkr fd;k tk, rk vkl&ikl

vr% nkuk dk vkil e yxHkx io /ukRed lglEcU/ ik;k x;k] ijUr bl e ;g vFk ugh fy;k Fkk fd bue dk;&dkj.k lEcU/ Hkh gkA Hkyk Hkkjr e Automobiles d c<u ij Japan Accidents fd;kA vr% bue vkil e dkb dk;&dkj.k lEcU/ ugh gA

db ckj ,lk n[kk x;k g fd dk;&dkj.k lEcU/ u gkr g, Hkh lglEcU/ ik;k x;k gA bld dkj.k bl idkj g—

(i) lglEcU/ dk ek=k l;kxo'k gkukA

(ii) lglEcU/ dk fuj.kd gkukA

(iii) nkuk pj ijLij fdlh vU; pj l ifrfØ;k dj jg gkA

vr% Correlation does not mean causation it may mean causation.

4. lkjk'k (Summary) :

lkf[;dh e lg&lEcU/ dk fl¼kUr cgr gh egÙoi.k gA tc ,d pj e ifjoru gku ij mlh fn'kk e dk foijhr fn'kk e ifjoru gk rk ;g ekuk tkrk g fd nkuk pjk e lg&lEcU/ gA nk fn'kk e ifjoru gku ij lglEcU/ /ukRed gkrk g vkj foijhr fn'kk e ifjoru gku ij ;g ½.k gkrk gA lg&lEcU/ e[;r;k rhu idkj dk gkrk g—(i) /ukRed vFkok ½.kkRed] (ii) ljy] vkf'kd rFkk cgx.kh lglEcU/] (iii) j[kh; rFkk vj[kh; lglEcU/A 0;kogkfjd thou e o lkf[;dh l ;g lEcU/ fl¼kUr dk cgr egRo g] bld }kjk vkfFkd rFkk 0;kikfjd leL;kvk dk fo'y"k.k oKkfud :i l fd;k tkrk gA lg&lEcU/ dk Kkr dju d fy, e[;r;k fuEu fof/;k dk i;kx fd;k tkrk gA (i) fo{ki fp=k fof/] (ii) lk/kj.k fcUn j[kh; jhfr] (iii) dky fi;jlu dk lglEcU/ x.kkd] (iv) fLi;jeu dk vufLFkfr lglEcU/ x.kkd o (v) lxkeh fu/kj.k x.kkd l ;g Kkr fd;k tkrk g fdlh pj d eY; e gku oky ifjoru e fdruk ;kxnku nlj pj d ifjoru n jg gA lg&lEcU/ dk;&dkj.k dh vkj dkb ldr ugh djrkA

5. iLrkfor iLro (Recommended Books) :

(i) Introduction to Statistics – by Dr. R.P. Hooda

(ii) Statisical Method - By S.P. Gupta

(iii) Business Statistics - by S.C. Sharma, R.C. Jain

(iv) Business Statisics - by Oswal, Aggarwal Sharma

6. vH;kl d fy, i'u %

(1) lg&lEcU/ dh ifjHkk"kk nhft, vkj lkf[;dh fo'y"k.k e mldh egÙkk dk foopu dhft,A

(2) lg&lEcU/ Kkr dju dh fofHkUu fof/;k dh 0;k[;k dhft,A

(3) lg&lEcU/ dh ek=kk o idkj D;k&D;k g \ dky fi;lu dk lg&lEcU/ x.kkd oQl ikIr fd;k tkrk gA

(4) fuEufyf[kr ij fVIi.kh fyf[k,—

(i) lglEcU/ dh ek=kk

(ii) fo{ki fp=k

(iii) fu/kj.k x.kkd

(iv) lEHkk; foHkze

(5) Find the Coefficient of Correlation between the values of *X* and *Y* from the series given below :

X	78	89	97	69	59	79	68	61
Y	125	137	156	112	107	136	123	108

Use 69 as assumed Mean for *X* and 112 for *Y*.

B.Com. Part-II

Paper - BC–204 : Business Statistics *Revised By :* **Dr. Heera Lal Sharma**

Lesson No. : 6

Regression Analysis

l jpuk (Structure)

1. ifjp; (Introduction)
2. mí'; (Objective)
3. fo"k; dk iLrrhdj.k
    - 3.1. irhixeu dk vFk ,o ifjHkk"kk
    - 3.2. irhixeu fo'y"k.k dh mi;kfxrk
    - 3.3. lg&lEcU/ ,o ifrixeu e vUrj
    - 3.4. j[kh; irhixeu
        - 3.4.1. irhixeu j[kk,
        - 3.4.2. irhixeu j[kkvk d dk;
        - 3.4.3. irhixeu j[kkvk dh jpuk dh jhfr;k
        - 3.4.4. irhixeu lehdj.k
    - 3.5. irhixeu x.kkd
        - 3.5.3. irhixeu x.kkdk dk chtxf.krh; eki
        - 3.5.2. irhixeu x.kkdk l lglEcU/ x.kkd dk fu/kj.k
        - 3.5.3. irhixeu x.kkdk d x.k
    - 3.6. vuekku dh iekki =kfV
4. lkjk'k
5. iLrkfor iLroQ
6. vkRecks/ d fy; i'u

1. ifjp; (Introduction) :

lg&lEcU/ dk fl¼kUr nks pj eY;ks e lEcU/k dh ek=kk ,o fn'kk dk crkrk g] ijUr bll ;g Li"V ugh gkrk g fd dkSu&lk pj dkj.k g (cause) g rFkk dkSu&lk pj ifj.kke (effect) gA dkj.k ,o ifj.kke lc/ dk Li"V :i l O;Dr dju d fy; irhixeu fo'y"k.k dk i;kx fd;k tkrk gA ;fn ge ;g tkuu d bPNd gk fd ,d J.kh d fdlh fuf'pr eY; d vkèkkj ij vfJr J.kh d rRloknh eY; dk lokÙke vueku D;k g rk ge irhixeu fo'y"k.k dh lgk;rk yuh gkxhA mnkgj.k d fy, ;fn foKkiu ,o foØ; e lg&lEcU/ LFkfir gk tkrk g rk ge foKkiu dh ,d nh gb fuf'pr ek=kk ij foØ; dk vueku ifrixeu dh lgk;rk l yxk ldr gA

2. mís'; (Objectives)

bl vè;k; dk vè;;u dju d i'pkr vki ;g tku ldr g fd

(i) ,d J.kh d fdlh fuf'pr eY; d vk/kj ij vkfJr J.kh d rRloknh eY; (corresponding value) dk lokÙke vuekfur eY; D;k gkxkA

(ii) vkfFkd] O;kolkf;d o lkekftd {k=k e fofHkUu ?kVukvk d eè; lEcU/k dk fo'y"k.k djukA

(iii) vk/fud le; e irhixeu fo'y"k.k dk mi;kx iokueku ;k vUro"k.k ,o cfgo"k.k d midj.k d :i e djukA

(iv) dkj.k ,o ifj.kke lEcU/ dk Li"V :i l O;Dr djukA

3. fo"k; dk iLrqrhdj.k (Presentation of Contents) :

3.1. irhixeu dk vFk ,o ifjHkk"kk (Meaning and Definition of Regression) :

'kCndk'k d vulkj irhixeu ftl lekJ;.k Hkh dgk tkrk g dk vFk g—ihN gVuk (Act of going back on retuning back) bldk loiFke i;kx dj iQkfll xkYVu (Sir Francis Galton) }kjk viu y[k "Regression towards Mediocrity in Hereditary Stature" e fd;k FkkA viu bl y[k e mUgku Li"V fd;k fd lkekU;r% O;fDrxr Åpkb;k dk >dko vklr Åpkb dh vkj gkrk g] mld vulkj]

(i) yEc firkvk d i=k Hkh yEc gkr gA

(ii) NkV dn d firkvk d i=k Hkh NkV gkr gA

(iii) yEc firkvk d i=kk dh vklr Åpkb mud firkvk dh vklr Åpkb l de gkrh g rFkk

(iv) NkV dn d firkvk d i=kk dh vklr Åpkb mud firkvk dh vklr Åpkb l vfèkd gkrh gA

vr% O;fDrxr Åpkb;k dk >dko vklr Åpkb dh vkj gkrk g ,o bl iofÙk dk gh mUgku irhixeu dk uke fn;kA

ifjHkk"kk, (Definition) :

irhixeu dh e[; ifjHkk"kk, fuEufyf[kr g—

(i) ,e-,e- Cy;j d vuqlkj] ^^ey bdkb;k d :i e nk ;k nk l vf/d pjk d ikjLifjd vkSlr lEcU/k e eki dk ifrixeu dgk tkrk gA**

"Regression is the measure of average relationship between two or more variables in terms of the original units of data."

(ii) g"k d 'kCnk e] ^^irhixeu fo'y"k.k nk ;k nk l vf/d pjk d lEcU/k dh iÑfr o ek=kk dk eki djd Hkfo"; vueku dh {kerk inku djrk gA**

"Regression analysis measures the nature and extent of the relation between two or more variables, thus enables us to make predictions."

(iii) rkjk ;keu d vuqlkj] ^^nk ;k nk l vf/d dk;&dkj.k lEcU/k l lcf/r pjk d eè; lEcU/k Kkr dju d fy, nk jhfr vFk'kkL=k ,o O;kolkf;d 'kk/k l vR;f/d i;Dr dh tkrh g] irhixeu fo'y"k.k dgykrh gA**

One of the frequently used techniques in economics and business research, to find a relation between two or more variables that are related causally is regression analysis." **—Taro Yamane**

3.2. irhixeu fo'y"k.k dh mi;kfxrk (Utility of Regression Analysis) :

O;kolkf;d ,o vkfFkd {k=k e irhixeu fo'y"k.k dk i;kx leLr leL;kvk d lek/kku e fd;k tkrk gA icU/dk }kjk bl rduhd dk i;kx fu;=k.k mi dj.k d :i e fd;k tkrk gA bl rduhd d vk/kj ij Kkr fu"d"k mru gh vf/d fo'oluh; gkx ftruk vf/d lg&lEcU/k nk pjk d chp gkxkA lkf[;dh fo'y"k.k e irhixeu dk vè;;u cgr gh mi;kxh ,o egRoi.k g tk fuEu ckrk l Li"V g—

(i) iokueku (Forecasting) : irhixeu fo'y"k.k }kjk ,d LorU=k pj eY; d vk/kj ij vkfJr pj eY; dk iokueku yxk;k tk ldrk gA

(ii) lEcU/k dh iÑfr (Nature of Relationship) : irhixeu fo'y"k.k nk ;k nk l vfèkd pjk d lEcU/k dh iÑfr dk Li"V djrk gA

(iii) vkfFkd ,o O;kolkf;d vuql/kku e mi;kxh (Useful in Economic and Business Research) : bl rduhd d vk/kj ij vkfFkd] O;kolkf;d o lkekftd {k=k e fofHkUu ?kVukvk d eè; lEcU/k dk fo'y"k.k dj mi;kxh fu.k; fy, tk ldr gA

(iv) lc/ dk vueku (Estimation of Relationship) : irhixeu fo'y"k.k }kjk nk ;k nk l vf/d pjk d ikjLifjd lEcU/k dk eki vkilh l fd;k tk ldrk gA

(v) lglEcU/k dh ek=kk rFkk fn'kk rFkk Kku % bl rduhd d vk/kj ij lglEcU/k dh ek=kk rFkk fn'kk dk vueku yxk;k tk ldrk gA

3.3. lglEcU/k ,o irhixeu e vUrj (Difference between Correlation and Regression) :

lglEcU/k ,o irhixeu e vUrj ie[k :i l fuEuor g—

(i) dkj.k ,o ifj.kke lEcU/k (Cause Effect Relationship) : lg&lEcU/k nk pj eY;k e lno dkj.k&ifj.kke lEcU/k dk O;Dr ugh djrk g ijUr irhixeu nk pj eY;k d dkj.k ifj.kke lEcU/ dk vf/d rFkk Li"V :i ls 0;Dr djrk gA blesa LorU=k pj dkj.k rFkk vkfJr pj dk ifj.kke gksrk gA

(ii) **lEcU/ dh ek=kk ,oa iÑfr (Degree and Nature of Relationship)**— lg&lEcUèk nks ;k nks vf/d pjksa d lg&ifjoZru dh ?kfu"Brk dh tkap djrk g] tcfd ijhixeu fo'y"k.k dk lg&ifjoZru dh iÑfr dk Li"V djrk g rFkk ;g crykrk g fd ,d pj oQ vkSlr eY; oQ rRlaxdkjh nlj pj dk lHkkO; vkSlr eY; D;k gksxkA

(iii) **Hkkoh vuqeku (Prediction)**— ;g lEcU/ ls Hkkoh vuqeku ugha yxk;k tk ldrk g tcfd ifrixeu e pjksa e lEcU/ dh ek=kk ,o iÑfr dk Kku ikdj djoQ Hkkoh vuqeku yxk;k tk ldrk gA

3.4. js[kh; ijhixeu (Linear Regression) :

tc js[kk, ljy ;k lh/h gksrh g rks ijhixeu js[kh; dgykrk gA bu ljy ijhixeu js[kkvksa oQ lehdj.k ,d?kkrh; gksr g (Equations of the first degree)A nks pjksa oQ eY;ksa d eè; js[kh; ijhixeu dk vè;;u ljy js[kh; ifrixeu dgykrk gA rhu ;k rhu ls vf/d pjksa d fo'y"k.k d fy, i;Dr js[kh; ijhixeu dks cgx.kh js[kh; ijhixeu (Multiple Linear Correlation) d uke ls tkuk tkrk gA

3.4.1. ijhixeu js[kk, (Regression Lines) :

nks led Jf.k;ksa d fofHkUu eY;ksa d ikjLifjd vkSlr lEcU/ dks 0;Dr djus okyh loksZi;Dr js[kkvksa dks ijhixeu js[kkvksa d uke ls tkuk tkrk gA ;s ijhixeu js[kk, ,d lfeed J.kh d vkSlr eY;ksa ls lcf/r nljh J.kh d loksZÙke vkSlr eY;ksa dks 0;Dr djrh gA ;fn nks pj *x* rFkk *y* fn;s g, gksa] rks mlls lcf/r nks ijhixeu js[kk, gksrh g tks fuEuor g—

(i) ***X* on *Y* dh ijhixeu js[kk (Regression Line of *X* on *Y*)**—;g ijhixeu js[kk *Y* oQ fn;s g, eY;ksa d vk/kj ij *X* d lHkkfor eY;ksa dk vuqeku yxkrh gA

(ii) ***Y* on *X* dh ijhixeu js[kk (Regression of *Y* on *X*)**—;g ifrxeu js[kk *X* d fn;s g, eY;ksa d vk/kj ij *Y* d lHkkfor eY;ksa dk vuqeku yxkrh gA

3.4.2. ijhixeu js[kkvksa d dk;Z (Functions of Regression Lines) :

ijhixeu js[kkvksa d nks egÙoi.kZ dk;Z g tks fuEufyf[kr g—

(i) **loksZi;Dr vuqeku yxkuk (Best Estimate)**— bu js[kkvksa dh lgk;rk ls ,d J.kh d fn;s g, eY; d vk/kj ij nljh J.kh d rRlaoknh vkSlr dk eY; dk vuqeku yxk;k tk ldrk gA

(ii) **lg&lEcU/ dh ek=kk ,oa fn'kk dk Kku (Knowledge of Extract and Nature of Correlation)**— bu js[kkvksa }kjk nks Jf.k;ksa e lg&lEcU/ Hkh fuf'pr fd;k tk ldrk g ,o fn'kk dk vuqeku yxk;k tk ldrk gA

3.4.3. ijhixeu js[kkvksa dh jpuk dh jhfr;k (Methods of Drawing Regression Lines) :

ijhixeu js[kkvksa dh jpuk dh nks ie[k fof/;k g—

(i) fo{ki fp=k fof/ (Scatter Diagram Method)

(ii) U;ure oxZ fof/ (Least Square Method)

(i) fo{ki fp=k fof/ (Scatter Diagram Method)— bl jhfr d vUrxZr fo{ki fp=k e vfdr fofHkUu fcUnvk d eè; l ,d j[kk bl idkj [khph tkrh g fd yxHkx vk/ fcUn bl j[kk d Åij rFkk vk/ fcUn bl j[kk d uhp jg tk;A bl fof/ dk O;ogkj e cgr de iz;kx fd;k tkrk g D;kfd bl fof/ }kjk ikIr irhixeu j[kk ij [khpu okyk d fu.kZ; dk cgr vfèkd iHkko iMrk gA

(ii) U;ure oxZ fof/ (Least Square Method)— ifrixeu j[kkvk dh jpuk U;ure oxZ fof/ }kjk Hkh dh tk ldrh gA bl jhfr d vUrxZr X rFkk Y pj d vfdr eY;k d chp l xtjrh gbZ ,d ljy j[kk bl idkj [khph tkrh g fd bl j[kk l vfdr eY;k d fopyuk d oxk dk ;kx U;ure gksxkA

3.4.4. irhixeu lehdj.k (Regression Equations) :

irhixeu lehdj.k irhixeu j[kkvk dk gh chtxf.krh; Lo:i gA irhixeu lehdj.k nk led ekykvk d lekUrj ekè;k d lEcU/ e ,d J.kh e mld ekè; l fopj.k rFkk nljh J.kh d ekè; l mld fopj.k dh ryuk idV djr gA ftl idkj ifrxeu j[kk, nk idkj dh gkrh g mlh idkj irhixeu lehdj.k Hkh nk gkrh gA irhixeu lehdj.k fuEu idkj d g—

(i) X dk Y ij irhixeu lehdj.k (Regression equation of X on Y)— bl lehdj.k dh lgk;rk l Y (LorU=k pj eY;) d fn; eY;k d vk/kj ij X d lEHkkfor eY;k ij vueku yxku d fy, fd;k tkrk gA ;g lehdj.k fuEu idkj l O;Dr fd;k tkrk g—

$$X = a + by$$

;gk a rFkk b vpj (Constant) gA X dk Y ij irhixeu lehdj.k dk lglEcU/ x.kkd] rki foypu vkj lekUrj ekè;k d ekuk d :i e fuEufyf[kr <x ij fy[kk tk ldrk g—

$$x - \bar{x} = r.\frac{\sigma x}{\sigma y}(y - \bar{y})$$

or $$x - \bar{x} = bxy(y - \bar{y})$$

b x $Y = X$ dk Y ij irhixeu x.kkd gA

(ii) Y dk X ij irhixeu lehdj.k (Regression equation of Y on X)—bl lehdj.k d vk/kj ij X (LorU=k pj eY;) d rRlokn Y (vkfJr pj eY;) d loki;Dr ekè; eY; dk vueku yxk;k tkrk gA ;g lehdj.k fuEu idkj l O;Dr fd;k tkrk gA

$y = a + b$x ;gk a rFkk b vpj (Constant) gA ;g lehdj.k fuEu idkj l fy;k tk ldrk gA

$$y - \bar{y} = r.\frac{\sigma y}{\sigma x}(x - \bar{x})$$

or $$y - \bar{y} = bxy(x - \bar{x})$$

byx $= Y$ dk X dk irhixeu x.kkd gA

3.5. izrhixeu xq.kkad (Regresion Coefficient) :

izrhixeu xq.kkad og eY; n'kkZrk gS tks ,d Js.kh d pj eY;ksa esa bdkbZ ifjorZu gksus lss nljh Js.kh d pj eY;ksa esa vkSlru ifjorZu gksxkA ;g izrhixeu js[kkvksa d lEcU/k esa js[kk <yku (Slope) dk chtxf.krh; ekiu gSA

3.5.1. izrhixeu xq.kkdksa dk chtxf.krh; eki (Algebracial Measurement of Regression Coefficient) :

izrhixeu js[kkvksa dh rjg izrhixeu xq.kkad Hkh nks izdkj d gksrs gSa ftudk o.kZu bl izdkj g—

(i) *X* dk *Y* ij ifrixeu xq.kkd (Regression Coefficient of *X* on *Y*)—;g xq.kkd ;g crkrk g fd *Y* esa ,d bdkbZ ifjorZu gksus ij *X* esa fdruk ifjorZu gksxkA *X* ij *Y* d izrhixeu xq.kkad dk ldrk{kj *bxy* d }kjk fn[kk;k tkrk gSA bldk l=k fuEu izdkj g—

$$bxy = r.\frac{\sigma x}{\sigma y}$$

(ii) ok ij izrhixeu xq.kkd (Regression Coefficient of *Y* on *X*)—;g xq.kkd ;g crkrk g fd *X* esa ,d bdkbZ ifjorZu gksus ij *Y* esa fdruk ifjorZu gksxkA *X* ij *Y* d izrhixeu xq.kkad dk lkdsfrd *byx* d }kjk fn[kk;k tkrk gSA bldk l=k fuEu izdkj g—

$$byx = r.\frac{\sigma y}{\sigma x}$$

bxy rFkk *byx* d eY; dk vU; l=kksa d enn ls Hkh Kkr fd;k tk ldrk gSA

Example : From the following data obtain the two regression equations :

X	5	8	7	6	4
Y	3	4	5	2	1

Solution. **Computation for Regression Equations**

X	Y	X^2	Y^2	XY
5	3	25	9	15
8	4	64	16	32
7	5	49	25	35
6	2	36	4	12
4	1	16	1	4
$\Sigma X = 30$	$\Sigma Y = 15$	$\Sigma X^2 = 190$	$\Sigma Y^2 = 55$	$\Sigma XY = 98$

Regression Equation of *Y* on *X* :

$$yc = a + bx$$

a vkSj *b* oQ eY; Kkr djus oQ fy, nks lehdj.kksa dk iz;ksx fd;k tk,xkA

$$\Sigma y = Na + b\Sigma x \qquad \ldots(i)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \qquad \ldots(ii)$$

lehdj.k (i) o (ii) e fn, x, eY; j[ku ij

$$15 = 5a + 30b \qquad \ldots\text{(i)}$$

$$98 = 30a + 190b \qquad \ldots\text{(ii)}$$

lehdj.k (i) dk 6 l x.kk dju ij

$$90 = 30a + 180b \qquad \ldots\text{(iii)}$$

lehdj.k (iii) e l lehdj.k (ii) ?kVku ij

$$90 = 30a + 180b$$

$$98 = 30a + 190b$$

$$- \quad -$$

$$b = \frac{8}{10} = 0.80$$

$$-8 = -10b$$

$$-10b = -8$$

$$b = = 0.80$$

vc b dk eku lehdj.k (i) e j[ku ij

$$15 = 5a + 30(8)$$

or $$15 = 5a + 24$$

$$15 - 24 = 5a$$

$$-9 = 5a$$

$$a = \frac{-9}{5} = -18$$

vc gekj ikl a rFkk b nkuk d eku g ftUg Y on X lehdj.k cuxk—

$$yc = -1.8 + .8x$$

vc ge X on Y irhixeu lehdj.k Kkr djx—

Regression equation x on y is

$$x = a + by$$

a rFkk b dk eku Kkr dju oQ fy, fuEufyf[kr nk lehdj.kk dk i;kx fd;k tk,xkA

$$\Sigma x = Na + b\Sigma y \qquad \ldots\text{(i)}$$

$$\Sigma xy = a\Sigma y + b\Sigma y^2 \qquad \ldots\text{(ii)}$$

lehdj.k (i) o (ii) e fn, x, eY; j[ku ij

$$30 = 5a + 15b \qquad \ldots\text{(i)}$$

$$98 = 15a + 55b \qquad \ldots\text{(ii)}$$

lehdj.k (i) dk 3 l x.kk dju ij

$$90 = 15a + 45b \qquad \ldots\text{(iii)}$$

lehdj.k (iii) e l lehdj.k (ii) ?kVku ij

$$90 = 15a + 45b \qquad \ldots\text{(iii)}$$

$$98 = 15a + 55b \qquad \ldots\text{(ii)}$$

$$- \quad -$$

$$-8 = -10b$$

$$-10b = -8$$

$$b = \frac{8}{10} = 0.8$$

lehdj.k (i) e b dk eku j[ku ij

$$30 = 5a + 15(0.8)$$

or $$30 = 5a + 12$$

$$30 - 12 = 5a$$

or $$a = 3.6$$

Now put value of a and b in the regression equations of x on y :

$$xc = a + by$$

$$xc = 3.6 + 0.8y$$

Hence the regression equations will be

$$x \text{ on } y = xc = 3.6 + 0.8y$$

$$y \text{ on } x = yc = -1.8 + 0.8x$$

3.5.2. irhixeu x.kkdk l lglEcU/ x.kkd dk fu/kj.k (Determination of Correlation Coefficient by Regression Coefficients) :

tc nk irhixeu x.kkd Kkr gk rk budh lgk;rk l lglEcU/ x.kkd (x) Kkr fd;k tk ldrk gA lglEcU/ x.kkd] nkuk irhixeu x.kkdk dk x.kkÙkj eké; (Geometric Mean) gkrk gA nlj 'kCnk e nkuk irhixeu x.kkdk dk oxey gh lglEcU/ x.kkd dk fu/kj.k djrk gA bldk l=k fuEu idkj g—

$$\sqrt{bxy \times byx} = \sqrt{r\frac{\sigma x}{\sigma y} \times r\frac{\sigma y}{\sigma x}} = \sqrt{r^2} = r$$

Example : Find out the value of r if $bxy = 1$ and $byx = 0.64$.

Solution. $$r = \sqrt{bxy \times byx}$$

or $$r = \sqrt{1 \times 0.64}$$

or $$r = \sqrt{0.64}$$

$$r = 0.8$$

From the following data qutain the two regression and find out the value of coefficient of Correlation (r).

X	3	4	6	7	10
Y	9	11	14	15	16

Solution. **Calculation of Regression Zones by Direct Method**

S.N.	x	$\bar{x}=6$ $dx=(x-\bar{x})$	d^2x	y	$\bar{y}=13$ $dy=(y-\bar{y})$	d^2y	$(dx.dy)$
1	3	-3	9	9	-4	16	12
2	4	-2	4	11	-2	4	4
3	6	0	0	14	$+1$	1	0
4	7	$+1$	1	15	$+2$	4	2
5	10	$+4$	16	16	$+3$	9	12
$N=5$	$\Sigma x=30$	$\Sigma dx=0$	$\Sigma d^2x=30$	$\Sigma y=65$	$\Sigma dy=0$	$\Sigma d^2y=34$	$\Sigma dxdy=38$

Mean Values of Series 'X' and Series 'Y'

$$X = \frac{\Sigma x}{N} = \frac{30}{5} = 6 \qquad Y = \frac{\Sigma y}{N} = \frac{65}{5} = 13$$

Regression Coefficient (b)

Xon Y — Y on X

$$bxy = \frac{\Sigma dxdy}{\Sigma d^2y} = \frac{30}{34} = +0.88 \qquad bxy = \frac{\Sigma dxdy}{\Sigma d^2x} = \frac{30}{30} = +1$$

Regression Equation

X on Y — Y on X

$$x-\bar{x} = bxy(y-\bar{y}) \qquad y-\bar{y} = bxy(x-\bar{x})$$

$$x-6 = 0.88(y-13) \qquad y-13 = 1(x-6)$$

$$x-6 = .88y-11.44 \qquad y-13 = x-6$$

or $$x = 0.88-544 \qquad \text{or } y = x+7$$

$$r = \sqrt{(bxy)(byx)} \text{ or } \sqrt{.88\times 1} \text{ or } \sqrt{0.88} = +0.938$$

Thus $r = +0.938.$

Following figure relate to Demand and Price of a commodity :

Demand (kg)	20	22	24	26	28	30	32	34	36	38
Price Per (kg) Rs.	10	12	16	18	20	20	22	24	24	24

Calculate the regression coeficient and find out the two regression equations. Estimate the average price when the demand of the commodity is 31 kg.

Solution. **Calculation of Regression Coefficients**

S.N.	Demand – X			Price – Y			Products
	Demand in kg	Deviation from	Deviation Squared	Price in Rs.	Deviations from	Deviations Squared	of respective deviations
	(x)	$A=30$ (dx)	($d2x$)	(y)	$A=20$ (dx)	(d^2y)	($dxdy$)
1.	20	– 10	100	10	– 10	100	100
2.	22	– 8	64	12	– 8	64	64
3.	24	– 6	36	16	– 4	16	24
4.	26	– 4	16	18	– 2	4	8
5.	28	– 2	4	20	0	0	0
6.	30	0	0	20	0	0	0
7.	32	2	4	22	2	4	4
8.	34	4	16	24	4	16	16
9.	36	6	36	24	4	16	24
10.	38	8	64	24	4	16	32
Total	290	– 10	340	190	– 10	236	272
No. 10	Σx	Σdx	Σd^2x	Σy	Σdy	Σd^2y	$\Sigma dxdy$

Regression Coefficient (*b*)

X on *Y*

$$bxy = \frac{N.\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{N.\Sigma d^2y - (\Sigma dy)^2}$$

$$= \frac{10\times 272 - (-10)(-10)}{10\times 236 - (-10)^2}$$

$$= \frac{2720-100}{2360-100} = \frac{2620}{2260}$$

$$= 1.1593$$

Y on *X*

$$axy = \frac{N.\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{N.\Sigma d^2x - (\Sigma dx)^2}$$

$$= \frac{10\times 272 - (-10)(-10)}{10\times 340 - (-10)^2}$$

$$= \frac{2720-100}{3400-100} = \frac{2620}{3300}$$

$$= 0.7930$$

(Arithmetic mean)

$$\overline{X} = Ax + \frac{\Sigma dx}{N} = 30 + \frac{-10}{10} = 29 \qquad \overline{Y} = Ay + \frac{\Sigma dy}{N} = 20 + \frac{-10}{10} = 10$$

(Regression Equations)

$$x - \overline{x} = bxy(y - \overline{y})$$

or $(x-29) = 1.1593(y-19)$

or $x - 29 = 1.1539 - 22.0267$

or $x = 29 + 1.1593Y - 21.0267$

or $x = 6.9733 + 1.1593y$

$$y - \overline{y} = bxy(x - \overline{x})$$

or $(y-19) = 0.7939(x-29)$

or $(y-19) = 0.7939 - 23.0231$

or $y = 19 - 23.0231 + 0.7939x$

or $y = -4.0231 + 0.7939x$

When the demand of the commodity is 31 kg, then for determining the average price (y) we shall have to use regression equation of Y on X :

$$y = -4.0231 + 0.7939x \qquad \text{or} \quad y = -4.0231 + 0.7939 \times 31$$

$$\text{or} \qquad y = -4.0251 + 24.6109 \qquad \text{or} \quad y = 20.5878$$

So when the demand is 31 kg, then the prorbable would be Rs. 20.59.

Example : The following table gives the number of students having different heights and weights :

Heights in Inches	Weight (in the)				
	70 – 80	80 – 90	90 – 100	100 – 110	Total
45 – 50	6	10	4	–	20
50 – 55	4	10	10	1	25
55 – 60	4	8	15	8	35
60 – 65	4	3	2	11	20
Total	18	31	31	20	100

On the basis of the above data, calculate regression equation.

Solution. **Computation of Regression Coefficeints**

Heights (X)	(Y) dy ∑dx	70-80 -1	80-90 0	90-100 1	100-110 2	F	Fdx	fd2x
45-50	-2	2 6 12	0 10 0	-2 4 -8	-4 - 0	20	-40	80
50-55	-1	1 4 4	0 10 0	-1 1 10	-2 1 -2	25	-25	25
55-60	0	0 4 0	0 8 0	0 15 0	0 8 0	35	0	0
60-65	1	-1 4 -4	0 3 0	1 2 2	2 11 22	20	20	20
	f	18	31	31	20	100 N	-45 ∑dx	125 fd^2x
	fdy	-18	0	31	40	53 ∑fdy		
	fd^2y	18	0	31	80	129 fd^2y		
	fxdy	12	0	-16	20	16 ∑fdxdy		

Regression Coefficient

Regression of X on Y | Regression of Y on X

$$bxy = \frac{ix(\sum fdxdy.N - (\sum fdx.\sum fdy)}{iy[\sum fd^2 y.N - (\sum fdy)^2} \qquad bxy = \frac{iy[(\sum fdxdy.N - (\sum fdx.\sum fdy)]}{ix[\sum fd^2 x.N - (\sum fdy)^2]}$$

$$= \frac{5[16\times100-(-45\times53)]}{10[129\times100-(53)^2]} \qquad = \frac{10[16\times100-(-45\times53)]}{10[125\times100-(45)^2]}$$

$$= \frac{5[1600+2385]}{10[12900-2809]} \qquad = \frac{10[1600+2385]}{5[12500-2025]}$$

$$= \frac{5\times3985}{10\times10091} = \frac{19924}{100910} = 0.197 \qquad = \frac{10\times3985}{5\times10475} = \frac{38950}{52375} = 0.761$$

$$bxy = 0.197 \qquad byx = 0.761$$

Mean

$$\overline{X} = Ax + \frac{\Sigma fdx}{N} \times i \qquad \overline{Y} = Ay + \frac{\Sigma fdy}{N} \times i$$

$$= 57.5 + \frac{-45}{100} \times 5 \qquad = 8.5 + \frac{-53}{100} \times 10$$

$$= 57.5 - 2.25 = 55.25 \qquad = 8.5 + 5.3 = 90.3$$

(Regression Equations)

X on Y | Y on X

$$x - \overline{x} = bxy(y - \overline{y}) \qquad y - \overline{y} = bxy(x - \overline{x})$$

or $(x - 55.25) = 0.197(y - 90.3)$ | or $(y - 90.3) = 0.761(x - 55.25)$

or $x - 55.25 = 0.197y - 17.78$ | or $y - 90.3 = 0.761x - 42.05$

or $x = 55.25 - 17.79 + 0.197y$ | or $y = 90.3 - 42.05 + 0.761x$

or $x = 37.46 + 0.197y$ | or $y = 48.25 + 0.761x$

3.5.3. irhixeu x.kkdk oQ x.k (Properties of Regression Coefficients)

(i) irhixeu x.kkdk dk x.kkÙkj ekè; lglEcU/ x.kkd gkrk gA

$$r = \sqrt{bxy \times byx}$$

(ii) nkuk ifrxeu x.kkdk oQ fpUg ,d leku gkr gA ;k rk nkuk /ukRed (+ve) gku ;k ½.kkRed (–ve) gkxA

(iii) nkuk irhixeu x.kkdk dk x.kuiQy ge'kk bdkb l de ;k cjkcj gkrk gA

$$r^2 = byx.bxy \le 1 \qquad (\therefore -1 \le r \le +1 \Rightarrow r^2 \le 1)$$

(iv) lg lEcU/ x.kkd dk fpUg Hkh irhixeu x.kkdk dh rjg gkrk gA

If bxy and byx are –ve then r is –ve if bxy and byx are +ve then r is +ve.

(v) irhixeu x.kkdk ij eyfcUn (origin) e ifjoru dk dkbi iHkko ugh iMrk yfdu ieku (scale) e ifjoru gku dk iHkko iMrk gA , l e lek;ftr l=k dk fuEu idkj gkrk g—

$$bxy = r \cdot \frac{\sigma x}{\sigma y} \times \frac{ix}{iy}, \quad byx = r \cdot \frac{\sigma y}{\sigma x} \times \frac{iy}{ix}$$

where *ix* and *iy* are common factor of *x* and *y*.

Example : Calculate the regression equations from the following data :

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Solution. Computation of Regression Equations

x	dx $A=5$	d^2x	y	dy	d^2y $A=12$	$dxdy$
1	– 4	16	9	– 3	9	12
2	– 3	9	8	– 4	16	12
3	– 2	4	10	– 2	4	4
4	– 1	1	12	0	0	0
5	0	0	11	– 1	1	0
6	1	1	13	+ 1	1	1
7	2	4	14	+ 2	4	4
8	3	9	16	+ 4	16	12
9	4	16	15	+ 3	9	12
45	0	60	108	0	60	57

$$N = 9, \Sigma x = 45, \Sigma y = 108, \Sigma d^2x = 60, \Sigma d^2y = 60, \Sigma dxdy = 57$$

$$\overline{X} = \frac{\Sigma x}{N} = \frac{45}{9} = \text{or } 5, \quad \overline{Y} = \frac{\Sigma Y}{N} = \frac{108}{9} \text{ or } 12$$

Regression Coefficient (*b*)

X on *Y*

$$bxy = \frac{\Sigma dxdy}{N\sigma y^2} \text{ or } \frac{\Sigma dxdy}{\Sigma d^2 y}$$

$$= \frac{57}{60} = 0.95$$

Y on *X*

$$byx = \frac{\Sigma dxdy}{N\sigma x^2} \text{ or } \frac{\Sigma dxdy}{\Sigma d^2 x}$$

$$= \frac{57}{60} = 0.95$$

Regression Equation

X on *Y*

$$x - \overline{x} = bxy(y - \overline{y})$$

or $(x-5) = 0.95(y-12)$

or $x = .95y - 11.4 + 5$

Y on *X*

$$y - \overline{y} = bxy(x - \overline{x})$$

or $(y-12) = .95(x-5)$

or $y = 0.95x - 4.75 + 12$

or $x = 0.95y - 6.4$ or $y = .95y + 7.25$

Example : The marks obtained by seven students in Statistics and Accoutancy are as follows :

Marks in Statistics (out of 50)	46	42	44	40	43	41	45
Marks in Accountancy (out of 50)	40	38	36	35	39	37	41

Calculate two regression equations.

Solution. **Computation of Regression Equations**

Marks in Statistics			Marks in Accountancy			
Marks	**Deviation from 40**	**Square of Deviations**	**Marks**	**Deviation from 36**	**Squares of Deviation**	**Product of *X* & *Y***
X	(*dx*)	*d2x*	*Y*	*dy*	*d2y*	*dxdy*
46	6	36	40	4	16	24
42	2	4	38	2	4	4
44	4	16	36	0	0	0
40	0	0	35	– 1	1	0
43	3	9	39	3	9	9
41	1	1	37	1	1	1
45	5	25	41	5	25	25
Total	21	91		14	56	63
$N = 7$	Σdx	Σd^2x		Σdy	Σd^2y	$\Sigma dxdy$

Regression Coefficient (*d*)

X on *Y* *Y* on *X*

$$bxy = \frac{N.\Sigma dxdy - \Sigma dx.\Sigma dy}{N.\Sigma d^2 y - (\Sigma dy)^2} \qquad byx = \frac{N.\Sigma dxdy - \Sigma dx.\Sigma dy}{N.\Sigma d^2 x - (\Sigma dx)^2}$$

$$= \frac{7\times 63 - 21\times 14}{7\times 56 - (14)^2} \qquad = \frac{7\times 63 - 21\times 14}{7\times 91 - (21)^2}$$

$$= \frac{441 - 294}{392 - 196} = \frac{147}{196} = 0.75 \qquad = \frac{441 - 294}{637 - 441} = \frac{147}{196} = 0.75$$

$\therefore$ $bxy = 0.75$ $byx = 0.75$

Mean

$$\overline{x} = Ax + \frac{\Sigma dx}{N} = 40 + \frac{21}{7} = 43 \qquad \overline{x} = Ay + \frac{\Sigma dy}{N} = 36 + \frac{14}{7} = 38$$

Regression Equations

X on Y	Y on X
$x-\bar{x} = bxy(y-\bar{y})$	$y-\bar{y} = bxy(x-\bar{x})$
$(x-43) = 0.75(y-38)$	$(y-38) = 0.75(x-43)$
$x-43 = 0.75y-28.5$	$y-38 = 0.75x-32.25$
$x = 43-28.5+0.75y$	$y = 38-32.25+0.75x$
$x = 14.5+0.75y$	$y = 5.75+0.75x$

3.6. vueku dh iFke =kfV (Standard Error of the Estimate)

tlk fd vki pkgr g fd ge irhixeu e LorU=k pj dk eY; fn; gku ij] vkfJr pj d eY; dk lokÙke vueku yxkr gA gekjk vueku okLrfodrk d fdruk fudV g vFkok fdl lhek rd Bhd gA ;k fo'oluh; g rk ;g tkuu d fy, vueku dfjiek.k =kfV dh x.kuk dh tkrh gA vr% vueku dk iek.k =kfV vkfJr pj d okLrfod eY;k o lxfBr eY;k d fopyuk dk vklr eki gA nkuk irhixeu j[kkvk d vueku dh ieki =kfV fuEufyf[kr l=kk d ekè;e l Kkr dh tk ldrh g—

Standard error of estimate of X on Y	Standard error of estimate of Y on X
(i) $Sxy = \sqrt{\dfrac{\Sigma(x-xc)^2}{N}}$	(i) $Syx = \sqrt{\Sigma(y-yc)^2}$
(ii) $Sxy = \dfrac{\sqrt{\Sigma x^2 - a\Sigma x - b\Sigma xy}}{N}$	(ii) $Syx = \dfrac{\sqrt{\Sigma y^2 - a\Sigma y - b\Sigma xy}}{N}$
(iii) $Sxy = \sigma x\sqrt{1-r^2}$	(iii) $Syx = \sigma x\sqrt{1-r^2}$
Where σx = S.D. of x;	Where σx = S.D. of y;
r = Coefficient Correlation between X and Y	r = Coefficient Correlation between X and Y

Find the Standard error of estimate :

$$\sigma x = 4.4;\ \sigma y = 2.2 \text{ and } r = 0.8$$

Solution. We know that standard error of estimate of $X = Sx$ and Standard error of estimae of $Y = Sy$.

$$Sx = \sigma x\sqrt{1-r^2}$$

$$= 4.4\sqrt{1-0.8^2}$$

$$= 4.4\sqrt{1-0.64}$$

$$= 4.4\sqrt{0.36} = 4.4 \times 0.6 = 2.64$$

Similarly,

$$Sy = \sigma y\sqrt{1-r^2}$$

$$= 2.2\sqrt{1-0.8^2}$$

$$= 2.2\sqrt{1-0.64}$$

$$= 2.2\sqrt{0.36}$$

$$= 2.2 \times 0.6 = 1.32$$

Example : Obtain the equations of the lines of regressin and the standard errors of the estimate from the following data :

X	1	2	3	4	5
Y	6	8	7	6	8

Solution.

X	$dx(3)$	d^2x	y	$dy(7)$	d^2y	$dxdy$
1	– 2	4	– 6	– 1	1	2
2	– 1	1	8	1	1	– 1
3	0	0	7	0	0	0
4	1	1	6	– 1	1	– 1
5	2	4	8	1	1	2
$\Sigma x = 15$		$\Sigma d^2x = 10$		$\Sigma Y = 35$	$\Sigma d^2y = 4$	$\Sigma dxdy = 2$

$$\bar{x} = \frac{15}{5} = 3 \qquad \bar{y} = \frac{35}{5} = 7$$

$$\sigma x = \sqrt{\frac{10}{5}} = \sqrt{2} = 14 \qquad \sigma y = \sqrt{\frac{4}{5}} = \sqrt{8} = 9$$

$$r = \frac{\Sigma dxdy}{N\sigma x\sigma y} = \frac{2}{5\times14\times9} = \frac{2}{63} = 32$$

Regression equation of x on y Regression equation of y on x

$$(x-\bar{x}) = r\frac{\sigma x}{\sigma y}(y-\bar{y}) \qquad (y-\bar{y}) = r\frac{\sigma y}{\sigma x}(x-\bar{x})$$

$$(\bar{x}-3) = 32\frac{1.4}{9}(y-7) \qquad (\bar{y}-7) = 32\frac{.9}{1.4}(x-3)$$

$$(x-3) = \frac{.448}{.9}(y-7) \qquad (y-7) = \frac{.288}{1.4}(x-3)$$

$$(x-3) = .5y - 3.5 \qquad (y-7) = .2x - .6$$

$$x = .5y - .5 \qquad y = .2x + 6.4$$

y x o{ vuekfur eY; xc x y d vuekfur eY; yc

6 $.5 \times 6.5 = 2.5$ 1 $.2 \times 1 + 6.4 = 6.6$

8 $.5 \times 8.5 = 3.5$ 2 $.2 \times 2 + 6.4 = 6.8$

7 $.5 \times 7.5 = 3.0$ 3 $.2 \times 3 + 6.4 = 7.0$

$6 \; .5 \times 6.5 = 2.5$ $\qquad$ $4 \;\; .2 \times 4 + 6.4 = 7.2$

$8 \; .5 \times 8.5 = 3.5$ $\qquad$ $5 \;\; .2 \times 5 + 6.4 = 7.4$

Computation of Standard error of estimate

x	xc	$(x-xc)$	$(x-xc)^2$	y	yc	$(y-yc)$	$(y-yc)^2$
1	2.5	– 1.5	2.25	6	6.6	– 0.6	.36
2	3.5	– 1.5	2.25	8	6.8	1.2	1.44
3	3.0	0	0	7	7.0	0	0
4	2.5	1.5	2.25	6	7.2	– 1.2	1.44
5	3.5	1.5	2.25	8	7.4	– 0.6	.36
			9.00				3.60

$$Sxy = \frac{\sqrt{\Sigma(x-xc)^2}}{N} = \sqrt{\frac{9}{5}} = \sqrt{18} = 13 \qquad \ldots\text{(i)}$$

$$Sxy = \alpha x\sqrt{1-r^2} = 1.4\sqrt{1-32^2} = 1.4\times\sqrt{.898}$$

$$= 1.4 \times 9.47 = 13$$

$$Syx = \sqrt{\frac{\Sigma(y-yc)^2}{N}} = \sqrt{\frac{3.6}{5}} = \sqrt{.72} = .85$$

odfYid l=k }kjk %

$$sxy = \sigma y\sqrt{1-r^2} = 0.9\sqrt{1-.32^2} = 0.9\times\sqrt{.898} = 0.9\times.947 = 0.85$$

4. lkjka'k (Summary) :

irhixeu lkf[;dh; fo'y"k.k dh og jhfr g ftlds ekè;e l ,d pj d fdlh Kkr eY; l lcf/r mRloknh (Corresponding) nlj pj dk lEHkkO; eY; vuekfur fd;k tk ldrk gA irhixeu fo'y"k.k d vk/kj ij lkekftd] vkfFkd o O;kolkf;d {k=kk e fofHkUu ?kVukvk d ekè; lEcU/k dk fo'y"k.k djd ,d pj l lcf/r nljk vkfJr pj&eY; vuekfur fd;k tkrk gA O;kolkf;d ,o vkfFkd {k=k e bldk vR;f/d egRo gA icU/d bl rduhd dk i;kx fu;=k.k midj.k d :i e djr gA bl rduhd d vk/kj ij lefpr O;kolkf;d fu.k;u ljy gk tkrk gA bl rduhd d vk/kj ij Kkr fu"d"k mru gh vf/d fo'oluh; gkx ftruk vfèkd ?kfu"B lglEcU/ nk pjk d chp gkxkA irhixeu dkj.k ,o ifj.kke e lEcU/ LFkkfir djrk g vkj lEcU/ dh ek=kk ,o iofÙk dk Li"V djrk gA irhixeu x.kkd ;g n'kkr g fd ,d led J.kh d pj&eY;k e ,d bdkb ifjoru gku l lEc¼ nljh led J.kh d pj eY;k e vklru fdruk ifjoru gkrk gA irhixeu x.kkd dk eY; irhixeu j[kk d <yku dk chtxf.kfr; eki gkrk gA irhixeu fo'y"k.k }kjk iklr vueku ;Fkkr d fdruk utnhd g vkj fdruk fo'oluh; g ;g tkuu d fy, vueku dh ieki =kfV dk i;kx fd;k tkrk gA

5. iLrkfor iLroQ (Recommended Books)

(i) Introduction to Statistics – Dr. R.P. Hooda

(ii) Statistical Methods – S.P. Gupta

(iii) Business Statistics – Oswal, Aggarwal & Sharma, Ramesh Book Depot, Jaipur

(iv) Business Statistics – T.R. Jain

(v) Business Statistcis – S.C. Sharma, R.C. Jain

6. vkReeh; dk fy, i'u (Self Assessment Questions)

(1) irhixeu dk vFkZ ,o mldh vkfFkZd fo'y"k.k e mi;kfxrk crykb;A irhixeu lehdj.k fdl idkj fudky tkr g \ mnkgj.k ndj le>kb;A

(2) irhixeu vo/kj.kk dh O;k[;k dhft,A ;g lglEcU/ l fdl idkj fHkUu g \ bldh mi;kfxrk dh foopuk dhft,A

(3) O;k[;kRed fVIif.k;k fyf[k,—

(i) j[kh; irhixeu

(ii) irhixeu x.kkd

(iii) vueku dk iekf foHke

(4) Given the following data :

X	6	2	10	4	8
Y	9	11	5	8	7

Find two regression equationss and calculate standard error or estimate.

(5) You are given below the following information about advertising and sales :

	Adv. Exp. (Rs. Lakhs) X	Sales (Rs. Lakhs) Y
Mean	10	90
S.D.	3	12

Correlation Coefficient = 0.8

(i) Calculate two regression lines.

(ii) Find the likely sales when advertisement expenditure is Rs. 15 Lakhs.

(iii) What should be advertisement expenditure if the company to attain sales target of Rs. 120 Lakhs.

(6) The regression equations calculated from a given set of observation are given below :

$x = -0.2y + 4.2$ and

$y = 0.8x + 8.4$

Calculate :

(i) $\bar{x}$ and $\bar{y}$

(ii) Coefficient of Correlation (r)

(iii) Estimated value of y when $x = 4$.

B.Com. Part-II

Paper - BC–204 : Business Statistics *Revised By :* **Dr. Heera Lal Sharma**

Lesson No. : 7

lpdkd

(Index Numbers)

lajpuk (Structure)

1. ifjp;
2. mís';
3. fo"k; dk iLrqrhdj.k
   - 3.1. lpdkd dk vFkZ ,oa ifjHkk"kk
   - 3.2. lpdkdksa dh jpuk esa leL;k,¡
   - 3.3. lpdkd jpuk dh fof/;k¡
     - 3.3.1. ljy leqgh fof/
     - 3.3.2. eY;kuqikrksa dk ek/;
     - 3.3.3. Hkkfjr lpdkd
   - 3.4. vkn'kZ lpdkd oQ ijh{k.k
     - 3.4.1. le; mRØe.kh;rk dh ijh{k.k
     - 3.4.2. rRo mRØe.kh;rk ijh{k.k
     - 3.4.3. ?kfu; ijh{k.k
   - 3.5. thou fuokZg lpdkd
   - 3.6. lpdkd jpuk lac/h fofo/ leL;k,¡
     - 3.6.1. vk/kj ifjorZu
     - 3.6.2. vk/kj o"kZ ifjorZu
     - 3.6.3. f'kjkscU/u
4. lkjka'k
5. iLrkfor iLrosQ
6. vH;kl dss fy, i'u

1. ifjp; (Introduction) :

ifjoru lalkj dk fu;e gA thou o iÑfr d iR;d {k=k ge blds vuHko djr gA bl lalkj e dkb oLrq N Hkh fLFkj ugh gA le; pØ fujUrj xfr'khy g ,o bl xfr'khyrk e oLr, LokHkkfod :i l ifjofrr gkrh jgrh gA ifjoru dk ;g fu;e jktufrd] lkekftd] vkfFkd o O;kolkf;d txr ij Hkh 'kr&ifr'kr ykxw gkrk gA ik;% ge n[kr g fd O;kolkf;d o vkfFkd txr e Hkh fofHkUu i{kk tl ekx] ifr] mRiknu] eY; jk"Vh; vk;] ifr&O;fDr vk; vkfn e fujUrj ifjoru gkrk jgrk gA vkfFkd o O;olkf;d txr e fujUrj gku oky bu ifjoruk dk eki djuk vko';d gkrk g rkfd Hkfo"; d fy, lgh fu;ktu fd;k tk ld vkj LVhd fu.k; fy, tk ldA vr% bu lcd fy, lkf[;dh e ftl fof/ dk i;kx fd;k tkrk gA ;g lpdkd gkrk gA

2. mís'; (Objective) :

bl vè;k; dk vè;;u dju d fy, e[; mís'; bl idkj g—

(i) vFkZO;oLFkk e gku oky lkisf{k ifjoruk dh lkekU; iofÙk dk eki djukA

(ii) lpdkdk dh jpuk lca/h fofHkUu leL;kvk dk le>ukA

(iii) lpdkdk dh oKkfudrk dh tkp dju d fy, fofHkUu ijh{k.kk dk tkuukA

(iv) fdlh fo'k"k LFkku e lcf/r ox&fo'k"k ij iMu oky eY; ifjoruk d iHkko dk eki djukA

(v) lpdkd jpuk dh fofHkUu fof/;k dk le>kukA

3. fo"k; dk iLrrhdj.k (Presentation of Contents)

3.1. lpdkd dk vFk ,o ifjHkk"kk—

lpdkd ,d fo'k"k idkj d ekè; gkr g ftud }kjk LFkku J.kh (Spatial Series) vkj dky J.kh (Time Series) dh dUæh; iofÙk (Central Tendency) dk ekik tk ldrk gA O;ogkj e dkiQh rFkk fofHkUu ,o tfVy i,k, tkr gA ,lh n'kk e ryukRed vè;;u d fy, lpdkd dk gh i;kx fd;k tkrk gA tc fuji{k ifjoruk (absolute changes) dk Kkr djuk vlEHko gk rk lpdkd dh enn l lkisf{k ifjoruk (Relative Changes) dk Kkr fd;k tkrk gA Cy;j d vuqlkj] ^^lpdkd ,d fo'k"k idkj l ekè; gA** "Index Numbers are a specialised type of average."—Blair.

Lihxy d vuqlkj]

^^lpdkd ,d lkf[;dh; eki g] tk le;] LFkku ;k vU; fo'k"krk d vk/kj ij pj eY;k d leg e gku oky ifjoruk dk n'kkrk gA

"An Index Number is a statistical measure designed to show changes in variables or group of related variable with respect to time, geographical location or other characteristics." —Special.

la{ki e dgk tk ldrk g fd lpdkd ,d idkj d ekè; g] ftud }kjk eY; rFkk vU; vkfFkd rRok e gku oky ifjoruk dh dUæh; iofÙk;k dk vueku yxk;k tk ldrk gA

3.2. lwpdkdk dh jpuk esa leL;k,¡

(Problems in the Construction of Index Numbers) :

(1) **mís'; (Purpose) :** lwpdkdk dh jpuk djus ls igys mís'; dk Li"V :i ls tkuuk yuk pkfg, D;ksafd oLrqvksa dk puko] vk/kkj o"kZ dk puko vkfn lwpdkdk ds mís'; ij gh fuHkZj djrk gSA

(2) **vk/kkj o"kZ dk puko (Selection of Base Year) :** lwpdkad dh jpuk djus ds fy, ,d lkekU; ifjfLFkfr okyk vk/kkj o"kZ dk puko djuk iM+rk gSA ftldk lwpdkad lnSo 100 eku fy;k tkrk gSA ;g vk/kkj o"kZ dkQh iqjkuk ugh gksuk pkfg,A blds nks Hkkx esa ckaVk tk ldrk gSA

(i) Fixed Base,

(ii) Chain base

(i) **LFkk;h vk/kkj (Fixed Base) :** blesa ,d o"kZ fuf'pr fd;k tkrk gSA ftlds vk/kkj ij v/;;u o"kksZa dh rqyuk dh tkrh gS vkSj o"kksZa dk vkSlr eY; fudkyk tkrk gSA

(ii) **Ük`[kyk vk/kkj (Chain Base) :** blesa vk/kkj o"kZ dks ¼Ük`[kyk½ ds vk/kkj ij ekuk tkrk gSA

(3) **oLrqvksa ;k enksa dk puko (Selection of Commodities or Items) :** oLrqvksa vkSj dherksa esa vlekurk gksus ds dkj.k lwpdkad dh jpuk ds fy, lHkh oLrqvksa dks 'kkfey ugh fd;k tk ldrkA blesa oLrqvksa dk puko lcf/kr oxZ dh #fp] vknr] jhfr&fjokt vkSj vko';drk ds vuqlkj] J.khÑr rFkk iekf.kr o x.kk dks Hkh è;ku esa j[k dj fd;k tkrk gSA

(4) **eY;ksa dk puko (Selection of Prices) :** Fkksd eY; o iqQVdj nkuksa idkj ds gksrs gSA lwpdkad ds mís'; dks è;ku esa j[krs gq, eY;ksa dk iz;ksx fd;k tkuk pkfg,A

(5) **Hkkjksa dk puko (Selection of Weights) :** okLro esa lwpdkad esa 'kkfey gksus okyh lHkh oLrq,¡ cjkcj dh ugh gksrhA lgh ifj.kke ikIr djus ds fy, oLrqvksa dks mudh egÙkk ds vuqlkj Hkkj fn, tkrs gSA

(6) **eké; dk puko (Selection of Average) :** lwpdkad esa vf/kdrj eè;dk (Medium) vkSj x.kkÙkj eké; (Geometric Mean) dk iz;ksx fd;k gSA yfdu x.kkÙkj eké; dk vf/kd mi;qDr ekuk tkrk gS D;ksafd x.kkÙkj eké; vuqikrksa] ifr'krksa vkfn dh vkSlr Kkr djus ds fy, mfpr gSA

(7) **mi;qDr lw=k dk puko (Selection of a Suitable Formula) :** lwpdkdk dh jpuk ds fy, mi;qDr lw=k dk puko djuk vfr vko';d gSA ;g puko vkdM+ksa o mís'; ij fuHkZj djrk gSA iks- fd'kkj dk lw=k vkn'kZ lw=k ekuk tkrk gSA

3.3. lwpdkad jpuk dh fof/k;k¡

3.3.1. ljy legh fof/k

3.3.2. eY;kuqikrksa dk eké;

3.3.3. Hkkfjr lwpdkad

lpdkdk jpuk dh fof/;k

(Methods of Constructing Index Numbers)

सूचकांक
(Index Numbers)

- **(1) सरल या अभारित सूचकांक (Simple or Unweighted Index Numbers)**
  - (i) सरल समूही विधि (Simple Aggregative Method)
  - (ii) मूल्यानुपातों का माध्य (Average of Price Relative)
- **(2) भारित सूचकांक (Weighted Index Numbers)**
  - भारित सूचकांक (Weighted Aggregative Method)
  - मूल्यानुपातों का भारित माध्य (Weighted Average of Price Relatives

3.3.1. ljy legh fof/ (Simple Aggregative Method) :

;g fof/ lpdkd cuku dh lcl ljy fof/ gA bld vUrxr ipfyr o"k d fy, fun'kkd dh x.kuk fuEu l=k }kjk dh tkrh g—

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

Where, ΣP_{01} = Price Index of the Current Year

ΣP_1 = Total of Prices of Current Year

ΣP_0 = Total of Prices of Base Year

Example 1 : From the following data constructed and index for 1991 taking 1990 as base :

Commodities	Price in 1990 Rs.	Price in 1991 Rs.
A	100	120
B	75	110
C	110	160
D	180	250
E	135	260

Solution. Construction of Price Index

Commodities	Price in 1990 Rs.	Price in 1991 Rs.
A	100	120
B	75	110
C	110	160
D	180	250
E	135	260
	$\Sigma P_0 = 600$	$\Sigma P_1 = 900$

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

$$= \frac{900}{600} \times 100$$

$$= 150$$

It means that as compared to 1990 to 1991, there is a net increase of 50% in prices of commodities.

Example II. Construct index numbers with the help of the data given below taking 1988 as base year.

Year	Price (Rs.)
1985	7
1986	8
1987	9
1988	10
1989	15

Solution.

Year	Price (Rs.)	Index numbers or Price Relatives (1988 – 100)
1985	7	$\frac{7}{10} \times 100 = 70$
1986	8	$\frac{8}{10} \times 100 = 80$
1987	9	$\frac{9}{10} \times 100 = 90$
1988	10	$\frac{10}{10} \times 100 = 100$
1989	15	$\frac{15}{10} \times 100 = 150$

3.3.2. eY;kuikrk dk ekè; (Average of Price Relatives) :

bl fof/ d fy, oLrvk ;k enk d eY;kuikrk (Price Relatives) dk Kkr fd;k tkrk gA lekUrj ekè; (Arithmetic Mean) dk i;kx dju ij l=k %

$$P_{01} = \frac{\Sigma \frac{P_1}{P_0} \times 100}{N}$$

N = Number of items or commodities

$$\log P_{01} = \frac{\Sigma \log \frac{P_1}{P_0} \times 100}{N}$$

$$= \frac{\Sigma \log P}{N}$$

where $$P = \frac{P_1}{P_0} \times 100$$

$$P_{01} = \text{Antilog}\left[\frac{\Sigma \log P}{N}\right]$$

Example III. From the following data construct the Price Index Number with the average price as base Rate per rupee.

Year	Wheat	Rice	Oil
I	10 kg	4 kg	2 kg
II	8 kg	2.5 kg	2 kg
III	5 kg	2 kg	1 kg

Solution. lfo/k dh nf"V l igy oLr dh ek=kk dh leku bdkbZ dher l cnyuk t:jh gA ;gk ij dher dk ifr fDoVy cnyk x;k gA

$$\text{Price Relative} = \frac{\text{The Price of current year}}{\text{Average Price}} \times 100$$

Comm-odity	Unit	Average Price P_0	Average Price P_1 $\frac{P_1}{P_0}\times100$	Price Relative	IInd Year P_2	IInd Year P_2 $\frac{P_2}{P_0}\times100$	IIIrd Year P_3	IIIrd Year P_3 $\frac{P_3}{P_0}\times100$
Wheat	Per Qtl.	14.17	10	$\frac{10}{14.17}\times100$ = 70.57	12.50	$\frac{1250}{14.17}\times100$ = 88.21	20	$\frac{20}{14.17}\times100$ = 141.14
Rice	Per Qtl.	38.33	25	$\frac{25}{38.33}\times100$ = 65.22	40	$\frac{40}{38.33}\times100$ = 104.36	50	$\frac{50}{38.33}\times100$ = 130.45
Oil	Per Qtl.	66.67	50	$\frac{50}{66.67}\times100$ = 75	50	$\frac{50}{66.67}\times100$ = 75	100	$\frac{100}{66.67}\times100$ = 150.00
Total of Relatives				210.79		267.57		421.59
Arrange of Relatives				70.26		89.19		140.53

Example IV. Construct Chain Index Numbers from the following data :

Years	1991	1992	1993	1994	1995
Prices	25	30	45	60	75

Solution.

Years	Prices	Link	Relatives	Chain Indices Chained to 1991
1991	25	100	100	
1992	30	$30/25\times100=120$		$100\times120/100=120$
1993	45	$45/30\times100=150$		$120\times150/100=180$
1994	60	$60/45\times100=133.33$		$180\times133.33/100=239.99$
1995	75	$75/60\times100=125$		$239.99\times125/100=300$

Link relatives = Price of Current Year/Price of Previous year is 100

3.3.3. Hkkfjr lpdkd (Weighted Index Numbers) :

okLro esa lHkh oLrqvksa dk egRo leku ugha gksrkA blfy, lpdkdksa dh jpuk djrs le; oLrqvksa os egRo dh nf"V ls fdlh fuf'pr vk/kkj ij Hkkj dk bLrseky djuk pkfg,A bud fuekZ.k dh nks fof/k;ka bLrseky dh tkrh g—

(i) Weighted Aggregative Method

(ii) Weighted Average of Price/Aggressive Method

(i) Hkkjr lefgd fof/k (Weighted Aggregative Method) : Hkkj nu dh vud fof/k;ka gSA

lpdkad d fy, fuEufyf[kr fof/k;ksa dk iz;ksx esa yk;k tkrk g—

(I) ykLis;j fof/k (Lespeyre's Method)

Formula :
$$P_{01}=\frac{\Sigma P_1Q_0}{\Sigma P_0Q_0}\times 100$$

(II) ikls fof/k (Pascle's Method)

$$P_{01}=\frac{\Sigma P_1Q_0}{\Sigma P_0Q_1}\times 100$$

(III) Mkjfo'k o ckmys fof/k (Dorbish and Bowley's Method)

Formula :
$$P_{01}=\frac{\frac{\Sigma P_1q_0}{\Sigma P_0q_0}+\frac{\Sigma P_1q_0}{\Sigma P_0q_0}\times100}{2}$$

(IV) fiQ'kj fof/k (Fisher's Method)

Formula :
$$P_{01}=\sqrt{\frac{\Sigma P_1q_0}{\Sigma P_0q_0}\times\frac{\Sigma P_1q_1}{\Sigma P_0q_1}}\times100$$

(V) o`yh fof/ (Kelly's Method)

$$P_{01} = \frac{\Sigma P_1 q}{\Sigma P_0 q} \times 100$$

Where Q = Given Weights or Standard Quantity

Example V : Calculate Fisher's Ideal Index from the following data.

Commodity	Base Year Price Per Unit Rs.	Base Year Total Exp. Rs.	Current Year Price Per Unit Rs.	Current Year Total Exp. Rs.
I	2	40	5	75
II	4	16	8	40
III	1	10	2	24
IV	5	25	10	60

Solution.

Commodity	Base Year Price Per Unit (P_0)	Base Year Qty. (Q_0)	Current Year Price Per Unit (P_1)	Current Real Exp. / Price Per Unit kg	P_0Q_0	P_1Q_0	P_0Q_1	P_1Q_1
I	2	$\frac{40}{2}=20$	5	$\frac{75}{5}=15$	40	100	30	75
II	4	$\frac{16}{4}=4$	8	$\frac{40}{8}=5$	16	32	20	40
III	1	$\frac{10}{1}=10$	2	$\frac{24}{2}=12$	10	20	12	24
IV	5	$\frac{25}{5}=5$	10	$\frac{60}{10}=6$	25	50	30	60
					$(\Sigma P_0Q_0) = 91$	$(\Sigma P_1Q_0) = 202$	$\Sigma(P_0Q_1) = 92$	$(\Sigma P_1Q_1) = 199$

$$P_{01} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_0}{\Sigma P_1 q_0}} \times 100$$

$$= \sqrt{\frac{202}{91} \times \frac{199}{92} \times 100} = \sqrt{43} \times 100$$

$$= 2.191 = 219.1$$ **Ans.**

3.4. vkn'k lpdkd d ijh{k.k (Test of an Ideal Index) :

;fn lpdkd fuEufyf[kr ijh{k.kk ij lgh mrjrk g rk vkn'k dgk tk,xk vU;Fkk ughA

3.4.1. le; mRØEirk ijh{k.k (Time Reversal Test)

3.4.2. y{; mRØEirk ijh{k.k (Factor Reversal Test)

3.4.3. pØh; ijh{k.k (Circular Test)

3.4.1. le; mRØEirk ijh{k.k (Time Reversal Test)

fiQ'kj d dFkkulkj bl ijh{k.k dk vfHkik;% g fd lpdkd jpuk dk l=k ,lk gkuk pkfg, tk ryukRed 0;k[;k d nk fcUnvk d eè; leku vuikr 0;Dr dj] pkg mue l fdlh dk Hkh vk/kj eku fy;k tk,A

bldk lehdj.k d :i e fl¼ fd;k tk ldrk gA

$$P_{01} \times P_{02} = 1$$

$$P_{01} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_1 q_1}}$$

$$P_{10} = \sqrt{\frac{\Sigma P_1 q_1}{\Sigma P_1 q_1} \times \frac{\Sigma P_1 q_0}{\Sigma P_1 q_0}}$$

$$\therefore \qquad P_{01} \times P_{10} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_0}{\Sigma p_1 q_1} \times \frac{\Sigma p_1 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_1 q_0}{\Sigma p_1 q_0}} = 1$$

3.4.2. y{; mRidj.k ijh{k.k (Factor Reversal Test)

fiQ'kj d vuqlkj] nk le;k d vUrj ifjoru l vlxr iQy u ikIr gk mlh idkj ;g Hkh lHko gkuk pkfg, fd eY; ,o ek=kdk d ifrLFkkiu dju ij Hkh vlxr ifj.kke u vk,A vFkkr nkuk fu"d"kk d vkil e x.kk dju ij okLrfod eY; vuikr ikIr gkA

bldk lehdj.k d :i e bl idkj fy[kk tk ldrk g %

$$P_{01} \times P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

$$Q_{01} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$= \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$$

$$= \sqrt{\frac{(\Sigma p_1 q_1)^2}{(\Sigma p_0 q_0)^2}} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

3.4.3. pØh; ijh{k.k (Circular Test) :

pØh; ijh{kk mRØEirk ijh{k.k dk foLrr :i gA tc nk l vf/d o"k fn, gk rk fdlh lpdkd dh x.kuk fof/ pØh; ijh{k.k djxhA

$$P_{01} \times P_{12} \times P_{20} = 1$$

P_{01} = igy ox dk vk/kj ekudj cuk f}rh; o"k dk lpdkd

P_{12} = f}rh; o"k dk vk/kj ekudj cuk rrh; o"k dk lpdkd

P_{20} = rrh; o"k dk vk/kj ekudj cuk iFke o"k dk lpdkd

lpdkdk d vf/dkj iQkeyk bl ijh{k.k dk lr"V ugh djr gA fiQ'kj dk vkn'k l=k Hkh bl ijh{kk ij db ckj lgh ugh mrjrkA ;g v/kfjr ;k fuf'pr Hkkfjr lePp; ;k ljy x.kkÙk ekè; l fudky x, lpdkdk e bLreky fd;k tkrk gA

Example VI. Calculate the index number from the following data using Fisher's ideal formula and show how it satisfies the time and factor reversal Test.

Commodity	Base Year		Current Year	
(oLr,)	Price	Qty.	Price	Qty.
A	5	25	6	30
B	4	20	5	
C	2	18	4	20

Solution.

Commodity	Base Price p_0	Year Qty. q_0	Current Price p_1	Year Qty. q_1	p_0q_0	p_0q_0		p_1q_1
A	5	25	6	30	125	150	150	180
B	4	20	5	16	80	100	64	80
C	2	18	4	20	36	72	40	80
					Σp_0q_0 = 241	Σp_1q_0 = 322	Σp_0q_1 = 254	Σp_1q_1 = 340

$$P_{01} = \sqrt{\frac{\Sigma p_0 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

$$= \sqrt{\frac{322}{241} \times \frac{340}{254}} \times 100$$

$$= \sqrt{1.788} \times 100$$

$$= 1.337 \times 100 = 133.7$$

Time Reversal Test :

$$P_{01} \times P_{10} = 1$$

$$= \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times \sqrt{\frac{\Sigma p_0 q_1}{\Sigma q_1 p_1} \times \frac{\Sigma q_0 p_0}{\Sigma q_1 p_0}}$$

$$= \sqrt{\frac{322}{241} \times \frac{340}{254}} \times \sqrt{\frac{254}{340} \times \frac{241}{322}}$$

$$= \sqrt{\frac{322}{241} \times \frac{340}{254} \times \frac{254}{340} \times \frac{241}{322}} = \sqrt{1} = 1$$

So, Fisher's ideal formula satisfies the Time Reversed Test.

Factor Reversal Test :

$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

$$= \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times \sqrt{\frac{\Sigma p_0 q_1}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$= \sqrt{\frac{322}{241} \times \frac{340}{254} \times \frac{254}{241} \times \frac{340}{322}}$$

$$= \sqrt{\frac{340}{241} \times \frac{340}{241}} = \frac{340}{241}$$

Here $$\frac{340}{241} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

Here also, the Fisher's ideal Index number also satisfies the Factor Reversal Test.

3.5. thou fuokg lpdkd (Cost of Living Index Number)

,d fo'k"k oxZ ij eY;k e ifjoruk dk iHkko Kkr dju d fy, thou fuokg lpdkd dk i;kx fd;k tkrk gA thou fuokg lpdkd dk miHkkDrk eY; Hkh dgk tkrk gA depkfj;k d egxkb HkÙk (Dearness Allowance) dk fu/kj.k Hkh blh vk/kj ij fd;k tkrk g rFkk ljdkj Hkh viuh vkfFkd uhfr;k dk fu/kj.k blh vk/kj ij djrh gA miHkkDrk eY; lpdkd dh jpuk d fy, oLrvk dk ie[k ikp Jf.k;k e foHkkftr fd;k tkrk gA

(i) [kk| lkexh] (ii) oL=k] (iii) b/u ,o idk'k] (iv) edku dk fdjk;k] (v) fofo/

thou fuokg lpdkd oQN ekU;rkvk ij vk/kfjr g %

(i) ox fo'k"k d lHkh 0;fDr;k dk jgu&lgu yxHkx ,d tl ekuk tkrk gA

(ii) oLrvk d eY; ,d LFkku l nlj LFkku ij yxHkx ,d leku gA

(iii) ftu oLrvk dk 'kfey fd;k tkrk g o oLr, 0;fDr;k dk lgh jgu&lgu dk ifrfufèkRo djrh gA

(iv) ; lpdkd vkIr :i l ykx gkr gA i.k :i l ughA

miHkkDrk eY; lpdkd jpuk dh fof/;k (Methods of Construction of Consumer Price Index Number) :

(i) legh 0;; fof/ (Aggregative Expenditure Method)

(ii) ikfjokfjd vk; 0;kid fof/ (Family Budget Method)

legh 0;; fof/ }kjk lpdkd Kkr dju d fy, fuEu l=k dk i;kx fd;k tkrk g—

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \times 100$$

Where P_{01} = Consumer Price Index Number

ikfjokfjd vk;&0;kid fof/ }kjk lpdkd bl idkj Kkr fd;k tkrk g—

$$P_{01} = \frac{\Sigma pv}{\Sigma v}$$

Where P = Price Relatives

$$\text{Price Relative} = \frac{P_1}{P_0} \times 100$$

$V = p_0 q_0$

$\sum P_1$ = Total of Product of Price Relatives and Weights

ΣV = Total of Weights

3.6. lpdkd jpuk lc/h fofo/ leL;k,

(Various Problems Regarding Construction of Index Numbers) :

3.6.1. vk/kj ifjorlu (Base-Conversion) :

dHkh&dHkh vko';drkuqlkj fLFkj vk/kj d lpdkd dk J[kyk vk/kj o vk/kj dk fLFkj vk/kj e ifjofrr djuk iMrk gA bld lc/ e fu;e bl idkj g—

(i) fLFkj vk/kj l Ük[kyk vk/kj (From Fixed Base to Chain Base) :

Formula :

$$\text{Chain Base Index Number} = \frac{\text{Current Year Fixed Based Index Number}}{\text{Previous Year Fixed Base Index Number}} \times 100$$

(ii) Ük[kyk vk/kj l fLFkj vk/kj (From Chain Base to Fixed Base) : igy o"k dk fLFkj vk/kj dk eY; k[kyk okyk gh ekuk tkrk g ;fn igy o"k dk vk/kj ekuu d fy, dgk g rk lpdkd 100 ekuk tk,xkA

vxy o"k l lpdkd d fy, fuEu l=k dk i;kx gkxkA

Fixed Base Index Number

$$= \frac{\text{Current Year Chain base Index Number} \times \text{Previous Year Fixed Base Index Number}}{100}$$

Example VII. From the Fixed Base Index Numbers given below, prepare Chain base Index Nos.

Year	1988	1989	1990	1991	1992	1993
Index No.	100	110	120	125	140	160

Solution. **Construction of Chain Base Index Nos.**

Year	Fixed Base Index No.	Conversion	Chain Base Index Nos.
1998	100	—	100
1989	110	$\frac{110}{100}\times100$	110
1990	120	$\frac{120}{110}\times100$	109
1991	125	$\frac{125}{120}\times100$	104.17
1992	140	$\frac{140}{125}\times100$	112
1993	160	$\frac{160}{140}\times100$	114.28

Example VIII. From the Chain Base Index Numbers given below, prepare Fixed Base Index Nos.

Year	1988	1989	1990	1991	1992	1993
Index No.	90	105	102	95	99	120

Solution. **Construction of Fixed Based Index Nos.**

Year	Fixed Base Index Nos.	Conversion	Chain Base Index Nos.
1988	90	—	90
1989	105	$\frac{105\times90}{100}$	94.5
1990	102	$\frac{102\times945}{100}$	96.4
1991	95	$\frac{95\times96.4}{100}$	91.6
1992	99	$\frac{99\times916}{100}$	90.7
1993	120	$\frac{120\times90.7}{100}$	108.84

3.6.2. vk/kj o"k ifjoru (Base Shifting) :

tc fdlh fn, g, o"k d LFkku ij vU; fdlh o"k dk vk/kj ekudj lpdkdk dh x.kuk dh tkrh g rk bl vk/kj o"k dk ifjoru (Base Shifting) dgk tkrk gA

(i) tc lpdkd Kkr dju dk igy okyk vk/kj o"k vf/d ijkuk gku ijA

(ii) tc lpdkd fHkUu&fHkUu o"kk ij vk/kfjr gkr gkA ryuk dju d fy, ,d vk/kj cuk;k tkrk gA bld fy, nk fof/;k bLrekey dh tkrh gA

(i) lf{kIr fof/ (Formula) : New Base Index No.

$$= \frac{\text{Old Index No. of Current Year}}{\text{Old Index No. of New Base Year}} \times 100$$

(ii) iufuek.k fof/—bl fof/ d vulkj fy, o"k dk vk/kj ekudj lpdkdk dh x.kuk u, fLj l dh tkrh gA ;g fof/ tc i;kx dh tkrh g rc lpdkdk dh jpuk lekUrj ekè; vFkok eè;dk dh lgk;rk l dh xbZ gkA

3.6.3. f'kjkcU/u (Splicing)

dHkh&dHkh fuf'pr vk/kj ij Kkr fd, x, lpdkdk dk cUn dj fn;k tkrk vkj cUn gku oky o"k dk vk/kj eku dj u, lpdkd ekyk r;kj dh tkrh g rk ml f'kjkcU/ d uke l tkuk tkrk gA ,lh ifjfLFkfr e ub lpdkd dk ijkuh lpdkd J.kh l ,lk lek;ktu fd;k tk, fd ryukRed vè;;u lHko gk ldA

bld fy, fuEu l=k dk i;kx fd;k tkrk gA

Spliced Index No.

$$= \frac{\text{Old Index No. of Current Year} \times \text{Old Index No. of New Base Year}}{100}$$

Example IX. Given below are two sets of Index, one with 1980 as base the other with 1984.

First	Years	1980	1981	1982	1983	1984
	Index No.	100	120	145	172	200
Second	Years	1984	1985	1986	1987	1988
	Index No.	100	110	116	125	150

Splice the second set of Index Numbers from 1980.

Solution.

Years	Old Index No.	New Index No.	Splicing	Spliced Index No.
1980	00	—	—	—
1981	120	—	—	—
1982	145	—	—	—
1983	172	—	—	—
1984	200	100	$\frac{100\times200}{100}$	200
1985	—	110	$\frac{110\times200}{100}$	220
1986	—	116	$\frac{116\times200}{100}$	232
1987	—	125	$\frac{125\times200}{100}$	250
1988	—	150	$\frac{150\times200}{100}$	300

Example X. From the following data prepare Index Numbers for real wages of workers.

Years	1986	1987	1988	1989	1990	1991
Wages (in Rs.)	300	340	450	460	475	570
Price Index No.	100	120	220	230	250	300

Solution.

Years	Wages	Price Index	Real Wages $\frac{\text{Money Wage}}{\text{Price Index}}\times 100$	Real Wages Index No.
1986	300	110	$\frac{300}{100}\times 100 = 300$	100
1987	340	120	$\frac{340}{120}\times 100 = 28333$	$\frac{28333}{100}\times 100 = 94.44$
1988	450	220	$\frac{450}{220}\times 100 = 20455$	$\frac{20455}{300}\times 100 = 68.18$
1989	460	230	$\frac{460}{230}\times 100 = 200$	$\frac{200}{300}\times 100 = 66.67$
1990	475	250	$\frac{475}{250}\times 100 = 190$	$\frac{190}{300}\times 100 = 63.33$
1991	570	300	$\frac{570}{300}\times 100 = 190$	$\frac{190}{300}\times 100 = 63.33$

4. lkjk'k (Summary)

fun'kkd ;k lpdkd nk le;ckf/ vFkok nk LFkkuk e e fdlh rF; e g, ifjoru dk lki{k eki gA lpdkd ,d fo'k"k idkj d ekè; gkr g ftud }kjk dky J.kh ,o LFkku J.kh e gku oky lki{k ifjoruk dh lkekU; iofÙk dh eki dh tkrh gA fun'kkd lno l[;k e O;Dr fd; tkr gA fun'kkd e[;r;k pkj idkj d gkr gA (i) dher lpdkd] (ii) ek=kk lpdkd] (iii) oQy&eY; lpdkd o (iv) fo'k"k mí'; lpdkdA vkfFkd txr e lpdkdk dk vkfFkd ok;ekid ;U=k (Economic Barometers) dh lKk nh tkrh gA lpdkdk dk cuku l io bld fuek.k dk mí'; Hkyh&Hkkfr Li"V dj yuk pkfg, D;kfd fuek.k l[;k lHkh ifØ;k, mí'; ij gh fuHkj djrh gA fun'kkd dk fuek.k djr le; eY;k dk puko i;kIr lko/kuh rFkk lroQrk d lkFk djuk pkfg, D;kfd ifjoru bUgh d vk/kj ij eki tkr gA fofHkUu oLrvk ;k ink dk ryukRed egÙo dk idV dju gr fdlh lfuf'pr vk/kj ij Hkkjk (Weights) dk i;kx fd;k tkrk gA miHkkDrk eY; lpdkd ,d ox&fo'k"k d jgu&lgu e gku oky ifjoruk dk crkrk gA miHkkDrk eY; lpdkd dk i;kx ljdkj eY; fu;U=k.k] egxkb HkÙkk o U;ure etnjh bR;kfn dk fu/kj.k dju e djrh gA

5. iLrkfor iLroQ (Recommended Books)

(i) Introduction to Statistics – By Prof. R.P. Hooda

(ii) Statistical Methods – By S.P. Gupta

(iii) Business Statisics – By S.C. Sharma, R.C. Jain

(iv) Business Statistics – By T.R. Jain

(v) Business Statistics – By Oswal, Aggarwal & Sharma and Khurana

6. vH;kl d fy, i'u (Self Assessment Questions)

(1) lpdkdk dh ifjHkk"kk nhft, ,o mld mi;kxk ij idk'k Mkfy,A

(2) ^^lpdkd vkfFkd ok;eki ;U=k gA** bl dFku dh O;k[;k dhft, vkj ;g Hkh crkb, fd fdlh idkf'kr lpdkd dk i;kx djr le; fdl idkj dh lko/kfu;k è;ku e j[kuh pkfg, \

(3) fuEufyf[kr ij O;k[;kRed fVIif.k;k fyf[k,—

(i) viLiQhfr (Deflating)

(ii) vk/kj ifjoru (Base Shifting)

(iii) f'kjkcU/u (Splicing)

(iv) pØh; ijh{k.k (Circular Test)

(4) From the following information construct Index Number of prices taking Average price as the base.

Rate per Ten Rupees

Year	Wheat	Rice	Sugar
I	10 kgms	2.5 kgms	5 kgms
II	8 kgms	2.0 kgms	5 kgms
III	5 kgms	1.6 kgms	2 kgms

(5) Construct the fisher's Ideal Index from the following data :

	Base Year		Current Year	
Articles	Price per Unit (Rs.)	Total Expenditure (in Rs.)	Total Value (in Rs.)	Quantity (kgs.)
A	6	300	560	56
B	2	200	240	120
C	4	240	360	60
D	10	300	288	24
E	8	320	432	36

B.Com. Part-II

Paper - BC–204 : Business Statistics *Revised By :* **Dr. Heera Lal Sharma**

Lesson No. : 8

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dky J.kh dk fo'y"k.k

(Analysis of Time Series)

lajpuk (Structure)

1. ifjp; (Intorduction)
2. mís'; (Objective)
3. fo"k; dk iLrqrhdj.k (Presentation of Contents)
    - 3.1 dky J.kh dk vFkZ ,oa ifjHkk"kk
    - 3.2 dky J.kh dk egÙo
    - 3.3 dky J.kh d ldkVd
        - 3.3.1 iÑfr vFkok miufr
        - 3.3.2 vkorZ ;k ekSleh fopj.k
        - 3.3.3 pØh; mPpkopu
        - 3.3.4 vfu;fer mPpkopu
    - 3.4 vkorZ fopj.kk ,oa pØh; mPpkopuk esa vUrj
    - 3.5 iÑfr dk ekiu dh fof/;k
        - 3.5.1 Lor=k gLr pØ fof/
        - 3.5.2 v¼&ekè; fof/
        - 3.5.3 py ekè; fof/
        - 3.5.4 U;ure oxZ fof/
            - 3.5.4.1 ljy js[kh; iÑfr
            - 3.5.4.2 f}rh;?kkr d njclc oØ fof/
            - 3.5.4.3 pj ?kkrkdh; oØ fof/
    - 3.6 ekufi lehdj.k dk :ikUrj.k
    - 3.7 ekSleh fopj.kk dk ekiu dh fof/;k
        - 3.7.1 ljy vkSlr fof/
        - 3.7.2 cy ekè; fof/
        - 3.7.3 pky ekè; l vuikr fof/
        - 3.7.4 iÑfr vuikr fof/
        - 3.7.5 J[kyk eY;kuqokn fof/
    - 3.8 vLFkdkyhu mPpkopu dk eki
    - 3.9 vfuok;Z mPpkopu dk eki
4. lkjka'k
5. iLrkfor iLrd
6. vH;kl d fy, i'u

1. ifjp; (Introduction)

vkfFkd ,o O;kolkf;d {k=k dh leL;kvk d vè;;u e le; ;k dky dk vR;Ur egÙoi.k LFkku gA le; dh xfr'khyrk d dkj.k gh vkfFkd lkekftd jktufrd vkj O;kolkf;d {k=kk e vud egRoi.k ifjoru gkr jgr gA ; ifjoru fofHkUu {k=kk dk iR;{k ;k viR;{k :i l iHkkfor djr g] ;Fkk] mRiknu] fcØh] mi;kx jk"Vh; vki] tul[;k] vk;kr fu;kr vkfnA bl idkj d ifjorUkk dk vè;;u miHkkDrk O;kolk;h] Ñ"kd jktursk] m?kkxifr] i'kklu o vU; lHkh oxk d fy, vko';d o mi;kxh gA

2. mí'; (Objectives)

bl vè;k; dk vè;;u dju d ckn vki tku tk;x—

(i) dky&J.kh fo'y"k.k dk vFk ,o ifjHkk"kk

(ii) dky&J.kh dk iHkkfor dju oky l?kVdk d ckj e tkudkjh iklr djukA

(iii) iofÙk dk ekiu dh fofHkUu fof/;k dk fo'y"k.k djuk

(iv) dky&J.kh fo'y"k.k dh mi;kfxrk dk tkuuk

(v) ekSleh fopj.kk dk Kkr dju dh fof/;k dk eY;kdu djukA

(vi) dky&J.kh d fo'y"k.k l Hkkoh ?kVukvk dk io e vueku yxkukA

3. fo"k; dk iLrrhdj.k

3.1. dky&J.kh dk vFk ,o ifjHkk"kk (Meaning and Definition of Time Series Analysis)

vFk % lk/kj.k 'kCnk e le; d fdlh Hkh eki t l o"k] ekg rFkk fnu vkfn d vk/kj ij iLrr ledk d O;ofLFkr Øe dk dky J.kh dgr gA dky&J.kh d vUrxr nk idkj d pjeY; gkr gA—(i) LorU=k pj&eY; (Independent Variables) rFkk (ii) vkfJr pj&eY; (Dependent Variables)A le; LorU=k pj&eY; gkrk g ,o le; d lkFk&lkFk ledk d eY; e gku oky ifjoruk dk vkfJr ij&eY; dgr g] fuEu mnkjgj.k dky&J.kh d vFk dk Li"V djrk g%

Table – 1		Table – 2	
Time	Production (Tonnes)	Time	Export (in Crores)
2000	150	2000	15
2001	180	2001	18
2002	210	2002	22
2003	250	2003	30
2004	290	2004	45
2005	320	2005	70

dky J.kh dh ifjHkk"kk, (Definitions of Time Series) :

dky J.kh dh dN e[; ifjHkk"kk, fuEufyf[kr g %

(1) duh ,o dhfix d vulkj] le;kuqlkj fn, x, vkdMk d leg dk dkyJ.kh dgr gA

A set of data depending on the time is called a time series Kemry and Keeping.

(2) ØkDLVu ,o dkmMu d vuqlkj] ^dkyJ.kh eas vkdM+s le;kuqlkj Øec¼ gksrs gSaA

"A time series consists of data arranged choronologically'' Croxton and Conden.

(3) iz- cuj fgj"k d vuqlkj—¶dky J.kh fdlh dky vFkok le;U/ d Øfed fcUnvksa d rRleEokn (Corresponding) mlh pj d eY;ksa dh ,d vo LFkk gSA**

A time series is a sequence of values of the same variate corresponding to seccussive points in time'' Prof Warner Hiresh

(4) ;k yu pkd d vuqlkj—^,d dky dk fdlh vkfFkd pj vFkok fefJr pjksa ftudk le;U/ fofHkUu le;kof/;ksa ls gksrk g] d volk/uksa d ladyu d :i e ifjHkkf"kr fd;k tk ldrk gSA*

''A time series may be defined as a collection of readings belonging to different time pervids, of some economic variable or composite of variables'' Yu-Lun-Chou

3.2. dky J.kh dk egÙo (Importance of Analysis of Time Series)

dky J.kh d fo'y"k.k dk vkfFkd 0;kolkf;d ,o lekt d lHkh J=kksa e cgr egRo gSA dky J.kh d vè;;u dk e[; mí's; Hkkoh ?kVukvksa dk lgh vuekuu yxkuk d fy, vkfFkd rF;ksa e gkus oky ifjorukas dk le>uk rFkk eY;kadu djuk] dky J.kh fo'y"k.k d ie[k egRo bl idkj g—

(1) Hkrdkyhu 0;ogkjk dk Kku (Knowledge of Past Behaviour)

dky J.kh d fo'y"k.k l Hkrdkyhu 0;ogkjk dk Kku ljyrk l gks ldrk gSA bu 0;ogkjksa dk lgh fo'y"k.k djd egRoi.kZ fu"d"kZ fudkydj budk vk/kj ij vkfFkd 0;ogkjk dk fu;f=kr dj ldr gSA

(2) Hkfo"; d fy, vueku (To Forecast the Future Behaviour)

dky J.kh d fo'y"k.k l Hkkoh iofÙk;k o ifjorukas dk iokueku lEHko gkrk g vkj bl iokueku d vk/kj ij m|kx] 0;kikj] mRiknu] miHkkx o Ñf"k vkfn d ckj e iHkkoh ;ktukvksa dk fuek.k fd;k tkrk gSA

(3) 0;kikj pØ dk vueku (Expectation of Business Cyele)

vkfFkd txr e ik;% lef¼ (boom), voufr (recession), volkn (depression) rFkk iu#RFkku (recovery) dh pØh; xfr n[kus dks feyrh gSA dky J.kh d fo'y"k.k d vk/kj ij pØh; mPpkouk dk fo'y"k.k djd vkxkeh 0;kikj pØk dk iokueku yxk;k tk ldrk g vkj vu:i ;ktukvksa dk fuek.k fd;k tkrk gSA

(4) ryukRed vè;;u (Comparative Study)

fofHkUu dky Jf.k;ksa d ikjLifjd vè;;u l vudk fu"d"k fudky tk ldr g tl fofHkUu n'kksa dh eR; nj] pje nj] Ñf"k mit nj ifr gDV;j f'k{kk nj vkfn dk tkudj fofHkUu n'kksa d ckj e tkuk tk ldrk gSA

(5) lkoHkkfed mi;kfxrk (Uaiversal Utility)

dky Jf.k;k dh mi;kfxrk doy 0;olk;h o vFk'kkfL=k;k d fy, gh ugh cfYd ik;% lHkh {k=kk e g tl ljdkj Ñ"kd lkekftd jktufrd lLFkk, 'kkèkdrkvk oKkfudk d fy, vkfnA

(6) ifjoruk dk vè;;u (Study of Changes)

dky J.kh dh lgk;rk l thou d fofHkUu {k=kk e fo'k"kr% vkfFkd ,o 0;kolkfud {k=kk e gku oky ifjoruk dk vè;;u fd;k tk ldrk gA

3.3. dky J.kh d l?kVd (Components of Time Series)

dky J.kh dk vud ?kVd iHkkfor djr g vFkkr dky J.kh e vud dkj.kk l ifjoru gkrk gA bu ifjoruk dk gh dky J.kh d l?kVd dgr gA dky J.kh e e[;r% pkj idkj oxk e foHkkftr fd;k tk ldrk gA

3.3.1. iofÙk ;k miufr (Trend)

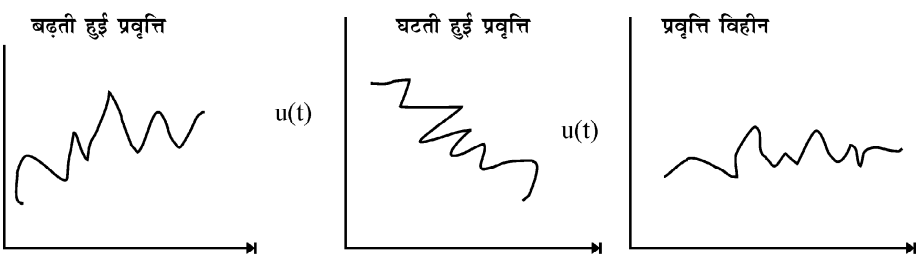
3.3.2. vkorl ;k ekleh fopj.k (Seasonal Variation)

3.3.3. pØh; mPPkkopu (Cyclical Fluctuations)

3.3.4. vfu;fer mPpkopu (Irregular Fluctuations)

3.3.1. iofÙk ;k miufr (Trend)

dky J.kh e nh?kdkyhu iofÙk dk vk'k; nh?kdkyhu e gA fdlh Hkh dky J.kh e yEc≤ e of¼ ;k deh dh LokHkkfod iofÙk gkrh gA mlh dk nh?kdkyhu iofÙk ;k miufr dgr gA mnkgj.kkFk Hkkjro"k e [kk|&inkFkk d eY;k e fnu&ifrfnu mrkj&p<ko gkr jgr g ijUr LorU=krk d ckn l vc rd n[kk tk, rk ;g dguk gkxk fd eY;k e p<u dh iofÙk crkb tk ldrh gA bl idkj ge dg ldr g fd nh?kdkyhu iofÙk og vifjoru iofÙk g tk lkekU; :i l mlh fn'kk e i;kIr vof/ rd jgrh gA ;fn dkb J.kh c<rh g rk ml c<rh gb iofÙk dgr gA ;fn dkb J.kh ?kVrh g rk ml ?kVrh gb iofÙk dgr gA ;fn dkb J.kh u c<rh g u ?kVrh g tk ml iofÙk foghu dgr gA



Fug-(i) Fug-(ii) Fug-(iii)

io`fÙk ekiu d mí'; (Purposes of Measuring Trend)

iÑfr ekiu d e[; :i l rhu mí'; gkr g %

1. vrhr e J.kh d fodkl vFkok voufr dk vè;;u dju d fy,A
2. nh?kdkyhu iokueku dju d fy,A

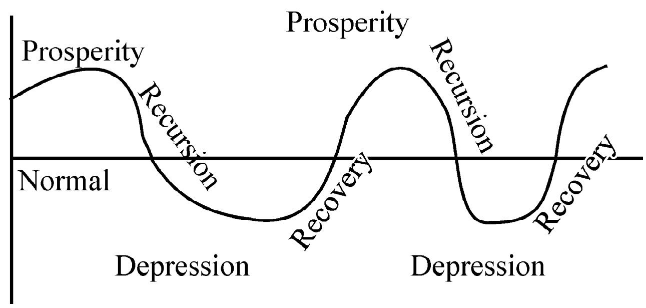
3. miufr ekiu dk rhljk mís'; mlls djuk g ftlls ledk d pØh; rFkk vYidkyhu mPPkkopuk dk Li"V fd;k tk ldA

3.3.2 vkorh vFkok ekSleh fopj.k (Seasocal Variations)

fdlh Hkh dky J.kh e ,d o"k d vUnj gkus oky ifjorukas dk ekSleh fopj.k dgk tkrk gA ; ,d ekg llrkg ,d fnu ;k ifr?kUVk gks ldr gA ,l ifjoru nks dkj.kks l gkr g tyok; vFkok jhfr&fjokt ekSleh ifjorukas dk iHkko mRiknu] fdlh Lo; [kp ij vyx&vyx iMrk gA mnkgj.k d fy, Hkkjr e 'kknh fookg d ekSle d dkj.k lkuk&pknh vkHk"k.k oL=k cru vkfn d nke c< tkr gA

3.3.3. lk/; mPPkopu (Cycfical Finctuations)

; ifjoru Hkh fu;rdkfyd gkr gA vr% ,d fuf'pr le; d ckn rd pyr g ij ekSleh ifjoru dh rjg budh vof/ ,d o"k ;k de ugh gkrhA ; ik;% dN o"kk rd pyr gA O;kikj dh Øekulkj pkj voLFkk, gkrh gA 1. lef¼ 2. ifrUrj 3. vilkn 4. iu#RFkkuA fuEu fp=k e bu pj.kk dk inf'kr fd;k x;k g—



3.3.4. vfu;fer mPpkopu (Irregular Fluctuations)

fu;fer mPPkkopuk d foijhr tc dkb ifjoru vkdfLed :i l ;k vfu;fer :i l gk tk; rk bl vfu;fer mPpkopu dgr gA bl idkj d ifjorUk dh u rk lEHkkouk gkrh g ,o u gh budh fuf'pr vof/A ; ifjoru ;¼ ck<] vdky] HkdEi] puko] vk|kfxd l?k"k ,o gMrky vkfn d dkj.k gkr gA

3.4. vkorh fopj.kk ,o pØh; mPpkopuk e vUrj (Difference between Seasonal Variations and Cyclical Variations)

1. **vof/ (Period) :** vkorh fopj.k dh vof/ ,d o"k l de gkrh g tcfd mPPkkopu 3 o"k l ydj 10 ;k 12 o"k rd gkr gA

2. **fu;ferrk (Regularity) :** vkorh fopj.kk dh vof/ vkj Øe nkuk e fu;ferrk gkrh gA pØh; mPpkopuk e lef¼] ifrlkj] volkn vkj iu:RFkku rk fuf'pr jgr g yfdu budh vof/ e vUrj gkrk gA

3. **ifjoru d dkj.k (Reasons for Variations) :** vkorh fopj.k tyok;] ekSle jhfr&fjokt vknr o iQ'ku vkfn e ifjoru l gkr g tcfd pØh; mPPkkopu eæk d ilkj ;k ldpu foØ; e of¼ ;k deh ljdkjh uhfr;k vkfn e ifjoru d dkj.k gkr gA

4. **idV gku dk Øe (Chronology of Reflection) :** ekSleh ifjorZu iR;sd 0;olk; esa vyx&vyx Øe ls idV gksrs gSa tcfd pØh; ifjoru iR;sd 0;olk; esa leku :i ls idV gksrs gSaA

3.5. iofÙk dk ekiu dh fof/;k (Methods of measuring Trends) :

nh?kZdkyhu iofÙk Kkr djus ds fy, fuEu fof/;ksa dk iz;ksx fd;k tkrk g—

3.5.1 **LorU=k gLr oØ fof/** (Free-Hand Curve Method)

3.5.2 **v¼ ekè; fof/** (Moving Average Method)

3.5.3 **pyekè; fof/** (Moving Average Method)

3.5.4 **U;wure oxZ fof/** (Method of Least Squares)

3.5.4.1 **ljy js[kh iofÙk** (Straight Line Method)

3.5.4.2 **f}rh;?kkr dk ijoy; oØ** (Second Degree Parabola Curve)

3.5.4.3 **pj?kkrkdh; oØ** (Exponential Curve)

3.5.1 Lor=k gLr oØ fof/ (Free-Hand Curve Method)

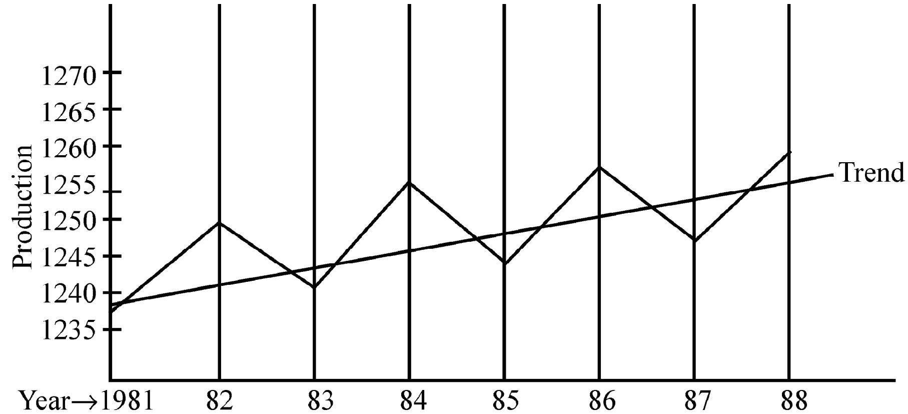
nh?kZdkyhu iofÙk dk Kkr djus dh ;g lcls ljy fof/ gSA bl fof/ ds vuqlkj igys dky J.kh dks eY;ksa dks fcUnqJ.kh; i=k ij vfdr djrs gSA blesa le; dks X-axis ij rFkk fn;s g;s pj eY;ksa dks Y-axis ij mfpr ekin.M ydj fn[kykrs gSA rRIk'pkr vkdMksa ds mrkj&p<ko dks è;ku esa j[krs g;s ,d ljfyr oØ [khpk tkrk gS tksfd dky J.kh ds mPpkopuks ds eè; ls gksrk gqvk xtjsA ;g oØ gh eDr gLr oØ gksrk gSA bl fof/ dk fujh{k.k }kjk oØ vUok;ktu (Curve fitting by inspection) dgk tkrk gSA ;g fof/ ljyre g] ykp'khy g rFkk vklkuh ls le> esa vku ;kX; gSA ;g jhfr i{kikri.kZ g ,o blesa 'k¼rk dk vHkko jgrk gSA

Example 1 :

From the following data fit a trend using "Free hand Carve Method"

Year	Production of Product X (in tonnes)
1881	1236
1982	1250
1983	1242
1984	1255
1985	1245
1986	1258
1987	1250
1988	1265

Solution : lcls igys ledks dk xkiQ esa Dotted Line }kjk fn[kkbA blds ckn ,d ljy js[kk bl idkj [khp fd ;g bu Dots ds eè; esa gksrk gqvk xtjsA



fp=k

Production and Trend of Product X

3.5.2 v¼ekè; fof/ (Semi Average Method)

bl fof/ d vulkj lcl igy djrk J.kh dk nk cjkcj Hkkxk e ckV fy;k tkrk g vkj mld ckn iR;d viu Hkkx dk vyx&vyx lekUrj x.kuk Kkr fd;k tkrk gA nkuk ekè;k dk ml le;kof/ d eè; d viu xkiQ ij vfdr fd;k tkrk gA bu nkuk fcUnvk d feyku l tk l[r j[kk curh g] ml iofÙk j[kk dgr gA

v¼ekè; fof/ dk vè;;u ge nk fLFkfr;k e djr g—

(i) tc J.kh e fn, x, o"kk dh le l[;k gk—

tc dky J.kh e fn; x, o"kk dh le l[;k gk] vFkkr 4, 6, 8, 10, 12 bR;kfn gk rk ml J.kh dk nk cjkcj Hkkxk e vklkuh l foHkkftr dj yx o nkuk Hkkxk dk lekUrj ekè; fudky yx vkj xkiQ iij ij vfdr dj yxA

(ii) tc J.kh e fn; x, o"kk dh fo"k; l[;k gk—

tc J.kh e fn, x, o"kk dh l[;k fo"ke gk vFkkr 5, 7, 9, 11, 13 bR;kfn gk rk ml nk Hkkxk e ckVu dh leL;k vk;xhA , l e J.kh d fcYdy chp d vd dk NkM nuk pkfg,A 'k"k ifØ;k ioor jgrh gA

x.k rFkk nk"k&v¼&ekè; jhfr l iofÙk dk vueku ljyrk l yxk;k tk ldrk g rFkk i{kikr dh Hkkouk l Hkh viHkkfor jgrh gA yfdu bldh lhek, Hkh g—(1) ;g fof/ j[kh; iofÙk d le; gh mfpr g (2) ;fn pje eY;k dh mifLFkfr d dkj.k lekUrj ekè; vokLrfod gk tk, rk ,lh fLFkfr e ;g jhfr mfpr ugh gA

Example : 2

Find the trend of the following data using semi-everage method—

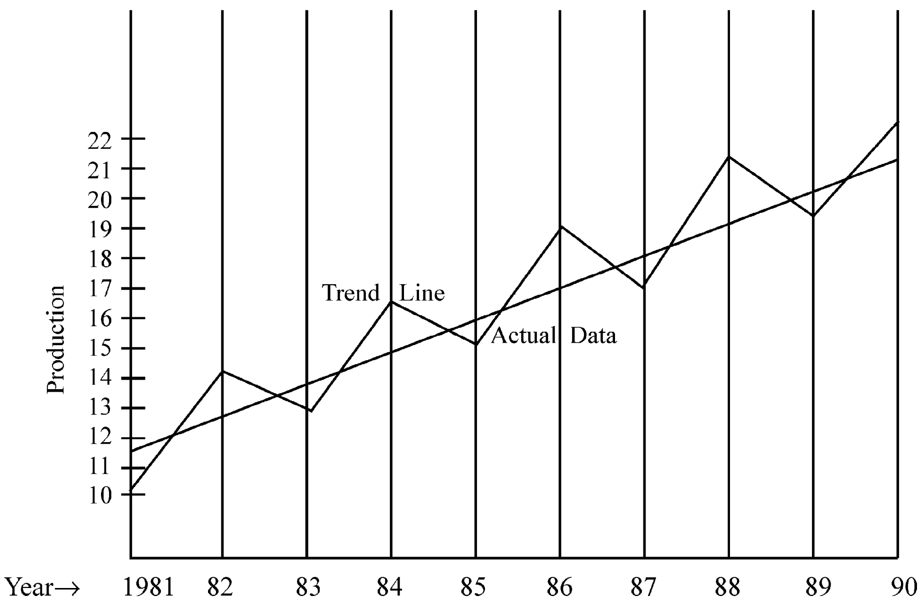
Year	81	82	83	84	85	86	87	88	89	90
Sales	10	14	12	16	14	18	16	20	18	22

Solution :

Year	Sales	Total		Semi-average
81	10			
82	14	$\frac{10+14+12+16+14}{5}=\frac{68}{5}$	=	136
83	12			
84	16			
85	14			
86	18			
87	16	$\frac{18+16+20+18+22}{5}=\frac{95}{5}$	=	18·8
88	20			
89	18			
90	22			

ukV % 13.6 o"k 83 d vkx ,o 18.8 o"k 88 d vkx fy[kh tk,xhA

Actual and Trend of Sales



3.5.3 py ekè; fof/ (Moving Average Method)

bl fof/ e ekè; py jhfr }kjk fy, tkr gA py ekè; fdru o"k dk gk] ;g bl ckr ij fuHkj djrk g fd dky J.kh d ledk e pØh; mrkj&p<ko fdru o"kk e gkrk gA py ekè; ik;% 3 o"kk l ydj 11 o"kk rd gkrk gA bldh x.kuk ifØ;k fuEufyf[kr g—

(i) igy ekè; dh vof/ fuf'pr djxA

(ii) ;g vof/ ;fn 5 o"k g rk dky J.kh d iFke ikp o"kk d eY;k dk tkMdj rhlj o"k d lkeu j[kxA

(iii) bld i'pkr iFke o"k dk NkMdj vxy ikp o"kk d eY; dk tkMdj pkFk o"k d lkeu j[kxA ;g fØ;k rc rd djr jgx tc rd J.kh d vfUre in rd u igp tk;A bl ifØ;k l tk led ekyk r;kj gkxh og Moving Totals dk inf'kr djxhA

(iv) py ekè; ;kxk dk 5 l (fu/kfjr vof/ l) Hkkx nxA tk HkkxiQy ikIr gkxk] ogh py ekè; gkxA ; py ekè; gh nh?kdkyhu iofÙk dk fn[kk,xA

;fn py ekè; vof/ le gk vFkkr 4 o"k g rk igy pkj o"kk d eY;k dk ;kx djd nlj o rhlj o"k d ekè; fy[kx vkj igy o"k d eY; dk NkMdj vxy pkj o"k d eY;k d ;kx dk rhlj o PkkFk o"k d chp fy[kxA

bld ckn tk ;kx nlj o rhlj o"k d chp rFkk rhlj o pkFk o"k d chp g mUg tkMdj rhlj o"k d vkx fy[kx vkj blh idkj vkx ifØ;k nkgjk;xA bl Add - in - pairs ;k Total centrerd djuk dgr gA bld ckn bl dfUær ;kx dk 8 l Hkkx ndj py ekè; Kkr djxA

py ekè; fof/ d x.k rFkk nk"k (Merits and Demerits of Moving Average Method) x.k (Merits)

(i) bl fof/ dh xf.krh; ifØ;k ljy gA

(ii) ;g ykpnkj gA

(iii) bl fof/ e vYidkfyu mPPkkopu ;k rk i.kr;k lekIr gk tkr g ;k dkiQh le; rd fujflr (eliminatc) fd; tk ldr gA

(iv) bl jhfr d i;kx e ekuo dh i{kikr Hkkouk dk dkb iHkko ugh iMrkA

nk"k (Demerits)

(i) bl jhfr dk i;kx iokueku yxku d fy, ugh fd;k tk ldrk gA

(ii) bl jhfr e lekUrj ekè; dk i;kx fd;k tkrk gA vr% lekUrj ekè; dh lhekvk dk iHkko miufr eY;k ij iM fcuk ugh jgxkA

(iii) bl fof/ dk ljyrk l fdlh xf.krh; l=k }kjk O;Dr ugh fd;k tk ldrkA

Example : 3

From the following data find out trend using3-yearly moving average.

Year	1981	1982	1983	1984	1985	1986	1987
Production	412	438	446	454	470	483	490

Solution :

Year (1)	Production (2)	Three-Yearly moving total (3)	Three yearly moving average (4)
1981	412		
1982	438	412 + 438 + 446 = 1296	1296 ÷ 3 = 432
1983	446	438 + 446 + 454 = 1338	1338 ÷ 3 = 446
1984	454	446 + 454 + 470 = 1370	1360 ÷ 3 = 457
1985	470	454 + 470 + 480 = 1407	1470 ÷ 3 = 469
1986	483	470 + 483 + 490 = 1443	1443 + 3 = 481
1987	490		

Examples :

From the the following data find out trend using 4-yearly moving average.

Years	Sales (in 000 units)
1981	2
1982	6
1983	4
1984	8
1985	10
1986	6
1987	12
1988	8
1989	16
1990	20

Solution.

Year	Sales	4-yearly moving totals	4-yearly moving average	Centing total	Centring average
1981	2				
1982	6	20	5	12	6
1983	4	28	7	14	7
1984	8	28	7	16	8
1985	10	36	9	18	9
1986	6	42	9	19.5	9.75
1987	12	56	10.5	24.5	12.25
1988	8		14		
1989	16				
1990	20				

3.5.4 U;ure ox fof/ (Method of Least Squares)

U;ure ox jhfr miufr d eki dh loJ"B jhfr gA ;g jhfr dN xf.krh; fo'k"krkvk dk ijk djr g tkfd fuEufyf[kr gA

(i) $\Sigma (Y - Yc) = 0$

(ii) $\Sigma (Y - Yc)^2$ = minimum

bUgh dkj.kk l bl uhfr dk ^U;ure ox jhfr* dh lKk nh xb gA bld lokfpr j[kk (Line of the Best fit) dgk tkrk gA bl fof/ d vulkj iofÙk Kkr dju d fy, fuEufyf[kr lehdj.k dk i;kx fiQj tkrk gA

$$Yc = a + bx$$

bl h lehdj.k e

Yc = iofÙk dk eku (computed trend value)

a = ‘a’ is a constant. It is intercept of Y i.e. it represents value of Y Variable when x = 0

b = ‘b’ is another constant and represents slope of trend fine

vUnj a ,o b dk eku fuEufyf[kr lehdj.kk l Kkr fd;k tkrk gµ

$$\Sigma Y = Na + b\Sigma X \quad \text{...(i)}$$

$$\Sigma XY = a\Sigma X + b\Sigma X^2 \quad \text{...(ii)}$$

3.5.4.1 ljy js[kh; iofÙk vUok;ktu (Fitting a Straight Line Trend)

$$Yc = a + bX$$

Example—

From the following data fit a straight line trend by using least square method/

Year	1977	1978	1979	1980	1981	1982
Sales (m' 000 Rs.)	10	12	15	16	18	19

Solution :

Year	Sales	X	XY	X^2
1977	10	0	0	0
1978	12	1	12	1
1979	15	2	30	4
1980	16	3	48	9
1981	18	4	72	16
1982	19	5	95	25
r = 6	90	15	257	55
	ΣY	ΣX	ΣXY	ΣX^2

The trend values can be obtained from the following equation

$$Yc = a + bX$$

To find out the values of a and b we have two equations

$$\Sigma Y = Na + b\Sigma X \quad \text{...(i)}$$

$$\Sigma XY = a\Sigma + b\,\Sigma X^2 \quad \text{...(ii)}$$

After putting the given value in the above equation we get

$$90 = 6a + 15b \quad \text{...(i)}$$

$$257 = 15a + 55b \quad \text{...(ii)}$$

Multiplying the equation (i) by 5 and equation (ii) by 2 we get

$$450 = 30a + 75\,b$$

$$-514 = 30a + 110b$$

$$-\,64 = 35b$$

$$\frac{64}{35} = b$$

$$b = 1.83$$

Putting the values of b in equation (i) we get

$$6a + 15\,b = 98$$

$$6a + 15\,(64/35) = 90$$

$$6a = 90 - 27.43$$

$$a = \frac{90 - 27.43}{6}$$

Hence $\quad Yc = 10.43 + 1.83\,(b)$

Estimating the trend values

$$XYc = a + bX$$

1977	0	10.43 + 1.83 (0) = 10.43
1978	1	10.43 + 1.83 (1) = 12.26
1979	2	10.43 + 1.83 (2) = 14.09
1980	3	10.43 + 1.83 (3) = 15.92
1981	4	10.43 + 1.83 (4) = 17.75
1982	5	10.43 + 1.83 (5) = 19.58

Example : Given below is the Statistics related to the number of accident in certain city.

Year	No. of accidents
1981	77
1983	88
1984	94
1985	85
1986	91
1987	98
1900	90

Fit a straight line trend using the method of least square and also find out monthly increase/decrease.

Solution :

Year	No. of Accidents	Deviations		
	Y	X	X^2	XY
1981	77	–4	16	–308
1983	88	–2	4	–176
1984	94	–1	1	–94
1985	85	+ 0	0	0
1986	91	+ 1	1	91
1987	98	+ 2	4	196
1990	90	+ 5	25	450
n = 7	623	1	51	159
	ΣY	ΣX	ΣX^2	ΣXY

$$Yc = a + Bx$$

Kkr dju d fy, a ,o b dk eku fuEu nk lehdj.kk l Kkr fd;k tk,xkA

$$\Sigma Y = Na + b\Sigma X$$

$$\Sigma XY = a\,\Sigma X + b\,\Sigma X^2$$

$$623 = 7a + 1b \qquad ....(i)$$

$$159 = a + 51\,b \qquad ....(ii)$$

lehdj.k (ii) dk 7 l x.kk dju ij 623 = 7a + b

$$1113 = 7a + 357\,b$$

$$-\ 490 = -\ 356\,b$$

$$\frac{-490}{-356} = b$$

$$1.38 = b$$

vc igy lehdj.k e b dk eY; j[ku ij

$$623 = 7a + b$$

$$623 = 7a + 1.38$$

$$621.62 = 7a$$

$$621.62/7 = a$$

$$88.803 = a$$

$$Yc = 88.803 + 1.38\ b$$

The value of b is + 1.38, hence monthly increase shall be monthly increase

$$= b/12 = 1.38/12 = 0.115 \text{ Ans.}$$

3.5.4.2 f}rh;?kkr d cjcyu }kjk miufr dh x.kuk

vkfFkd ,o O;kolkf;d {k=k e ,lh ifjfLFkfr;k gkrh g ftue ljy j[kk dky J.kh d vkdMk dh nh?kdkyhu iofÙk dk lgh :i l iLrrhdj.k ugh dj ikrhA ,lh n'kk e dky J.kh d ,d l vf/d Hkkx djd gj ,d d fy, fofHkUu lehdj.k ikIr fd, tkr gA bld fy, fuEu lehdj.k dk i;kx gkrk gA

$$Y = a + bx + cx^2 + dx^3 + ex^4 + ...$$

;fn ;g l=k x d f}rh;?kkr rd tk; rk ml ,dfUær f}rh;?kkr oØ (Parabolar of the Second Degree) vkj rrh;?kkr rd c<k;k tk, rk ml ,dfUær rrh;?kkr oØ dgr gA ;gk ge f}rh;?kkr oØ dk foopu djxA bldk lehdj.k gµ

$$Y = a + bx + cx^2$$

tgk a, b, c vpj eY; gA bu vpj eY;k dh x.kuk fuEu ilkekU; lehdj.kk (Normal equations) dk gy djd dh tk ldrh gA

$$\Sigma Y = Na + b\Sigma X + c\Sigma X^2 \qquad ....(i)$$

$$\Sigma XY = a\Sigma X + \Sigma X^2 \ + c\Sigma X^3 \qquad ....(ii)$$

$$\Sigma X^2 Y = a\Sigma X2 + b\Sigma X^2 + c\Sigma X^4 \qquad ....(iii)$$

Short-cut Method : (When $\Sigma X = 0$ and $\Sigma X^2 = 0$)

$$\Sigma Y = Na + a\Sigma X^2 \qquad ....(i)$$

$$\Sigma XY = b\Sigma X^2 \qquad ....(ii)$$

$$\Sigma X^2 Y = a\Sigma X^2 + c\Sigma X^4 \qquad ....(iii)$$

$$\text{Now } b = \frac{\Sigma XY}{\Sigma X^2} \quad a = \frac{\Sigma Y - C\Sigma X^2}{N}$$

$$C = \frac{N\Sigma X^2 Y - (\Sigma X^2 X \Sigma Y)}{N = \Sigma X^4 - (\Sigma X^2)^2}$$

Example : The following table relates to the urban population in the form of percentage to the total population

Cennus Year	1951	1961	1971	1981	1991
% Population	11.4	12.1	13.9	17.3	18.0

Fit a parcabolic $Y = a + bx + cx^2$ to these data. Estimate trend- value of population for the year 2001.

Solution :

f}&Hkkjrh; miufr&eY;k dk ifjdyu

Year	Y	X	XY	X^2	X^4	$Yc = a + bX + cX^2$
1951	11.4	–2	–22.8	4	16	11.0882
1961	12.1	–1	–12.1	1	1	12.5853
1971	13.9	0	0	0	0	14.3111
1981	17.3	+1	17.3	1	1	16.2653
1991	18.0	+2	36.0	4	16	18.4482
5	72.7	0	18.4	10	34	72.6981
N	ΣY	ΣX	ΣXY	ΣX^2	ΣX^4	ΣYc

I	II	III
$\Sigma Y = Na + c\Sigma X^2$	$\Sigma XY = b\Sigma X^2$	$\Sigma Y = a\Sigma X^2 + c\Sigma X^4$
72.7 = 5a + 10c	18.4 = 106	147 = 10a + 34c
72.7 = 5a + 10X. 1143	b = 1.84	145.4 = 10a + 20c
5a = 72.7 – 1.143		1.6 = 14c
a = 14.311		1 dk 2 l x.kk dju ij c = 0.1143

vc lehdj.k Y = 14.311 + 1.84 + 0.1143 e X d eku j[ku ij

X = – 2 = 14.311 + (1.84 X – 2) + (0.1143 X – 2^2)

X = – 1 = 14.311 – 3.68 + .4572 = 11.0882

X = – 1 = 14.311 + (1.84 X – 1) + (0.1143X – 1^2)

14.311 – 1.84 + 0.1143 = 12.5853

X = 0 = 14.311

blh idkj X = + 1 o X = + 2 d miuhr eY; Øe'k% 16.2653 o 18.4482 gkxA 2001 d fy, vueku : 2001 d fy, X cjkcj gkxkµ

$$X = \frac{2001 - 1971}{10} \leq \frac{30}{10} = 3$$

Hence

Y = 14.311 + (1.84 × 3) + (0.1143 × 3^2) = 20.8597 or 20.86 Approx

3.5.4.3 ij ?kkrkdh; oØ fof/ (Expenential Curve)

tc fdlh dky J.kh e ,d LFkk;h ifr'kr nj l deh ;k of¼ gkrh g rk ,lh fLFkfr e ?kuk'k pØ vFkok v¼&y?kx.kdh; oØ dk i;kx fd;k tkrk gA blds lehdj.k fuEu idkj gµ

$Y = ab^X$ or $\log Y = \text{Log } a + (\log b)$

vc a rFkk b dk eY; Kkr dju gr fuEu lehdj.kk dk gy djxA

$\Sigma \text{Log } Y = N \log a + \Sigma \text{Log } b . \Sigma X$ ...(i)

$\Sigma (X \log Y) = \text{Log } a \Sigma X + \text{Log } b \, \Sigma X^2$ ...(ii)

Short-cut-Method—;fn le; fopyu eè; ox dh fy, tkr g rk ΣX dk eku 'kU; gkxk or ΣX = 0 rFkk mijkDr lehdj.k fuEu idkj l gk tk,xA

$$\Sigma \text{Log } Y = N \log a + 0 \Rightarrow \log a = \frac{\Sigma \log Y}{N}$$

$$= a = AL \left[\frac{\Sigma \log Y}{N} \right]$$

$$\Sigma (X \log Y) = 0 + \text{Log } b \, \Sigma X^2$$

$$= \log b = \frac{\Sigma (X \log Y)}{\Sigma X^2}$$

$$b = AL \left(\frac{\Sigma X \text{ Log } Y}{\Sigma X^2} \right)$$

Example :

The sales of a company in lakhs of Rupees for the years 1997 to 2003 are given below :

Year	1997	1998	1999	2000	2001	2002	2003
Sales	32	47	65	92	132	190	275

Estimate sales for the year 2004 using an equation of the form $Y = ab^2$, where X = years and Y = sales.

Solution : Fitting equation of the form $y = ab^x$

Year	Sales Y	X	Log Y	X Log Y	X^4	Trend Values
1997	32	–3	1.5051	–4.5153	9	32.24
1998	47	–2	1.6721	–3.3442	4	45.96
1999	65	–1	1.8129	–1.8129	1	65.52
2000	92	0	1.9638	0	0	93.42
2001	132	+1	2.1206	+2.1206	1	131.10
2002	190	+2	2.2788	+4.5576	4	190.00
2003	275	+3	2.4393	+7.3179	9	270.70
		0	+4.3237	13.7926	28	

Exponital Trend : $Y = ab^X$ or $\text{Log } y = \log a + X \text{ Log } b$

By using short cut method $\Sigma \text{ Log } y - \Sigma \text{ Log } a$

or Log a = $\frac{\Sigma \log Y}{N} = \frac{13.7926}{7}$ = 1.9704

Similarly Σ X Log y = log b Σ X^2; log b = $\frac{\Sigma X.LogY}{\Sigma X^2} = \frac{4.3237}{28}$ or 0.154

By substituting the values in equation

Year	X	Log y = Log a + X log h	Trend Al I
1997	–3	1.9704 + (–3) (0.154) = 1.5084	32.24
1998	–2	1.9704 + (–2) (0.154) = 1.6624	45.96
1999	–1	1.9704 + (–1) (0.154 = 1.8164	65.52
2000	0	1.9704 + (0) (0.154) = 1.9704	93.42
2001	+1	1.9704 + (1) (0.154) = 2.1244	131.10
2002	+2	1.9704 + (2) (0.154) = 2.2784	190.00
2003	+3	1.9704 + (3) (0.154) = 2.4324	270.70
2004	+4	1.9704 + (4) (0.154) = 2.5864	385.90

The estimated sales for the year 2004 = Rs. 385.9 lakhs

3.6 miufr lehdj.k dk :ikUrj.k (Conversion of Trend Equation)

fdlh Hkh miufr lehdj.k dk :ikUrj.k fuEu rhu ?kVdk ij fuHkj djrk gA

(i) fopkjk/khu le; dk ey fcUn

(ii) inÙk eY;k dh bdkb;k (tl fd eY; okf"kd] ekfld] lkIrkfgd)

(iii) le; dh bdkb;k (tl o"k] lIrkg)

miufr lehdj.k dk gekjh lfo/k gr fuEu idkj l ifjofrr dj ldr gA

(i) ey fcUn ifjoru ;k vk/kkj ifjoru

(ii) okf"kd miufr lehdj.k dk ekfld miufr lehdj.k e :ikUrj.k ;g nk idkj l lEHko g tk fj;fr ij fuHkj gkxkA

(i) o"k Y d eY;k dk okf"kd ;kx n j[kk gkA

$$Yc = \frac{a}{12} + \frac{b}{144x}$$

(ii) tc Y dk eY; ekfld vk/kj ij gA

bl fLFkfr e a e dkb ifjoru ugh gkxk ijUr b ifjofrr gk tk;xkA

$$Y = a + \frac{b}{12}$$

3.7 ekSleh fopj.kksa dk ekiu dh fof/;k¡

ekSleh ifjorZu ls vfHkizk; O;kolkf;d fØ;kvksa esa ,sls lefid ifjorZuksa ls gS tks ifro"kZ gksrs jgrs gSA ,sls ifjorZu 12 eghuksa ds vUnj gksrs gSA ekSleh fopj.kksa dk fo'ys"k.k rHkh lEHko gS tc ge J.kh ds l?kVdksa ls nh?kZdkyhu iofÙk] pØh; mPpkopu o vfu;fer ifjorZuksa dks xk.k dj nsA ekSleh ifjorZuksa dk ekiu dh fof/;k¡ fuEufyf[kr gSA

3.7.1 py ek/; fof/

3.7.2 py ekè; fof/

3.7.3 py ekè; ls vuqikr fof/

3.7.4 iofÙk vuqikr fof/

3.7.5 Jª[kyk eY;kuqikr fof/

3.7.1 ljy vkSlr fof/ (Simple Average Method)

ekSleh fopj.kksa dk ekiu dh ;g lcls ljy fof/ gSA ;g fof/ mlle; vf/d mi;qDr gksrh gS tc vkdMksa esa iofÙk dk vHkko ik;k tkrk gSA

fØ;k

(i) nh xbZ dky J.kh dks ekl vFkok =kekfld ledks ds vk/kj ij Øekuqlkj fy[ksaA

(ii) fofHkUu o"kksZa ds iR;sd ekl ds vkdMksa dk ;ksx djsaA

(iii) mijksDr ;ksxksa dh lgk;rk ls iR;sd ekl dk vkSlr eY; Kkr djsaA

(iv) iR;sd ekl ds vkSlrksa dk ;ksx djds iqu% vkSlr Kkr djsaA

(v) blds ckn lkekU; ekè;k dks 100 ekudj gj ,d ekfld vkSlr ekè; fuEufyf[kr lw=k dh enn ls lwpdkad esa cny fn;k tkrk gSA

$$\text{Seasonal Variation Index} = \frac{\text{Average Value of the Month}}{\text{Average Value of the Year}} \times 100$$

Example :

Calculate Seasonal Variation and Index of the following

Months	2000	2001	2002
January	62	59	35
February	69	68	17
March	60	51	10
April	53	38	20
May	44	36	31
June	34	16	36
July	21	8	26
August	37	11	30
September	45	11	23
October	31	17	29
November	65	12	30
December	60	22	26

Months	2000	2001	2002	Total	Average	Seasonal Variation average general average	Seasonal Variation Index
January	62	59	35	156	52.0	17.5	150.7
February	62	68	17	154	51.3	16.8	148.6
March	60	51	10	121	40.3	5.8	116.7
April	53	38	20	111	37.0	2.5	107.2
May	44	36	31	111	37.0	2.5	107.2
June	34	16	36	86	28.7	–5.8	83.1
July	21	8	26	55	18.3	16.2	53.0
August	37	11	30	78	26.0	–8.5	75.3
September	45	11	23	79	26.3	–8.2	76.2
October	31	17	29	77	25.7	8.8	74.4
November	65	12	30	107	35.7	1.2	103.5
December	60	22	26	108	36.0	1.5	104.3

$$\text{General Average} = \frac{\text{Sum of Average}}{\text{Number of Seasons}} = \frac{414.3}{12} = 34.5$$

Seasonal Variation Index

$$\text{for January} = \frac{\text{Average of Jan.}}{\text{General Average}} \times 100 \text{ or } \frac{52}{34.5} \times 100 = 105.7;$$

$$\text{for February} = \frac{51.3}{34.5} \times 100 = 1486$$

$$\text{for March} = \frac{40.3}{34.5} \times 100 = 116.7;$$

$$\text{for April} = \frac{37}{34.5} \times 100 = 107.2 \text{ and soon}$$

3.7.2 py ekè; fof/ (Moving Average Method)

bl fof/ d }kjk ge iofÙk] vYidkfyd ifjoru vkor ,o vfu;fer lHkh idkj d fopj.kk dk fo'y"k.k dj ldr gA bUg fuEu fof/ }kjk ifjHkkf"kr djr gA

(i) lcl igy py ekè; jhfr }kjk iofÙk eY;k dk Kkr fd;k tkrk gA ;fn vkdM ekfld gk rk ckjg ekfld py ekè; vkj ;fn =kekfld gk rk pkj =kekfld py ekè; dh tkrh gA

(ii) iR;d ey vkdM (0) e l rRlEcU/h py ekè; eY; (T) ?kVkdj vYidkyhu fopyu (Short term fluctuations) ikIr dj fy, tkr gA

(iii) ,d iFkd rkfydk cukdj ekfld o =kekfld vof/;k d vYidkyhu fopyuk dk tkMdj rFkk mudk fopyuk dh l[;k l Hkkx nu ij lekUrj ekè; fudkyh tkrh gA ; lekUrj ekè; gh fofHkUu eghu dk =kekIk d ekleh fopj.k dgykr gA

(iv) fofHkUu ekfld ;k =kekfld l lEcfU/r vklrk dk tkMdj lkekU; vklr Kkr dh tkrh gA blds ckn ekleh lpdkdk dh x.kuk fuEu l=k l dh tkrh g %

Seasonal Variations = Quarterly Average – General Average

Example :

Find-out seasonal fluctuations by the method off moving average

Year	frekgh (Quarter)			
	I	II	III	IV
1986	30	40	36	34
1987	34	52	50	44
1988	40	58	54	48
1989	54	76	58	62
1990	80	92	86	82

Solution **rkfydk-1**

o"k	frekgh	0	pkj frekgh py ;kx	py ;kx dfUær	pkj frekgh py ekè; (T)	vYidkyhu mPp opu iii-vi	vkor fopyu (S)
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
1986	I	30	–	–	–	–	–
	II	40→	–	–	–	–	–
	III	36→	140→	284	35.5	+.5	–0.2
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
	IV	34	144→	300	37.5	–3.5	–5.9
1987	I	34→	156→	326	40.8	–6.8	–4.1
	II	52→	170→	350	43.8	+8.2	10.4
	III	50→	180→	366	45.8	+4.2	–0.2
	IV	44→	186→	378	47.3	–3.3	–5.9
1988	I	40→	192→	388	48.5	–8.5	–4.1
	II	58→	196→	396	49.5	+8.5	10.4
	III	54→	200→	414	51.8	+2.2	–0.2
	IV	48→	214→	446	55.8	–7.8	–5.9
1989	I	54→	232→	468	58.5	–4.5	–4.1
	II	76→	236→	486	60.8	+15.2	10.4
	III	58→	250→	526	65.8	–7.8	–0.2
	IV	62→	276→	568	71.0	–9.0	–5.9
1990	I	80→	292→	612	76.5	+3.5	–4.1
	II	92→	320→	660	82.5	+9.5	10.4
	III	86→	340	–	–	–	–
	IV	82	–	–	–	–	–

rkfydk-2

o"k	frekgh (Quarter)			
	I	II	III	IV
1986	–	–	+0.5	–3.5
1987	–6.8	+8.2	+4.2	–3.3
1988	–8.5	+8.5	+2.2	–7.8
1989	–4.5	+15.2	–7.8	–9.0
1990	+3.5	+9.5	–	–
;kx	–16.3	41.4	–0.9	–23.6
vkSl r	–4.1	10.4	–0.2	–5.9

3.7.3 py ekè; l vuqikr fof/ (Ratio to Moving Average Method)

ekSleh fopj.kk dh ekiu dh ;g lokf/d ipfyr fof/ gA bld vUrxr fuEufyf[kr fØ;k, djuh iMrh gA

(i) loiFke fn, x, vkdMk dk ekfld ;k =kekfld :i d o"kk d Øekuq lkj fy[kxA

(ii) ;fn vkdM frekgh :i e 0;Dr g rk 4 frekgh py ekè; Kkr djxA

(iii) bld ckn okLrfod frekgh eY; dk py ekè; ij vk/kfjr eY; l foHkkftr djd ifr'kr :i e 0;Dr djr gA

$$\text{Ratio to Moving Average} = \frac{O}{T} \times 100$$

Where O = Original Value; T = Moving Average

(iv) bld ckn fofHkUu vof/;k l lEcfU/r py eè;kuikrk dk ,d vyx rkfydk e 0;ofLFkr djd lekukUrj ekè; fudky tkr gA

(v) lHkh eè;kuikrk d ekè; dk tkM djd mldk lkekU; ekè; (General Average) fuEu tkrk gA

(vi) vUr e lkekU; ekè; dk vk/kj ekur g, fuEu l=k yxkdj py eè;kuikrk d lekUrj ekè; dk ekleh lpdkdk (Seasonal Indices) e cny fn;k tkrk gA

$$\text{Seasonal Indices} = \frac{\text{Quarterly Average}}{\text{General Average}} \times 100$$

Form the following data, calculate4 seasonal indices by Ratio to Moving Average Method.

Year	1 Quarter	II Quarter	III Quarter	IV Quarter
1995	68	62	61	63
1996	65	58	66	61
1997	68	63	63	67

Solution :

Calculation of Scasonal Indices by Ratio-to-Moving Average Method

Year	Quarter	Values (0)	4 Quarterly Moving Totals	2 Period Total Centralized	4 Quarterly Moving Average	Ratio to Moving Average
1	2	3	4	Total	(T)	$\left(\frac{O}{T}\times 100\right)$
1995	I	68	–	–	–	–
	II	62	–	–	–	–
			→ 254			
	III	61		→ 505	63.1	96.7
			→ 251			
	IV	63		→ 498	62.3	101.1
			247			
1996	I	65		→ 499	62.4	104.2
			→ 252			
	II	58		→ 502	62.8	92.4
			→ 250			
	III	66		→ 503	62.9	104.9
			→ 253			
	IV	61		→ 511	63.9	95.5
			→ 258			
1997	I	68		→ 513	64.1	106.1
			→ 255			
	II	63		→ 516	64.5	97.7
			→ 261			
	III	63	–	–	–	–
	IV	67	–	–	–	–

Calculation of Seasonal Indices

Year	1 Quarter	II Quarter	III Quarter	IV Quarter
1995	–	–	96.7	101.1
1996	104.2	92.4	104.9	95.5
1997	106.1	97.7	–	–
Total	210.3	190.1	201.6	196.6
A. Average	105.15	95.05	100.8	98.3
Seasonal Indices	105.4	95.2	101.0	98.5

$$\text{General Average} = \frac{105.15+95.05+100.8+98.3}{4} = \frac{399.3}{4} = 99.8$$

Calculation of Seasonal Indices

$$\text{Seasonal Indices for I Quarter} = \frac{105.15}{99.8} \times 100 = 105.36$$

$$\text{II Quarter} = \frac{95.05}{99.8} \times 100 = 95.24$$

$$\text{II Quarter} = \frac{100.8}{99.8} \times 100 = 101.00$$

$$\text{IV Quarter} = \frac{98.3}{99.8} \times 100 = 98.5$$

3.7.4 iofÙk vuikr fof/ (Ratio to Treand Method)

;g fof/ dky J.kh d x.kd fun'k (Multiplicative Model) ij vk/kfjr gA ekleh fopj.kk dk ekiu dh bl fof/ e fuEufyf[kr fØ;k, djuh iMrh g %

(i) loiFke U;ure ox fof/ }kjk nh?kdkyhu miufr Kkr dh tkrh gA

(ii) x.kkRed ekMy dk i;kx djd lHkh vof/;k d iR;d ey vkdM (0) dk mll lEcfUèkr (T) eY; l Hkkx ndj Hktui{y dk 100 l x.kk djd iofÙk vuikr fudky fy, tkr gA l=kulkj

$$\text{Ratio to Tren} = \frac{0}{T} \times 100.$$

(i) iR;d ekfld vFkok =kekfld vof/ d lHkh oxk d iofÙk eY; vuikrk dk lekUrj ekè; fudkyk tkrk gA

(ii) lHkh ekfld ;k =kekfld d ekè;k dk tkMdj mudk lkekU; ekè; (General Average) fudkyk tkrk gA bld ckn fuEu l=k yxkdj ekleh lpdkd Kkr fd, tkr gA

$$\text{Seasonal Index} = \frac{\text{Quarterly Average}}{\text{General Average}} \times 100$$

Example

Calculate seasonal Indices by the ratio to moving average method from the following data

Year	1 Quarter	II Quarter	III Quarter	IV Quarter
2003	40	35	38	40
2004	42	37	39	38
2005	41	35	38	42

Year	Quarter	Sale	4 Qurly Moving Totals	Qurly Moving Average	Centring Total	Centring Average	Percentage Moving Avg= $\frac{\text{Orginal}}{\text{Trend}} \times 100$
2003	I	40					
	II	35					
			153	38.25	77	38.50	$\frac{38}{38.5} \times 100 = 98.7$
	III	38					
			155	38.75	78	39.00	$\frac{40}{39} \times 100 = 102.56$
	IV	40					
			157	39.25	78.75	39.38	$\frac{42}{39.38} \times 100=106.66$
	I	42	158	39.50	78.5	39.25	$\frac{37}{39.25} \times 100 = 94.27$
	II	37	156	39.0	77.75	38.88	$\frac{39}{38.88} \times 100=100.31$
	III	39	155	38.75	77	38.50	$\frac{38}{38.5} \times 100 = 98.7$
	IV	38	153	38.25			
					76.25	38.12	$\frac{41}{38.12} \times 100=107.55$
2005	I	41	152	38.00	77	38.50	$\frac{35}{38.5} \times 100 = 909$
	II	35	156	39			
	III	38					
	IV	42					

Calculation of Seasonal Indesx

(Percentage to Moving Average)

Year	1 Quarter	II Quarter	III Quarter	IV Quarter	
2003 2004 2005	– 106.66 107.55	– 94.27 90.90	98.7 100.31 –	102.56 98.7 –	
Total	214.21	185.17	199.01	201.26	Total
Average	107.10	92.59	99.50	100.63	399.82
Seasonal	$\frac{107.1\times400}{399.82}$	$\frac{92.59\times100}{399.82}$	$\frac{99.5\times400}{399.82}$	$\frac{100.63\times400}{399.82}$	400
Index	= 107. 15	= 92.63	= 99.55	= 100.67	

3.7.5 Ük[kyk eY;kuikr fof/ (Link Relative Method)

dEiuh fopj.k Kkr dju dh ;g lcl dfBu fof/ gA bl fof/ d i;kx dh ifØ;k fuEu idkj gA

(i) loiFke iR;d ekg dh l[;k dk mld igy oky ekg d eY;kuikr d :i e j[kr gA l=kuqlkj

$$\text{Link Relative} = \frac{\text{Value for Current Year}}{\text{Value for Previous Year}} \times 100$$

(ii) bld ckn k[kyk eY;kuikrk dk ekè; Kkr djr gA

(iii) bu k[kyk eY;kuikr ekè;k dk iFke le;kof/ dk vk/kj ekudj k[kyk fun'kkdk (Chair Relatives) e cnyk tk;xkA l=kuqlkj

$$\text{Chain Relatives} = \frac{\text{Chain Relative of Last Season} \times \text{L. R. of Current Seasons}}{100}$$

(iv) iFke vof/ d fudky x; fun'kkd vkj 100 d vUrj dk ½rvk dh l[;k dk Hkkx ndj ifr ½r ekè; vUrj Kkr fd;k tk,xkA

(v) bld ckn k[kyk fun'kkdk e lek;ktu djuk iMrk gA

(vi) vUr e bl idkj ikIr l;kftr k[kyk lpdkd (Adjusted or Corrected Chain Relatives) dk :ikUrj ekè; fudkyk tkrk gA lkekU; ekè; dk vk/kj ekur g, fuEu l=k dh enn l ekleh lpdkd fudky tkr gA

$$\text{Seasonal Indicets} = \frac{\text{Connected Chain Relatives}}{\text{General Average}} \times 100$$

Eample:

Find out seasonal variation by the Link Relatives Method from the following data

Year	I	II	III	IV
1992	45	54	72	60
1993	48	56	63	56
1994	49	63	70	65
1995	52	65	75	72
1996	60	70	84	77

Solution :

o"k	Ük[kyk eY;kuikr			
Year	I	II	III	IV
1992	–	120	133	83
1993	80	117	113	89
1994	88	129	111	92
1995	80	125	115	96
1996	83	117	120	79
;kx	331	608	592	439
vkIr	82.8	121.6	118.4	87.8
k[kykuikr	100	$\frac{100\times1216}{100}=1216$	$\frac{12161\times1184}{100}=1440$	$\frac{1440\times87.8}{100}=1265$
I'kkf/r k[kykuikr	100	121.6.1.2 = 120.4	144.0-2.4 = 141.6	126.5–3.6 = 122.9
vkor Ipdkd	82.5	99.4	116.8	101.3

3.8 vYidkyhu mPpkopu (mPpkou) dk eki (Short Term (Oscillations)

le; J.kh e vYidky e tk mPpkopu (mrkj&p<ko) gkr jgr g mUg vYidkyhu mPpkopu dgr gA mnkgj.k d rkj ij oLrvk d mRiknu e of¼ ;k deh d lkFk oLrvk d eY; e mrkj&p<ko gkrk jgrk gA bldk vè;;u dju d fy, nh?kdkyhu ifjoruk dk gVk fn;k tkrk gA vFkkr J.kh e l nh?kdkyhu mPpkopuk dk fudky nu ij tk dN 'k"k cpxk ogh vYidkyhu mPpkopu gkxkA

vYidkyhu mPpkopu = eY; led – miufr

Short Term Oscillations = Original Data – Trend Value

vYidkyhu mPpkopu fu;fer v?kRok vfu;fer gk ldr gA

3.9 vfu;fer mPpkopu dk ekiu (Measurement of Irregular Variations)

vfu;fer mPpkopu dk ekiu dh dkbZ fof/k ugha gA dky Js.kh esa lHkh iofÙk] ekSleh fopj.k o vYidkyhu mPpkopu ?kVkus ds i'pkr~ ds 'ks"k jgrk gS dgh vfu;fer mPpkopu gksrk gA

4. lkjka'k

vkfFkd leL;kvksa ds vè;;u esa dky J.kh dk cgr gh egRoi.kZ LFkku gA blh le; lhek lEc¼ eY;ksa dks ,d J.kh ds :i esa fy;k tkrk gS rks ;g dky J.kh dgykrh gA dky J.kh dk e[; mís'; Hkkoh ?kVukvksa dk lgh vuqeku yxkus ds fy, vkfFkd rF;ksa esa gksu okys ifjorukas dk le>uk rFkk eY;kadu djuk gA dky J.kh dk vusd ?kVd iHkkfor djrs gSa bUgsa dky J.kh ds l?kVd dgrs gA ;s e[;r% pkj idkj ds gksrs gA (i) nh?kdkyhu iofÙk (ii) vYidkyhu mPpopu (iii) pØh; mPpopu o (iv) vfu;fer ;k no ifjoruA dky J.kh ds pkjksa l?kVdksa dk fo'ys"k.kRed ekiu nks fu;eksa ij vk/kfjr gA μ(i) ;kxkRed fun'ku o (ii) x.kkRed fun'kuA dky J.kh esa nh?kdkyhu iofÙk dk vk'k; nh?kdkyhu ifjoruksa ls gA dky J.kh esa vYidky esa rks mrkj&p<ko gksrs jgrs gSa mUgsa vYidkyhu mPpkopu dgrs gA fu;fer mPpkopuksa ds vfrfjDr dHkh&dHkh dky J.kh esa vfu;fer mPpkopu dh nf"Vxkpj gksrs gSa ftUgsa no ;k vfu;fer mPpkopu dgrs gA vr% ge dg ldrs gSa fd dky J.kh esa miufr (nh?kdkyhu o vYidkyhu) dk ekiu djus dh fofHkUu jhfr;ksa dks i;kx djds Hkkoh iokueku yxk;s tk ldrs gSa tks O;kolkf;d fu;ktu esa lgk;d gksrs gA

5. iLrkfor iLrds (Recommended Books)

(i) Introduction to Statistics – By Dr. R.P. Hooda

(ii) Statistical Method – By S.P. Gupta

(iii) Business Statistics – By Oswal, Aggarwal and Sharma

(iv) Business Statistics– By T.R. Jain

(v) Business Statistics – S.C. Sharma, R.C. Jain

(vi) Business Statistics – Dr. Shukal anmd Sahai

6. vH;kl ds fy, i'u

(1) ,d O;olk;drk rFkk vFk'kkL=kh ds fy, dky J.kh fo'ys"k.k dh mi;ksfxrk le>kb,A dky J.h ds fofHkUu l?kVdksa dh Hkh O;k[;k dhft,A

(2) dky J.kh esa iofÙk dk ekiu dh fofHkUu fof/k;ksa dh O;k[;k djsA iR;sd ds x.k o nks"k crk;sA

(3) U;wure oxZ fof/k }kjk js[kh;] f}?kkrh; rFkk ?kkrkdh; iofÙk dk fiQV djus dh ifØ;k le>kb,A

(4) ekSleh lpdkd dk D;k vFk gS\ blds ekiu dh fofHkUu fof/k;ka le>kb,A

(5) ekSleh lpdkd dks Kkr djus dh pyu ekè; fof/k dks le>kb;A

(6) Fit a straight line trend by the method of leand squares and estimate the production for the year 2003 and 2005

Year	1997	1998	1999	2000	2001	2002
Production in Lakh Tonnes	25	40	47	59	68	80

Also convert your annual Trend Equation into monthly trend equation

(7) Calculate Seasonal Indices by Link Relative Method

Link Relatives

Quarter	2001	2002	2003	2004	2005
I	–	80	88	80	83
II	120	117	129	125	117
III	133	113	111	115	120
IV	83	89	93	96	79

B.Com. Part-II

Paper - BC–204 : Business Statistics *Revised By :* **Dr. Heera Lal Sharma**

Lesson No. : 9

ikf;drk fl¼kUr

(Probability Theory)

l jpuk (Structure)

1. ifjp;
2. mí';
3. fo"k; dk iLrqrhdj.k

   3.1 ikf;drk dk vFk ,o ifjHkk"kk

      3.1.1. ikf;drk dh 'kkL=kh; ifjHkk"kk

      3.1.2. lki{k vkofÙk ikf;drk ifjHkk"kk

      3.1.3. vkRe pru ikf;drk ifjHkk"kk

      3.1.4. ikf;drk dh vk/fud ifjHkk"kk

   3.2. ikf;drk dk egRo

   3.3. ?kVuk,

   3.4. ikf;drk ifjdyu d fu;e

      3.4.1. ;kx ie;

      3.4.2. x.ku ie;

      3.4.3. 'kr;Dr ikf;drk

      3.4.4. u;u ie;

4. lkjk'k
5. iLrkfor iLrd
6. vH;kl d fy, i'u

1. ifjp; (Introduction)%

bl lEHkkouk ;k ikf;drk 'kCn dk i;kx viu 0;kogkfjd thou e ik;% djr jgr g lqQyrk bl idkj d okD; tl ^'kk;n vfirk ch-dke- iFke o"k dh ijh{kk e ikl gk tk;* gk ldrk g vkt x, nkigj e fnYyh pyk tkmQ* vfer rFkk lfer nkuk cgr egurh fo|kFkh gA mud viuh d{kk e iFke vku dh lEHkkouk leku g*] ;fn vkil e ckrphr djr le; i;kx e yk; tkr gA bu lHkh okD;k e ^'kk;n gk ldrk g*] ^lEHkkouk*] 'kCnk dk i;kx fd;k x;k g ftl ?kVuk ?kVu dh vfuf'pr irhr gkrh gA bue fdlh Hkh okD; l ?kVuk l fuf'pr :i l ?kVu dk ldr ugh feyrkA ikf;drk fl¼kUr dk blh idkj dh vfuf'prkvk dk rdi.k <x l le>u e fo'k"k ;kxnku gA

ikf;drk fl¼kUr dk fodkl l=kgoh 'krkCnh l ikjEHk gvk gA ikLdy (Pascal), iQeV (Fermet) rFkk xyhfy;k vkfn ifl¼ xf.krKk e bl fl¼kUr d fodkl d fy, ikjfEHkd dk; fd;kA vBkjgoh rFkk mUuhloh 'krkCnh d xf.krKk o l[;k'kkfL=k;k u ikf;drk fl¼kUr d fodkl dk tkjh j[kkA chloh 'krkCnh e uur rFkk fiQ'kj (R. A. Fisher) u ikf;drk fl¼kUr ij vk/kfjr U;kn'k fl¼kUr (Sampling Theory) dk fodkl fd;k ftldk 0;ogkj e cgr egRo gA

2. mí'; (Objective)%

bl vè;k; dk vè;;u dju d ckn vki eku tk;x—

(i) ikf;drk dk vFk ,o bldk fodkl

(ii) vfuf'prrk d okrkoj.k e Hkkoh vueku yxkukA

(iii) fu.k;u e ikf;drk fl¼kUr dk ijh{k.k djukA

(iv) fofHkUu idkj dh ikf;drk dh x.kuk dju d fy, fofHkUu fu;ek dk ekuukA

(v) vfrfjDr tkudkjh d vk/kj ij 0;kogkfjd 0;kikfjd leL;kvk dk lek/ku djukA

3. fo"k; dk çLrqrhdj.k (Presentation of Contents)%

3.1. vFk ,o ifjHkk"kk (Meaning and Definition) :

ikf;drk 'kCn dh ifjHkk"kk dfBu g D;kfd l[;k'kkfL=k;k e bl fo"k; e erHkn gA e[;r% ;g er fuEu rhu idkj d g—

3.1.1. ikf;drk dh 'kkL=kh; ifjHkk"kk ;k er (Classical definition or approach of probability) :

;g fl¼kUr bl ekU;rk ij vk/kfjr g fd fdlh i;kx }kjk ikIr leLr ifjp; (outcomes), ikLifjd vioth (mutually exclusive) rFkk lekU; :i l ?kfVr gku oky (equally likely) gA mnkgj.kkFk] ;fn ge ,d iÙkk rk'k dh xMMh (Pack of cards) l fudky rk ;g dky jx dk gkxk ;k yky jx dk—nkuk gh ,d lkFk ugh vk ldr vFkkr rk'k d iÙk d fudky tku ij dkyk o yky iÙkk vkuk nkuk viorh ?kVuk, gA ,l gh dN vU; mnkgj.k g— flDd o lVy (die) dk mNkyuk] fdlh Fky l fofHkUu jxk dh xn fudky ij vku okyh fo'k"k jx dh xn bR;kfnA ikf;drk dh ;g lcl ikphu rFkk ljy vo/kj.kk g xf.rh; rd ij vkèkkfjr gku d dkj.k bl xf.kr; lEHkkouk Hkh dgr gA ykly d vulkj] ¶Probability is the ratio of number of favourable events to the total number of equally likely events.¸—Laplace.

Liyl }kjk nh xb ifjHkk"kk l ;g Li"V g fd fdlh ?kVuk d ?kfVr gku dh ikf;drk fudkyu d fy, bl fuEu nk ckrk dk Kku gkuk pkfg, %

(i) Number of favourable cases

(ii) Total number of equally likely cases

l=kulkj—Probability 'P' of occurrence of event 'E' is given by

$$P = P(E) = \frac{\text{Number of favourable cases}}{\text{Total No. of equalty likely cases}}$$

blh idkj Probability of Non-Occurrence is event 'E' is

$$q \text{ or } p(E) = \frac{\text{Number of case not favourable}}{\text{Total No. of equalty likely cases}}$$

;gk q dk vFk gµ ?kVuk d u ?kVu dh lEHkkouk

ge'kk ;kn jg fd ?kVuk d ?kVu dh lEHkkouk (p) rFkk u ?kVu dh lEHkkouk (q) dk ;kx gkrk gA fdlh nh gb ?kVuk d ?kVu dh lEHkkouk ;k ikf;drk lno 'kU; (0) rFkk ,d (1) eè; e gkrh gA ;fn ?kVuk dk gkuk ik;% fuf'pr gk rk mldh ikf;drk ,d gkxhA xf.krh; Hkk"kk e ;fn dkb ?kVuk 'a' ckn vudy de l rFkk 'b' ckj ifrdy e ?kV ldrh g vkj bue l dkb ?kVuk ?kV ldrh g rk mld vudy ?kVu dh ikf;drk $p = \frac{a}{a+b}$ d cjkcj gkxhA

Illustration 1.

What is the probability of getting an even number in a throw of an unblassed die.

,d rk'k iQdu ij lc vdk d vku dh ifk;drk D;k gkxhA

gy **(Solution)** %

,d ikl d mNkyu ij leku :i l ?kVu okyh Ng ?kVuk, g—1, 2, 3, 4, 5 rFkk 6A bu ?kVukvk e l 3 le vd g vFkkr rhu vudy ?kVuk, gA vr% vdk d vku dh ikf;drk $\frac{1}{2}$ gkxhA

$$P = \frac{\text{Number of favourable cases}}{\text{Total No. of equalty likely cases}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

mnkgj.k 2.

nk iklk ,d lkFk mNky x;A nkuk iklk e 4-4 vku dh ikf;drk Kkr dhft,A iklk dh mNky e 10 dk ;kx vku dh Hkh ikf;drk Kkr dhft,A

gy % nk iklk d mNkyu ij dy ifj.kkek (outcomes) dh l[;k = 6 ´ 6 = 36

iklk dh mNky e 4-4 vku dh dy l[;k = 1

$$\therefore\ p = \frac{1}{36}$$

nk ikl mNkyu ij 10 dk i;kx nk idkj vk ldrk gA (i) 6, 4, (ii) 4 : 6

ifj.kkek dh dy l[;k = 36

$$\text{vr\%}\ p = \frac{2}{36} = \frac{1}{18}$$

ikf;drk d 'kkL=kh; fl¼kUr dh lhek,
(Limitations of Classifcal Theory of Probability) :

ikf;drk d 'kkL=kh; fl¼kUr dh ie[k lhek, fuEufyf[kr g—

(i) bl fl¼kUr d vulkj ikf;drk Kkr dju d fy, leku :i l ?kVu okyh lHkh ?kVukvk dh l[;k Kkr djuh iMrh gA ,lk djuk iR;d fLFkfr l lEHko ugh gA mnkgj.kkFk] fdlh fo|kFkh d ijh{kk e iFke J.kh l mÙkh.k gku dh ikf;drk Kkr dju d fy, bl vo/kkj.k ;k ifjHkk"kk dk i;kx djuk dfBu g D;kfd ;g ekU;rk mfpr u gkxh fd fo|kFkh iFke J.kh yxk ;k ugh yxk] dh ikf;drk leku gA vr% bl fl¼kUr dk {k=k dkiQh lhfer jg tkrk gA

(ii) ikf;drk dk 'kkL=kh; fl¼kUr ,l i'uk—¶,d 0;fDr dh eR;q 60 o"k dh vk;q d ckn gkxh¸ dh ikf;drk fudkyu e vleFk gA

3.1.2. lki{k vkofÙk ikf;drk fl¼kUr (Relative Frequency Theory of Probability) :

bl fl¼kUr d vulkj ;fn dkb ?kVuk '*n*' e l '*a*' ckj ?kVrh g tk bldh vkofÙk $\frac{a}{n}$ gkxhA

tc dN ijh{k.kk dh l[;k vuUr (in finite) gk rk $\frac{a}{n}$ l tk eku tk,xk] ml gh yxxk vkofÙk dh lhek dgxA bl fuEu idkj l 0;Dr fd;k x;k g—

$$p(\text{A}) = \underset{n \to \infty}{\text{Limit}}\ \frac{a}{n}$$

ekU;rk, **(Assumptions) :** ;g fl¼kUr e[;r% nk ekU;rkvk ij vk/kfjr g—

(i) voykdu ;k ijh{k.k no fun'ku fof/ (Random Sampling method) ij vk/kfjr gku pkfg, rkfd iR;d ?kVuk d ?kVu d leku volj gkA

(ii) ijh{k.k (Trial) vud ckj gkuk pkfg, vFkkr voykdu l'k; l vf/d gkA

lki{k vkofÙk fl¼kUr rFkk 'kkL=kh; fl¼kUr n[ku e nkuk leku ekye gkr g yfdu bue cgr vUrj gA 'kkL=kh; er loekU; iko¶frd fu;ek ij vk/kfjr g og rd lxr g] blfy, bl Lo;&fl¼ ikf;drk Hkh dgk tkrk g vkj ble i;kx dh vko';drk ugh gkrhA tcfd lki{k vkofÙk lEekfur (probability) fl¼kUr e ?kVuk dk vud ckj nkgjk;k tkrk g vkj blfy, bl okLrfod vFkok i;kxkRed lEHkkouk Hkh dgk tkrk gA ;g rd d LFkku ij i;kx ij vk/kfjr gA

3.1.3. vkRepru ikf;drk (Subjective or persenalistic Probaility) :

fiNy dN n'kdk l dN xf.krK rFkk l[;k 'kkL=kh ikf;drk dk ,d vifjHkkf"kr lck/ ekuu yx gA mud vulkj ikf;drk j[kkxf.kr l fcUn (point) rFkk j[kk (line) dk Hkkfr vifjHkkf"kr gA o;fDrd ikf;drk fl¼kUr d vulkj fdlh ?kVuk d ?kfVr gku dh ikf;drk dk 0;fDrxr fo'okl ek=k le>k tkrk gA fdlh 0;fDr dk fdlh ?kVuk d ?kfVr gku dk fo'okl ml 0;fDr dk miyC/ rF; (evidence) ij vk/kfjr gkrk gA bl fl¼kUr e ,d 0;fDr fdlh ?kVuk dk mld ?kVu dh lEHkkouk d vulkj pt. zero 0 vkj ! d chp Hkkj iknku djrk gA mnkgj.kkFk] ,d fooQ; eutj viu fo'okl d vk/kj ij pky o"k e fooQ; ;k VkjxV (Target) iklr gku dh lEHkkouk 0.20 ;k 0.98 vkfn 0;Dr dj ldrk gA bldk vFk Li"V g fd Hkkj 0.20 gku ij fooQ; vueku ijk gku dh cgr de lEHkkouk g rFkk Hkkj 0.98 dk vFk g fd fooQ; VkjxV Lrj ij gku dh cgr vf/d lEHkkouk gA bl fl¼kUr dk e[; nk"k ;g g fd ble Objectivity dh deh g vFkkr fofHkUu 0;fDr fdlh ,d ?kVuk dk fofHkUu Hkkj n ldr gA

3.1.4. ikf;drk dh vk/fud ifjHkk"kk (Modern Approach of Probaility Theory) :

ikf;drk d mi;Dr rhuk fl¼kUrk dh dy lhek, gA blfy, ,d u; fl¼kUr dk ifriknu gvk—ftl ikf;drk dh vk/fud ifjHkk"kk dgk tkrk gA bldk ifriknu lu 1933 e :l d xf.krK ,-,u- dkleksxkso u fd;kA ble dN ekU;rkvk (axioms) dk o.ku g] ftu ij ikf;drk dk vueku vk/kfjr g] tk fuEu idkj g—

(i) leLr U;kn'k d ?kfVr gku dh lEHkkouk gA

(ii) fdlh ?kVuk d ?kVu dh ikf;drk lno 0 l 1 d eè; jgrh gA

(iii) ;fn ^v* rFkkk ^o* ijLifjd viorh ?kVuk, (mutually exclusive events) gk] rk ^v* vFkok ^o* ?kVuk d ?kfVr gku dh ikf;drk fuEu idkj Kkr dh tk;xh—

$$P(A \text{ or } B) = P(A) + P(B)$$

ikf;drk d mi;Dr pkjk fl¼kUrk d x.k o nk"k g rFkk 0;ogkj e fof'k"V leL;k d lek/ku e fdlh Hkh eki dk tk lcl vf/d mi;Dr gk i;kx fd;k tk ldrk gA

ikf;drk dk egRo (importance of Probability) :

ikf;drk fl¼kUrk dk fopkj vkfn dky l pyk vk jgk gA yfdu bld fof/or vè;;u dk ikjEHk 17oh 'krkCnh e LVkfj;k l lEcfU/r i'uk dk xf.krh; mÙkj nu d fy, gvkA rnijkUr bldk mi;kx volj lEcU/h leL;kvk tl fLDd d mNkyu iklk iQdu] rk'k dh xMMh l iÙk fudkyu lEcU/h leL;kvk dk mÙkj fudkyu d fy, gvkA fdUr vktdy ikf;drk dk mi;kx mu lHkh {k=kk e gkrk g tgk ?kVuk, vfuf'pr gkrh gA vkt d oKkfud ;x e lEHkkouk fl¼kUr dk fuf'prrk (certainty) d LFkkuki Uu d :i e tkuk tku yxk gA lEHkkouk dh mi;kfxrk dk fuEufyf[kr rF;k l tkuk tk ldrk g—

(i) lkf[;dh fu;ferrk fu;e (Law of Statistical Regularity) o xrkd tMrk dk fu;e (Law of Intertin of Large Numbers) lHkkfork fl¼kUr ij vk/kfjr gA

(ii) fu.k; fl¼kUr (Decision theory) Hkh ikf;drk d vk/kjHkr fu;ek ij vk/kfjr gA

(iii) ikf;drk fl¼kUr volj lEcU/h leL;k dk lekiu dju e mi;kxh gA

(iv) ikf;drk d ijh{k.k (Test of Significance) dh lEHkkfor fl¼kUr ij gh vk/kfjr gA

(v) lkFkdrk fl¼kUr vkfFkd o O;kolkf;d leL;kvk d lek/ku e i;kx fd;k tkrk gA

(vi) ikf;drk dk o;fDrd er ,lh ifjfLFkfr;k e vf/d mi;kxh g ftue ikf;drk dk okLrfod :i l ugh ekik tk ldrk gA

3.3. ikf;drk x.kuk dju dh oQN egRoi.k ckr (Some Important Terms to the Calculation of Probability) :

(1) no i;kx (Random Experiment) : ,d ,lk i;kx ;k ijh{k.k ftl leku ;k lxr n'kkvk e ckj&ckj dju l dkb Hkh ifj.kke (outcome) vk; yfdu ;g lHkh lEHko ifj.kkek e l ,d gh rk bl no i;kx dgxA

(2) ijh{k.k rFkk ?kVuk (Trial and Event) : ,d no i;kx dk lEiUu djuk ijh{k.k rFkk bld fdlh ifj.kke rFkk ifj.kkek d leg dk ?kVuk dgr gA mnkgj.kkFk] ,d flDd dk tu ckj&ckj mNkyk tkrk g] rk ifj.kke ,d tlk ugh gkrkA ge NV ;k fpÙk e l dkb Hkh ry ikIr dj ldr gA bl idkj d flDd dk VkI djuk no i;kx ;k ijh{k.k g rFkk iVV ;k fpÙk dk vkuk ,d ?kVuk gA

(3) leku :i l ?kfVr ?kVuk, (Equally Likely Events) : bl ekU;rk d vulkj leLr ?kVuk, leku gkuh pkfg,A vFkkr dkb ?kVuk vU; ?kVukvk l vf/d ckj u ?kV tk rHkh lEHko g tc ijh{k.k no vk/kj ij gkA mnkgj.kkFk ;fn ,d Fky e 10 xn gj ,d vyx jx dh xn gk] vkj ;fn mue l dkb ,d xn randamoly fudkyh tk;] rk lHkh xnk d

fudky tku dh ikf;drk leku (vFkkr $\frac{1}{10}$) gkxhA

(4) ikjLifjd viorh ?kVuk, (Mutually Exclusive Events) : ikjLifjd viorh ?kVuk, rk g ftud ?kVu ij vU; ?kVukvk d ?kVu dh lEHkkouk lekIr gk tkrh gA mi;Dr dk vFk g fd ikjLifjd viorh ?kVuk, ,d lkFk ugh ?kV ldrhA mnkgj.kr;k ;fn ,d ikIk (die) iQdk tk, rk N% i{kk (sides) e l dkb Hkh ,d mQij vk ldrk g vkj vU; lHkh i{kk d mQij vku dh lEHkkouk lekIr gk tkrh gA vr% o ikjLifjd viorh ?kVuk, gA

(5) loxkgh ?kVuk, (Collectively Exclustive Events) : leLr ifrdy o vudy ?kVuk okyh ?kVukvk dk ;kx dy ?kVukvk d ?kVu d cjkcj gkrk gA mnkgj.kkFk] ,d flDd dk mNkyu ij doy iVV ;k fpÙk gh lEHko gk ldr gA vr% o loXkkgh ?kVuk, gA

(6) **LorU=k ?kVuk,** **(Independent Events) :** ;fn fdlh ?kVuk d ?kVu dk u ?kVu dk iHkko fdlh vkx dh ?kVuk d ?kVu ;k u ?kVu ij ugh iMrk rk ,lh ?kVuk, LorU=k ?kVuk, dgykrh gA mnkgj.kkFk] ;fn ,d rk'k dh xMMh l ,d irk no fun'ku fof/ l fudkyk tk; rk ;g

iÙkk iku dk gkxk bl ckr dh ikf;drk P (A) = $\frac{13}{52}$ gkxhA bld ckn ;fn ml iÙk dk okfil xMMh e j[k fn;k tk, vkj fiQj ,d iÙkk (randomly) fudkyk tk, rk vc Hkh ml nckjk

fudky x, iÙk d iku dk gku dh ikf;drk $\frac{13}{52}$ gh jgxhA vr% igyh rFkk nljh ?kVuk, LorU=k :i l ?kfVr gk jgh gA

(7) **vkfJr ?kVuk,** **(Dependent Events) :** o ?kVuk, LorU=k ?kVukvk d foijhr g] D;kfd bu ?kVukvk dh n'kk e ,d ?kVuk dk ?kfVr gkuk nljh ?kVuk d ?kVu dk iHkkfor djrk gA mnkgj.kkFk] rk'k dh xMMh e pkj cxe gkrh gA ;fn xMMh e l ,d iÙkk [khpk tk; rk

ml d cxe gku dh ikf;drk $\frac{3}{51}$ gkxhA

(8) **ljy ,o l;Dr ?kVuk,** **(Simple and Compound Events) :** ljy ?kVuk d vUrxr ge ,d gh ?kVuk d ?kVu vFkok u ?kVu dh ikf;drk fudkyr gA mnkgj.kr;k % ,d Fky e l ftle 2 liQn rFkk 10 yky xn g] ,d yky xn fudky tku dh ikf;drk] nk ikl (die) ,d lkFk iQdu ij 12 dk ;kx vku dh ikf;drk vkfnA l;Dr ?kVuk d vUrxr nk ;k nk l vf/d ljy ?kVuk, lfEefyr gkrh g] tl—;fn ,d Fky e 5 yky o 10 dkyh xn g vkj ;fn nk&nk xn nk ckj fudkyh tk, rk igyh ckj nk yky o nljh ckj nk dkyh xnk d vku dh ikf;drkA

3.4. ikf;drk ifjdyu d fu;e (Rules of Calculating Probability) :

ikf;drk d ifjdyu e fuEufyf[kr nk i;kx cgr egRoi.k g—

3.4.1. ;kx&ie; (Addition Theorem)

3.4.2. x.ku ie; (Multiplication Theorem) :

bldk lf{kIr o.ku uhp fd;k x;k g—

3.4.1. ;kx&ie; (Additions Theorem) : bl fu;e d vulkj] ;fn nk ?kVuk, A rFkk B ikjLifjd viothr gk rk A rFkk B d ?kfVr gku dh ikf;drk A rFkk B d ?kVu dh ikf;drk dk ;kx gkxkA

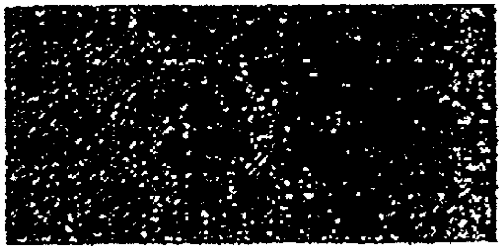
l=ku lkj %

$$P\,(A \text{ or } B) = P\,(A) + P\,(B)$$

;k P (A È B) + P (A) + P (B)

;gk A È B dk A Union B dgxA

Overlapping Events



Proof : Let N denote the total number of equally likely cases of which m_1 are favourable to an event A, and m_2 to an event B. Thus we get

$$P(A) = \frac{m_1}{N} \quad P(B) = \frac{m_2}{N}$$

Since the events are mutually exclusive, therefore, the number of cases favourable to the event A + B is $m_1 + m_2$

$$\therefore \quad P(A+B) = P(A \text{ or } B)$$

or

$$\frac{m_1 + m_2}{N} = \frac{m_1}{N} + \frac{m_2}{N}$$

$$= P(A) + P(B)$$

Hence the proof

Illustration 3.

A card is drawn from a pack of 52 cards. Calculate the probability of getting either a King or a Queen.

Solution : There are 4 kings and 4 Queens in a pack of 52 cards.

Probability that a card drawn is a king = $\frac{4}{52}$ and probability that the card drawn is a queen = $\frac{4}{52}$

Since the events are mutually exclusive, the probability that card drawn is either a king or a queen.

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{8}{52}$$

$$= \frac{2}{13}$$

;kx dk fu;e ;fn ?kVuk, ioh jkT; l ikLifjd viorh u gk (Addition Rule of events are not mutually exclusive) :

;kx ie; }kjk ikIr ifj.kke mlh n'kk e lk{; gksu tcfd ?kVuk, ikjLifjd :i dk vioorh gkA ;fn ?kVuk, leku (common) g rk ;kx ie; l l'kk/u djuk iMrk gA

Here the additional law can be stated a follows :

The Probability of the occurrence of either A or B or both in equal to the probability that event A occurs, plus the probability that event B occurs minus, the probability that both events occurs. Symbolically, in can be written as

$$A (A È B) = P(A) + P(B) - P(A È B)$$

The following diagram will make it more clear.

Overlapping Events

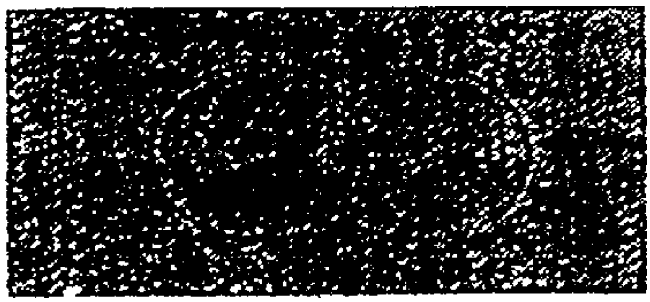


Illustration 4.

What is the probability of drawing a 'heart' or a king card from a pack of cards ?

Solution :

Probability of drawing a 'heart' card is—

$$P(A) = \frac{13}{52}$$

Similarly, probability of drawing a king card is—

$$P(B) = \frac{4}{52}$$

But 13 cards of heart also include a king. Hence, King of heart is common andhas been counted twice. Thus, events are mutually exclusive.

\ $$P(A \text{ and } B) = \frac{1}{52}$$

So the required probability by applying addition rule is—

$$P(A \text{ or } B) = P(A) + P(B) - (A \text{ and } B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}$$

Illustration 5.

A bag contains 30 balls numbered from 1 to 30. One ball is drawn at random. Find the probability that the number of the drawn ball will be multiple of 3 or 7.

Solution : Let A and B denote the events of being multiple of 3 and 7 respectively.

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

\ $$P(A) = \frac{10}{30} = \frac{1}{3}$$

And multiples of 7 are 7, 14, 21, 28

\ $$P(B) = \frac{4}{30}$$

Since 21 is a multiple of 3 as well as 7, the drawing of the ball entails the occurrence of both the events A and B and hence, the probability of getting a number which is muliple of 3 or 7 is

$$P\,(A \text{ or } B) = P(A) + (B) - P\,(A \text{ and } B)$$

$$= \frac{10}{30} + \frac{4}{30} - \frac{1}{30} = \frac{13}{30}$$

3.4.2. x.ku i;kx (Multiplications Theorem) :

ikf;drk x.kuk dh bl ie; d vulkj ;fn A vkj B nk Lo=kUr ?kVuk, g rk nkuk d ,d lkFk ?kVu dh ikf;drk bu ?kVukvk dh 0;fDrxr ikf;drk dh x.kk d cjkcj gkxhA l=kku lkj—

(P (A and B) = P (A) ´ P(B)

In case of Three events A, B C it will be P (A and B and C) = P (A) ´ P(B) ´ P(C)

Proof : If event 'A' can take place in my ways of which favourbale cases are n_1 and event 'B' can take place in m_2 ways of which favourable cases are n_2, then both events can simultaneously take place. The total number of possible cases will be mix m_2 and total number of favourable cases will be n_1 ´ n_2 ways (According to counting rule).

Since both the events are independent so favourable cases will be—

$$\frac{n_1 \times n_2}{m_1 \times m_2} = \frac{n_1}{m_1} \times \frac{n_2}{m_2}$$

Hence, $$P\,(A \text{ and } B) = P\,(A) + P\,(B)$$

Since $$P(A) = \frac{n_1}{m_1} \text{ and } P\,(B) = \frac{n_2}{m_2}$$

The multiplication rule will be extended to three events as follows :

$$P\,(A \text{ and } B \text{ and } C) = P\,(A) + P\,(B) + P\,(C)$$

Illustration 6.

A university has to select an examiner from a list of 50 persons, 20 of them women and 30 men, 10 of them knowing Hindi and 40 not, 15 of them being teachers and the scmaiming 35 not. What is the probability of the university selecting a Hindi Knowing woman teacher ?

Solution :

$$\text{Probability of selecting a woman} = \frac{20}{50}$$

$$\text{Probability of selecting a teacher} = \frac{15}{30}$$

Probability of selecting a Hindi knowing candidate

$$= \frac{10}{50}$$

Since all of the above events are independent, the probability of the university selecting a Hindi Knowing woman.

$$= \frac{20}{50} \times \frac{15}{30} \times \frac{10}{50} \times \frac{3}{125}$$

3.4.3. 'kr;Dr ikf;drk (Conditional Probability when events are dependents) :

tc ?kVuk, vkfJr gkrh g rk x.ku&ie; dk i;kx ugh fd;k tk ldrk D;kfd ,l h fLFkfr e ,d ?kVuk d ?kVu dk iHkko nljh ?kVuk d ?kVu ij iMrk gA ;fn E_1 vkj E_2 l vkfJr ?kVuk, g rk E d ?kVu d 'kr;Dr ikf;drk bl idkj Kkr dh tk,xh—

$$P(E_2/E_1) = \frac{P(E_1E_2)}{P(E_1)}$$

or $$P(E_1 E_2) = P(E_1) + P(E_2/E_1)$$

rFkk E_1 d ?kVu dh 'kr;Dr ikf;drk

$$P(E_1/E_2) = \frac{P(E_1E_2)}{P(E_2)}$$

or $$P(E_1 E_2) = P(E_2) + P(E_1/E_2)$$

Illustration 7.

Find the probability of drawing a king, a queen and ace in this order from a pack of 52 cards in three successive drawn presuming that carans drawn are not replaced.

Solution :

We know there are 4 kings, 4 queens and 4 ace in pack of cards.

Protability of drawing a king card is :

$$P(E_1) = \frac{4}{52}$$

Probability of drawing a queen card after a king card is—

$$P(E_2/E_1) = \frac{4}{51}$$

Similarly, probability of drawing an ace after drawing a king and a queen card is

$$P(E_2/E_1E_2) = \frac{4}{50}$$

As events are dependent so the required probability of drawing a king, a queen and an ace in that order is—

$$P(E_1E_2E_3) = P(E_1)\ ´\ P(E_2/E_1)\ ´\ P(E_2/E_4E_2)$$

$$= \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} = \frac{8}{16575}$$

udkjkRed ikf;drk (Negative Probability)—tc i'u e db Lor=k :i l ?kfVr gku okyh ?kVukvk dh ikf;drk nh gb gk vkj mu e l de l de ,d ?kVuk d ?kVu dh ikf;drk fudkyuh gk rk ;g udkjkRed ikf;drk d }kjk fudkyh tkrh gA l=k—

Prob. of happening of at least one of events = 1 – P (Happening of none of the event)

Illustration 8.

A canddiate is selected for interview of management trainers for 3 companies. For first company there are 15 canddiates, for the second there are 12 condidates and for the third there are 11 candidates. What are the chances of his getting at least at one of the company ?

Solution :

The probability that the candidates gets the job at least at one company.

= 1 – probability that the candidates does not get the job in any company.

Prob. That the candidates does not get the job in the first company :

$$= 1 - \frac{1}{15} = \frac{14}{15}$$

Prob. that the candidates does not get the job in the 2nd company :

$$= 1 - \frac{1}{12} = \frac{11}{12}$$

Prob. that the candidates does not get the job in the 3rd company :

$$= 1 - \frac{1}{11} = \frac{10}{11}$$

Since the events are independent therefore the prob. that the candidates does not any job in any company of the three.

$$= \frac{14}{15} \times \frac{11}{12} \times \frac{10}{11} = \frac{7}{9}$$

Hence the required probability $= 1 - \frac{7}{9} = \frac{2}{9} = 0.22$.

3.4.4. o;t ie; (Baye's Theorem)—

bl ie; dk ifrikknu vxt xf.krK Fkkel c;t (1702 – 1761) u fd;k FkkA blfy, ;g ie; o;t ie; d uke l tkuh tkrh gA bl ie; d }kjk 'kr;Dr ikf;drk dh x.kuk dju d fy, fd;k tkrk gA ;g ie; fuEu idkj 0;Dr dh tk ldrh g—

Let E_1, E_2 ....E_2 be n mutually exclusive events whose union an arbitrary event in the universe such that $P(A)\ ^{1}\ 0$

Given that $P(A/E_1)$ and $P(E_1)$ are known (Here $i = 1, 2, ...n$)

$$P(E_1/A) = \frac{P(A/Ej)P(Ej)}{\Sigma^1 P(Ej)P(A/Ej)} \qquad \text{for } j = 1, 2, ...n$$

This throrem is frequently used as a mechanism for revising the probability of an event after observing information about a process. The initial prob. is reffered to as prior probability and the revised as posteri or probability.

Illustration 8.

A firm produces steel pipes in three plants with daily production volumes of 500, 1000 and 2000 units respectively. According to past experience it is known that the fractions of defective output produced by the three plants are 005, .008 and 010 respectively. If a pipe is selected from a days total production and found to be defective. Find out (i) from which plants the pipe comes. (ii) What is the probability that it come from the first plant ?

Solution :

This problem is solved with the use of Baye's theorem.

Let A_1 = Production volume of first plant.

A_2 = Production volume of Second plant.

A_3 = Production volume of third plant.

E = A defective item.

$$P(A_1) = \frac{500}{3500} = \frac{1}{7}\ ;\ P(A_2) = \frac{1000}{3500} = \frac{2}{7}\ ;\ P(A_3) = \frac{2000}{3500} = \frac{4}{7}$$

$$P(A_1\ Ç\ E) = P(A_1)\ P(E/A_1) = \frac{1}{7}\ (.005) = \frac{.005}{7}$$

$$P\ (A_2\ Ç\ E) = P\ (A_2)\ P(E/A_2) = \frac{2}{7}\ (.005) = \frac{.016}{7}$$

$$P(A,\ Ç\ E) = P\ (A_3)\ P(E/A_3) = \frac{4}{7}\ (.010) = \frac{.040}{7}$$

Therefore sum of the probabilities

$$P(E) = \frac{.005}{7} + \frac{.016}{7} + \frac{.040}{7} = \frac{.061}{7} \text{ and}$$

$$(A_1/E) = \frac{P(A_1 \cap E)}{P(E)} = \frac{.005/7}{.061/7} = \frac{.5}{61}$$

$$(A_2/E) = \frac{P(A_2 \cap E)}{P(E)} = \frac{.016/7}{.061/7} = \frac{16}{61}$$

$$(A_3/E) = \frac{P(A_3 \cap E)}{P(E)} = \frac{.040/7}{.061/7} = \frac{40}{61}$$

As P (A_2/E) has the heigh dest probability, it is most likely that the defective item had been down from the third plant.

(ii) Prob. that the pipe came from first plant is

$$\frac{\frac{1}{7}\times.005}{\left(\frac{1}{7}\times.005\right)+\left(\frac{2}{7}\times.008\right)+\left(\frac{4}{7}+.010\right)} = \frac{.0007}{.0007+.00023+.0057} = \frac{.0007}{.0087} = .0805$$

Hence required prob. is .0805.

4.0. lkjk'k (Summary) :

bl vè;k; e geu ikf;drk dh dN vk/kjHkr vo/kj.kkvk dh 0;k[;k dh g fd dl ikf;drk fo'y"k.k fuokpu e egRoi.k lpuk, inku djrk gA fu.k;u d le; bl fofHkUu fodYik e l lokÙke dk puko djuk gkrk g vkj ;g fodYi Hkfo"; d lkFk tM gkr gA ;fn vfuf'pr ?kVukvk dh vfu'prrk dk vdk e 0;Dr fd;k tk,] rk vdk e 0;Dr fd; g, eY;

dk ge ikf;drk dgr gA ikf;drk dk dkb loekU; ifjHkk"kk ugh gA budk oxhZdj.k pkj Hkkxk e fd;k tk ldrk g—(i) Dykfldy ;k xf.krh; ifjHkk"kk] (ii) lkf[;dh; vFkok lki{k vkofÙk ifjHkk"kk] (iii) vkRepru ikf;drk o (iv) Hkkoxr nf"Vdk.kA fdlh ?kVuk d ?kfVr gku dh ikf;drk lno 0 l 1 d chp gkrh gA ;fn ?kVuk vo'; gh ?kfVr gkxh rk bldh ikf;drk '1' gkxh o ;fn ?kVuk dk ?kVuk vlEHko g rk '0' gkxhA ?kVuk d fy, vudy ifjfLFkfr;k dh l[;k dk lHkh lEHkkfor ifjfLFkfr;k dh l[;k l Hkkx fn;k tkrk gA ;g ml ?kVuk dh ikf;drk gkrh gA oct ie; dh lgk;rk l ikFkfed ikf;drk dk lEiy lpukvk (Sample Imformations) d vk/kj ij l'kkf/r fd;k tkrk gA

5. iLrkfor iLrd (Recommonded Books) :

(i) Introduction to Statistics—Dr. R. P. Hooda

(ii) Business Statistics—T.R. Jain

(iii) Statistical Methods—S. P. Gupta

(iv) Business Statistics—S.C. Sharma, R.C.Jain

(v) Business Statistics—Oswal, Aggarwal, Sharma, Khanna.

6. vH;kl d fy; i'u (Self Assument Questions) :

(1) Write notes on :

- (i) Random Experiment
- (ii) Trial and Event
- (iii) Equally likely Events
- (iv) Mutually Exclusive Events
- (v) Exhansitive Events
- (vi) Dependent Events.

(2) What do you understand by measurement of probability ? Explain.

(3) What is the chance that a vowel selected at random in a book of English is an 'O' ?

(4) A university has to select an examiner from a list of 50 persons, 20 of them women and 30 men, 10 of them knowing hindi and 40 not; 15 of them being teachers and the remaining 35 not. What is the probabaility of the university selection a Hindi knowing women teacher ?

(5) Explain various apprunders to probability.

(6) State and Prove.

- (a) Addition theorem
- (b) Multiplication theorem
- (c) Bayes theorem.

(7) The probability that a company executive will travel by plane is 2/3 and he will travel by train is 1/15. Find the probability of his travelling by plane or train.

B.Com. Part-II

Paper - BC–204 : Business Statistics ***Revised By :*** **Dr. Heera Lal Sharma**

Lesson No. : 10

ikf;drk forj.k (f}in] ik;lu ,o lkekU;)

Probability Distribution (Binomial, Poisson and Normal)

ljpuk (Structure)

1. ifjp;
2. mí';
3. fo"k; dk iLrrhdj.k
   - 3.1 ikf;drk forj.k
   - 3.2. ikf;drk fooj.k dh mi;kfxrk
   - 3.3. ikf;drk forj.k d idkj
     - 3.3.1. f}in forj.k
       - 3.3.1.1. f}in forj.k dk ikf;drk iQyu
       - 3.3.1.2. f}in forj.k dh fo'k"krk,
       - 3.3.1.3. f}in forj.k dh fiQfVx
     - 3.3.2. ik;lu fooj.k
       - 3.3.2.1. ik;lu forj.k dk ikf;drk iQyu
       - 3.3.2.2. ik;lu forj.k f}in forj.k dk lhekUr :i
       - 3.3.2.3. ik;lu forj.k dh fiQfVx
     - 3.3.3. lkekU; forj.k
       - 3.3.3.1. lkekU; forj.k dh ekU;rk,
       - 3.3.3.2. lkekU; forj.k dk egRo
       - 3.3.3.3. lkekU; ooQ dh fo'k"krk,
       - 3.3.3.4. lkekU; ,o f}in forj.k e vUrj
       - 3.3.3.5. f}in] ik;lu ,o lkekU; forj.k e lkekU;
       - 3.3.3.6. lkekU; ooQ }kjk {k=kiQy vFkok ikf;drk Kkr dju dh fof/
4. lkjk'k
5. iLrkfor iLrd
6. vH;kl d fy, i'u

1. ifjp; (Introduction)%

vkofÙk forj.kk dh jpuk nks idkj ls dh tk ldrh g%

1. okLrfod voyksdu d vk/kj ij vkofÙk forj.k (Observed frequency distribution).

2. ikf;drk ;k iR;kf'kr ;k l¼kfUrd vkofÙk forj.k (Probability of expected frequency distribution).

okLrfod voyksdu ij vk/kfjr vkofÙk forj.kk e ik;% vuql/ku ls miyC/ leadk ;k lkexh dks i;kx gkrk gA bu forj.kk dks fuEu mnkgj.k ls le>k tk ldrk gA

,d vuql/kudrk u foKkiu 0;; d lEcU/ e 100 dEifu;ka ls vkdM ,d=k fd; ftUg mlu fuEu lkj.kh d }kjk 0;Dr fd;k—

foKkiu 0;; dk Lrj (o"k 1994-95)

foKkiu 0;; (Rs.)	iQek dh l[;k
10 gtkj rd	20
10 l 30 gtkj	30
30 l 60 gtkj	30
60 gtkj l mQij	20
;kx	100

ik;% ,df=kr ledk dk fo'y"k.k fofHkUu lkf[;dh; ;U=kk (Statistical tools) tl leklUrj ekè;] iHkko fu;ru] fo"kerk] i;'kh"kRo (Kurtosis) vkfn d ekè;e l djr gA ;|fi mi;Dr lkf[;dh; fof/;k ledk dk le>u] fo'y"k.k dju rFkk egRoi.k fu"d"k fudkyu l gekjh cgr enn djrh g] rFkkfi lex (Population) dh fo'k"krkvk d ckj e lgh&lgh fu"d"k fudkyu d fy, ,d vPNh vkj oKkfud fof/ dh t:jr gA ikf;drk forj.k ;k l¼kfUrd vkofÙk forj.k ,lh gh ,d oKkfud jhfr g ftldk o.ku geu bl ikB e fd;k gA

2. mí'; (Objectives) :

bl vè;k; dk vè;;u dju d fuEufyf[kr mí'; g—

(i) ikf;drk forj.k d ckj e tkudkjh gkfly djukA

(ii) ikf;drk forj.k dh mi;kfxrk dk le>ukA

(iii) ikf;drk forj.k fof/;k l fdlh ifjfer l[;k (Finite Number) oky pj dh lki{k vkofÙk;k Kkr djukA

(iv) mfpr iokueku iklr djd fu.k; yu e tkf[ke rFkk vfuf'prrk dk de djukA

(v) fofHkUu ikf;drk forj.kk dk vkil e lc/ tkuukA

(vi) ikf;drk forj.kk dh fiQfVx djukA

3. fo"k; dk iLrqrhdj.k (Presentation of Contents)

3.1. ikf;drk forj.k (Probability Distribution)

ikf;drk forj.k l vk'k; ,d tlh xf.krh; jhfr l g ftldh lgk;rk l fdlh ifjfer l[;k (Finite number) oky pj dh lki{k vkofÙk;k Kkr dh tk ldrh gA bl forj.k e vkofÙk;k okLrfod fujh{k.k ;k i;kxk }kjk ikIr u djd dN fuf'pr ekU;rkvk d vk/kj ij ikIr dh tkrh gA mnkgj.k d rkj ij ;fn ge ,d fu"i{k flDdk 500 ckj mNky rk vk'kku lkj 50 ifr'kr ijh{k.kk e fpr vk;kx rFkk 50 ifr'kr ifjfLFkfr;k e ;g iVV ;kfu 250 ckj fpr rFkk 250 iVV vk;xkA yfdu tc okLrfod i;kx fd;k tk; rk gk ldrk g fd 240 ckj fpr o 260 ckj iVV vk;A iR;kf'kr vkofÙk;k e vUrj vkuk LokHkkfod (Natural) gkrk gA ;fn ge ijh{k.kk (Trails) dh l[;k c<kr tk; rk okLrfod vkofÙk;k ikf;d vkofÙk;k d lehi vkrh tk;xhA

3.2. ikf;drk forj.k dh mi;kfxrk (Utility of Probability Distribution)

ikf;drk vkofÙk forj.k vk/fud lkf[;dh d vk/kj LrEHk gA lkf[;dh fo'y"k.k e ;k foLrkj vud idkj l mi;kxh fl¼ gkr gA ikf;drk vkofÙk forj.k d e[; mi;kx fuEu idkj g—

1. ikf;drk forj.kk rFkk okLrfod vkofÙk forj.kk e vUrj Kkr djd ;g irk yxk;k tk ldrk g fd nkuk e vUrj U;kn'k d mPpkopuk d dkj.k g vFkok fdUgh vU; dkj.kk l gA
2. ikf;drk forj.k foodi.k yu e cgr lgk;d gA
3. tu le; o /u d vHkko e vul/kudrk okLrfod led ,df=kr dj iku e Lo; dk vleFk le>u yx rk mld fy, fodYi d :i e ik;f'kr ;k l¼kfUrd vkofÙk forj.k cgr egRoi.k fl¼ gkrk gA
4. bl idkj d fooj.kk d }kjk mfpr iokueku ikIr djd fu.k; yu okyk tkf[ke rFkk vfuf'prrk ij dN lhek rd fu;=k.k ik ldrk gA

3.3. ikf;drk forj.k d idkj (Types of Probability Distribution)

ikf;drk forj.k vud idkj d gkr g fdUr lkf[;dh; fo'y"k.k e fuEu rhu idkj d forj.kk dk lokf/d i;kx gkrk g—

(i) f}in forj.k (Binomial Distribution)

(ii) ik;lu forj.k (Poisson Distribution)

(iii) lkekU; forj.k (Normal Distribution)

3.3.1. f}in forj.k (Binomial Distribution)

f}in forj.k dk ifriknu fLoLl xf.krK tEl cukyh (James Bernoulli 1654-1705) u fd;kA bl forj.k dk ojukyh forj.k ;k ie; dgu dk dkj.k mld }kjk bldk ifriknu gh gA ;g ie; lu 1700 e fodflr gb yfdu lu 1713 e bldk idk'ku gvkA f}in forj.k ,d [kf.Mr vkofÙk forj.k gkrk gA ,lh ifjfLFkfr;k l tgk ,d ijh{k.k ;k i;kx d doy nk gh ifj.kke (outcomes) fudky—lQyrk o vlQyrk bl ie; ;k forj.k dk mi;kx e yk;k tk ldrk gA lQyrk l gekjk vfHkik; g fd ble dN x.k fo|eku g rFkk vlQyrk l vk'k; g fd mijkDr x.kk dk vHkko gA cjukyh ifoQ;k (Bemoulli Process) dh xf.krh; ie; cgr Li"V ekU;rkvk d vk/kj ij fodflr dh xb g tk fuEufyf[kr g—

1. no i;kxk vFkok U;kn'k dh l[;k fuf'pr gkrh g vFkkr (n) ijh{kkvk dh l[;k ifjfer o fuf'pr gkrh gA

2. no ijh{k.k e doy nk ?kVuk, ?kVrh g vkj o ?kVuk, ikjLifjd vioth (Mutually Exclusive) gkrh g ftud ifj.kke lQyrk o vlQyrk d :i e 0;Dr djr gA

3. ijh{k.k LorU=k gkr g vFkkr fdlh ,d i;kx d ijh{k.k dk iHkko vkx fd, tku oky i;kxk d ifj.kkek ij ugh iMrkA

4. ?kVuk d ?kVu (lQyrk) dk 'p' l o u ?kVu (vlQyrk) dk 'q' l 0;Dr fd;k tkrk gA $p + q = 1$ gkrk g blfy, $q = 1 - p$ gkxkA

3.3.1.1 f}in forj.k dk ikf;drk iQyu (Probability Function of Binominal Distribution)

If X denotes the number of successes in n trials satisfying the above conditions, then X is a random variable which can take the values 0, 1, 2, ...n, since in n trials we many get no success, one succes, two successes, ...or the n successes.

mijkDr l Li"V g fd ge 'kU; l n rd dh lQyrkvk dh ikf;drk fudkyuk pkgr gA blds fy, f}in forj.k dk mi;kx e yk;k tk ldrk g rFkk forj.k e f}in foLrkj l=k $(q + p)^n$ dk i;kx bfPNr ikf;drk ikIr dju e cgr mi;kxh gA $(q + p)^n$ dk foLrkj dju ij x = 0, 1, 2,..... d fofHkUu ekuk dh ikf;drk ikIr gkrh gA

l=k dk foLrkj bl idkj gkrk g—

$$(q + p)^n = q^n + {}^n c p^{n-1} p^1 + {}^n c p^{n-2} p^2 + ... + {}^n c r q p^{n-c} p_2{}^r + ...p^n$$

1, ${}^n c_1$, ${}^n c_2$ vkfn dk f}in x.kkd dgk tkrk gA

;fn ge fofHkUu ifj.kkek dh vkofÙk;k Kkr djuk pkg rk $(q + p)^n$ d LFkku ij $N(q + p)^n$ dk i;kx fd;k tkrk gA

The general expression for the probability of r successes is given by :

$$p(r) = p\,(x = r) = {}^n\mathrm{C}r.P^r.\, q^{n-r};\ r = 0, 1, 2, ..., n.$$

Regarding Binomial expansion keep the following in mind—

(i) Putting r = 0, 1, 2, ..., n in the above function we get the probabilities of 0, 1, 2, ...n successes respectively in n trials.\

(ii) Total Prob. is unity *i.e.*,

$$q^n + {}^n crp^{n-1} + ncrq^{n-2}\, p^2 + ...pn = 1$$

or $(q + p)^n = 1$

(iii) The Binomial distribution is completely determined if its permachters n and p are known.

(iv) Since the random variable X takes only integral values, Binomial distribution is a distcrete probability distribution.

The values of Binomial co-efficients for different values of n can be obtained conveniently from the Pascal's triangle given below.

PASCALS TRIANGLE

Value of n	Binomial Co-efficients	Sum (2^n)
1.	1 1 2	2
2.	2 1	4
3.	1 4 3 3 1	8
4.	1 6 4 1	16
5.	1 5 15 10 10 5 1	32
6.	1 6 20 15 6	64

It can be easily seen that, taking the first and last terms as 1, each term in the above table can be obtained by adding the two terms on either side of it in the pveceding line.

Constants of Binomial Distribution :

Mean $= \bar{x} = np$

Standard Deviation $= s = \sqrt{npq}$

Variance $= s^2 = m2 = npq$

Moments $= m$

$m_1 = 0 m_2 = \text{variance} = npq$

$m_3 = npq\,(q - p)$

$m_4 = 3n^2p^2q^2 + npq\,(1 - 6pq)$

$$b_1 = \frac{\mu^2 3}{\mu^3 2} = \frac{(q - p^2)}{npq}$$

iFk'kh"kRo Kurtosis $= b_2 = \frac{\mu^4}{\mu^2 2}$ or $3 + \frac{1 = 6pq}{npq}$

3.3.1.2. f}in forj.k dh fo'k"krk,¡ %

(i) f}in forj.k dk vkdkj *p* rFkk *q* ij fuHkZj gS *n* d fLFkj gksu ij ;fn *p* c<rk g rk f}in forj.k dk >dko nkb vkj gk tkrk gS ;g rFkk fuEu fp=kk }kjk Li"V gk tkrk g—

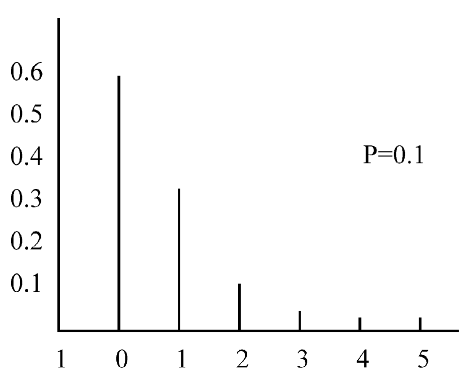


Diagram-A

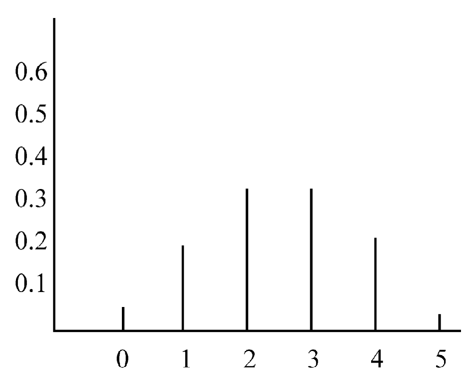
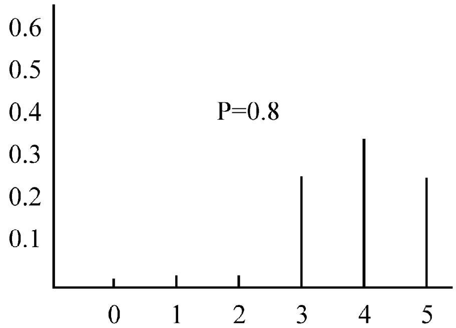


Diagram-B



(ii) f}in forj.k dk Hkf;"Bd X dh ml value dk leku gkrk g ftldh ikf;drk lcls vf/kd gkrh gA ekuk p=5 vkj p = 0.5 gk rk Hkf;"Bd 4 gkxkA

(iii) ;fn n dk fLFkj j[kk tk; rk p dk c<u ij ek/; (mean) ,o Hkf;"Bd (mode) nkuk c<r gA

(iv) ;fn p dk eku fLFkj j[kk tk; rk p f}in forj.k n dk c<u ij nkb vkj ljdrk g vkj iQSyk tkrk gA

(v) ;fn Bdjko dh ikf;drk $\frac{1}{2}$ dk vFkkr $p = q = r$ gk rk f}in forj.k i.kr;k lefer (Symmetrical) gkxkA

Illustration No. 1

If the chance that the vessel arrives safely at a port is $\frac{9}{10}$, find the chances that out of 5 vessels expected at least 4 will arrive safely.

Solution :

Probability that a vessel arrived safely at the port = p = $\frac{9}{10}$

$$q = 1 - p = 1 - \frac{9}{10} = \frac{1}{10}$$

By Binomial probability law, the probability that out of 5 vessels, x vessels arrive safely at the port is given by—

$$p(x) = {}^5c_x q^{5-x} p^x = {}^5c_x \left(\frac{9}{10}\right)^n \left(\frac{1}{10}\right)^{5-n}$$

Probability that at least 4 vessels will arrive safely is given by :

p (4) + p (5). So first we will calculate

$$p(4) = 30_4 \left(\frac{9}{10}\right)^4 \left(\frac{1}{10}\right)^{5-4} = \left(\frac{9}{10}\right)^4 \left(\frac{1}{10}\right)^1$$

$$= \left(\frac{1}{2}\right)\left(\frac{9}{10}\right)^4 = 0.328 \text{ and then}$$

$$p(5) = {}^3\text{cs} \left(\frac{9}{10}\right)^4 \left(\frac{1}{10}\right)^{5-3} = \left(\frac{9}{10}\right)^5 = 0.590$$

$$= p(4) + p(5) = 0.328 + 0.590$$

$$= 0.918$$ **Ans.**

Illustration No. 2

Out of 800 families with 4 children each, what percentage would be expected to have (a) 2 boys and 3 girls (b) at least one boy, and (c) at the most 2 girls. Assume equal probailities for boys and girls.

Solution :

(i) Probability of getting a boy $= \frac{1}{2} = p$

Probability of getting a girl $= \frac{1}{2} = q$

Probability of getting 2 boys and 2 girls

$$= n\, c_r\, q^{n-r}\, p^r$$

$$= {}^4c_2 \left(\frac{1}{2}\right)^{4-2} \left(\frac{1}{2}\right)^2$$

$$= \frac{4 \times 3 \times 21}{2 \times 1 \times 21}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$$

$$= 6\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{6}{16} = \frac{3}{8}$$ **Ans.**

Percentage of families expected to have 2 boys and 2 girls.

$$= \frac{3}{8} \times 100 = 375\%$$

(ii) Probability of getting at least one boy means sum of the probabilities of one boy and 3 girls 2 boys and 2 girls, 3 boys and 1 girl and no girl and 4 boys.

$$p(1) + p(2) + p(3) + p(4) = {}^4c_1 \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 + {}^4c_2 \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$$

$$+ {}^4c_3 \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 + {}^4c_4 \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^4$$

$$= 4\left(\frac{1}{2}\right)^4 + 6\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^4 + 1\left(\frac{1}{2}\right)^4$$

$$= \frac{4}{16} + \frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{4+6+4+1}{16}$$

$$= \frac{15}{16} \quad \textbf{Ans.}$$

(iii) Probability of getting at the most 2 girls means the sum of the probabilities of getting no girl, one girl and 2 girls.

$$p(\text{no girl}) = {}^4c_4 \left(\frac{1}{2}\right)^{4-4} \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$p(\text{one girl}) = {}^4c_3 \left(\frac{1}{2}\right)^{4-3} \left(\frac{1}{2}\right)^3 = 4 \times \frac{1}{16} = \frac{4}{16}$$

$$p(\text{two girls}) = {}^3c_2 \left(\frac{1}{2}\right)^{4-2} \left(\frac{1}{2}\right)^2 = 6 \times \frac{1}{16} = \frac{6}{16}$$

$$\text{Sum of probabilities} = \frac{1}{16} + \frac{4}{16} + \frac{6}{16} = \frac{1+4+6}{16} = \frac{11}{16}$$

\ Percentage of families expected to have at most 2 girls

$$= \frac{11}{16} \times 100 = 68.75\% \qquad \textbf{Ans.}$$

3.3.1.3. f}in forj.k dh fiQfVx (Fitting a Binomal Distribution) :

The following procedure will be adopted for this purpose—

(1) loiFke p ,o q dk eku Kkr djkA ;fn bue l dkb ,d nh g rk nljh value bl lEcU/ }kjk ikIr dj ldr g—$p = (1 - q)$, and $q = 1\ (1 - p)$;fn p o q d eku leku ugh g rk f}in forj.k vlhfer (Skewed) gkxkA

(2) bld ckn cukyh dh ie; $(q + p)^2$ dk foLrkj djk vFkok bl [kkydj fy[kkA

(3) vUr e Binomial Expansion dh gj term dk n (dy vkofÙk;k) l x.kk djk rk gj ox dh vkofÙk;k iklr gk tk;xhA

Illustration No. 3.

Four coins are tossed at a time, 240 times, Number of heads of observed at each throw is recorded and the results are given below. Find the expected frequencies. What are the theoretical values of mean and standard deviation ?

Number of heads at a throw	Frequency
0	10
1	60
2	95
3	65
4	10

Solution :

Probability of getting one head in a single throw of one coins is $\frac{1}{2}$

\ $p = \frac{1}{2}, q = \frac{1}{2}, N = 240, n = 4$

By expending $240\left(\frac{1}{2}+\frac{1}{2}\right)^4$ we shall get the expected frequencies of 1, 2, 3, 4 heads.

Number of Heads (x)	Frequency = N (nc, $q^{n-r}p^r$) (Expected)
0	240 ´ $^4c_0 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 = 15$
1	240 ´ $^4c_1 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 = 60$
2	240 ´ $^4c_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = 90$
3	240 ´ $^4c_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 = 90$
4	240 ´ $^4c_4 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 = 15$
	Total = 240

The mean of the above distribution is $np = 4$ ´ $\frac{1}{2} = 2$

Standard deviation = $\sqrt{npq} = \sqrt{4\times\frac{1}{2}\times\frac{1}{2}} = 1$

Illustration No. 4

The probability of defective bulb in a total of 100 bulbs is 0.2 Find the X, S.D. (c) moment. Co-efficient of skewness and kurtosis of the distribution.

Solution :

As we have, $p = 0.2$

\ $q = 1 - 0.02 = 0.8$

$n = 100$ given

(i) $x = np = 100 \times .2 = 20$

(ii) $s = \sqrt{npq} = \sqrt{100 \times 2 \times 8}$

$= \sqrt{16} = 4$

(iii) Moment of Co-efficient of skewness :

$$= \sqrt{\beta_1} = \frac{q-p}{\sqrt{apq}}$$

$$= \frac{0.8-0.2}{\sqrt{100 \times .2 \times .8}}$$

$$= \frac{0.6}{\sqrt{16}} = \frac{0.6}{4}$$

$= 0.1.5$

(iv) Kurtosis $b^2 = 3\ \frac{1.\sigma pq}{npq}$

$$= 3 + \frac{1 - 6 \times 0.2 \times 0.8}{100 \times 0.2 \times 0.8}$$

$$= 3 + \frac{1 - 0.96}{16} = 3 + 0.002$$

$= 3.002$

Because $b_2 > 3$ the curve is *k* pto kurtic

3.3.2 ik;lu forj.k (Poisson Distribution) :

ik;lu forj.k d fodkl dk J; ifl¼ iQp xf.krK ik;lu (Simeon Poisson) dk tkrk g ftUgksu bldk ifrikfnu lu 1837 e fd;kA bl forj.k dk i;kx mu ifjfLFkfr;k e fd;k tkrk g ftue fdlh ?kVuk d ?kVu dh lEHkkouk (*p*) dk eku cgr gh de gkrk g D;kfd ?kVuk dnkfpr (rarely) gh ?kVrh gA bu ?kVukvk l cu forj.k [kf.Mr (Discrete) io{fr d gkr gA bl forj.k e doy ?kVuk d ?kVu dk lekUrj ek/; ($\bar{x}$) ekye gkr gh ;g forj.k cuk;k tk ldrk gA ik;lu forj.k e lekUrj ekè; dk (m) l 0;Dr djr g tk fd fiNy vuHko l ikIr gkrk gA

List of some Pratical Situations where Poisson distribution can be used :

(i) Number of telephone calls arriving at a telephone switch board in unit time (say, per minute).

(ii) To count the number of bacterias per unit (Biology).

(iii) To count the number of radio-active disintegrations of a radio-active element per unit of time (Physics).

(iv) Number of customers arriving at the super market; say per hour.

(v) The number of suieides reported in a particular day in a particular city or town.

(vi) Number of typographical errors per page in a typed material or the number of printing mistake per page in a book

On the basis of above list it can be stated that the Poisson Distribution has found application in a variety of fields such as Queucing Theory, Insurance, Physics, Biology, Business, Economics and Industry.

3.3.2.1. ik;lu forj.k dk ikf;drk i{yu

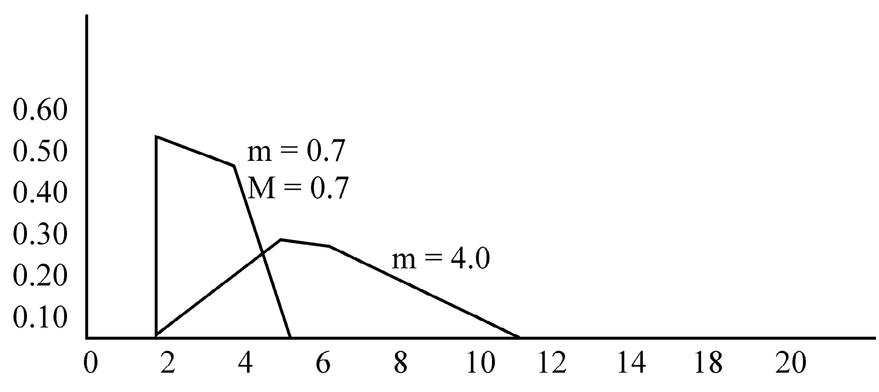
We can express the Poisson Distribution as follows :

$$P(r) = \frac{e^{-m} m^r}{r^1}$$

Here c = 27183, m = arithmatic mean

r = Number for which probable frequency is to be calculated *i.e.*, r = 0, 1, 2, ...

The above can be expanded as follows :



$$c^{-m}\left(1+m+\frac{m^2}{2!}+\frac{m^3}{3!}+\frac{m^r}{r!}+......\right)$$

ik;lu forj.k mijkDr fp=k l Li"V gk tkrk g—

This Poisson Distribution can be written in tabular form as follows :

No. of Successes (x)	Probabilities P(x)
0	e^{-m}
1	me^{-m}
2	$\frac{m^2 e^{-m}}{2!}$
3	$\frac{m^3 e^{-m}}{3!}$
4	$\frac{m^4 e^{-m}}{4!}$
:	: :
:	: :
r	$\frac{m^r e^{-m}}{r!}$

mQij nh xb lkj.kh l ge ikf;drk ikIr djr g vxj bu foHkUu ?kVukvk d ?kVu dh ckjEckjrk tkuuk pkgr rk Poisson Formula dh nj en (term) dk N (Total number of observations) l x.kk djxA

Constants of the Peisson Distribution :

1. lekUrj ekè; $= x = m = np$

2. iekk fopyu s = $\sqrt{m}$

3. Variance = m

4. Moment =

(i) $m_1 = 0$

(ii) $m_2 = m$

(iii) $m_3 = m$

(iv) $m_4 = m + 3m^2$

(v) $b_1 = \frac{\mu^2 3}{\mu^2 3} = \frac{m^2}{m^3} = \frac{1}{m}$

(vi) $b_2 = \frac{\mu^4}{\mu_2^2} = 3 + \frac{1}{m}$

3.3.2.2. ik;lu forj.k f}in forj.k dk lhekUr :i
(Poission Distribution as a Limiting Case of Bionomial Distribution) :

Poission distribution may be obtained as a limiting of Binomial probability distribution under the following conditions :

(i) n_1 the number of trials is indefinitely large *i.e.*, n = ¥

(ii) p_1 the constant probability of successes for each trial is indefinited small, *i.e.*, p ® 0.

(iii) np = m_1 (say), is finite.

Probability function of the Poisson Distribution

$$p(x) = p(x = r) = \frac{c^{-m} m^r}{r!}$$

and probability function of the Binomial Distribution :

$$P(r) = {}^n crq^{n-r} p^r$$

put $$p = \frac{m}{n}$$

\ $$q = 1 - p = 1 - \frac{m}{n}$$

$$p(r) = \frac{n(n-1)....(n-r+1)}{r!}\left(\frac{m}{n}\right)^r \times \left(1-\frac{m}{n}\right)^{n-r}$$

$$= \frac{2\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)....\left(1-\frac{r-1}{n}\right)m'}{r!}\left(\frac{\left(1-\frac{m}{n}\right)^n}{\left(1-\frac{m}{n}\right)^n}\right)$$

For fixed r, as n ® ¥

$\left(1-\frac{1}{n}\right),....,\left(1-\frac{r-1}{n}\right)\left(1-\frac{m}{n}\right)^r$ all tend to one and $\left(1-\frac{m}{n}\right)^n$ to e^{-m}. Hence it is represented in the limiting case.

$$p(r) = \frac{e^{-m} m^r}{r!}$$

3.3.2.3. ik;lu forj.k fof/ dh fiQfVx

How to want to fit a poisson distribution

If we want to fit a poisson distribution to a given frequency distribution, we compute the mean X of the given distribution and take it equal to the mean of the fitted distribution *i.e.*, we take m = X. Once m is known tha various probabilities of the Poisson distribution can be obtained the general formula being.

$$p(r) = p(x = r) = \frac{e^{-m} m^{r}}{r!} \quad r = 0, 1, 2, ...$$

If N is the total observed frequency, then the expected or theoretical frequencies of the Poisson distribution are given by N, X $p(r)$ and therefore, we will have

$$N(P_0) = Ne^{-m}$$

$$N(P_1) = N(P_0) \times \frac{m}{1}$$

$$N(P_2) = N(P_1) \times \frac{m}{2}$$

$$N(P_3) = N(P_2) \times \frac{m}{2}, \text{ etc.}$$

fuEu mnkgj.kk l ik;lu forj.k dk 0;kogkfjd i;kx o mi;kx le>k tk ldrk gA

Illustration No. 5.

If 3% of the electric bulls manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective.

Solution :

Prob. of getting a deffective bulb is

$$P = \frac{3}{100}, \text{ Size of sample } n = 100$$

$$X = m = np = 100 \times \frac{3}{100} = 3$$

We known that $\quad P(r) = \frac{e^{-m} m^{r}}{r!}$

$\therefore$ Prob. of 5 defective bulbs is—

$$p(5) = \frac{e^{-3} 3^{5}}{5!}$$

$\because \quad e^{-3} = .04979$ – [sec table of e^{-m} from any book of Statistics]

$$\text{blhfy,} = \frac{.04979 \times 243}{5 \times 4 \times 3 \times 2 \times 1} = 0.1008$$

Illustration No. 6.

It is known from past experience that in a cetain plant there are on the average 4 industrial accidents per month. Find the probability that in a given year there will be less than 4 accidents. Assume Poisson distribution (e^{-1} = 0.0183).

Solution :

We are given at = 4

$$\text{By P. Dist. ® } (x = r) = \frac{e^{-m} m'}{r!} = \frac{e^{-4} - 4'}{r!} \quad ....x$$

The distribution that there will be less than 4 accidents is given by

$p\ (x < 4) = p\ (x = 0) + p\ (x = 1) + p\ (x = 2) + p\ (x = 3)$

$$e^{-4} \left[1 + 4 + \frac{4^2}{2!} + \frac{4^2}{3!}\right] \quad \text{(From } x)$$

e^{-4} [1 + 4 + 8 + 10.67]

e^{-4} ´ 23.67 = 0.0183 ´ 23.67 = 0.4332. **Ans.**

Illustration No. 7

The number of defects per unit in a sample of 330 units of manufactured product was found as follows :

Number of defects	0	1	2	3	4
Numbers of units	214	92	20	3	1

Fit a Poission distribution to the data.

Solution :

Fitting of Poission Distribution

X	f	fX	Expected frequencies N - $p(x)$
0	214	0	N.p(0) = .6447 ´ 330 = 212.75
1	92	92	N.p(1) = N(po) ´ m = 212.7 ´ .430 = 93.4
2	20	40	N.p(2) = N(p1) ´ $\frac{m}{2}$ = 93.4 ´ .439 = 20.52
3	3	.9	N.(P3) = N(p2) ´ $\frac{m}{3}$ = 3.0
4	1	4	N.(P4) = N(p3) ´ $\frac{m}{4}$ = 0.35
	N.=330	Σfx=145	

$$\overline{X} = \frac{\Sigma fx}{N} = \frac{145}{330} = 0.439$$

$$p(X = 0) = e^{-m} = e^{-.439} = 0.6447$$

Thus the expected frequencies as per Poission distribution are :

Number of defects :	0	1	2	3	4
Number of units :	212.75	93.4	20.50	3.00	0.35

Some Important Questions for Practice :

(1) What are the conditions for the Binomial distribution ? Discuss its properties.

(2) What is Poission distribution ? Discuss its uses and properties.

(3) Show that Binomial distributional is the limitting form of binomial distribution.

(4) Compare the Normal, Poissional and Binomial Distribution.

(5) The normal rate of infection of a certain disease in animals is known to be 25%. In an experiment with 6 animals in injected with a new vaccine it was observed that none of the animals caught infection. Calculate the probability of the observed result.

$\left(\frac{729}{4096}\right)$ **Ans.**

(6) Suppose that in key punching of 80 column IBM cards, the arithmatic mean number of mistakes per card is 03. What percent of cards will have (i) no mistake (ii) one mistake, and (iii) two misttakes.

[(i) = 74%, (ii) = 22%, (iii) = 3%) **Ans.**

(7) If 8 coins are tossed 256 times, find the expected frequencies of getting various heads.

Ans.

Heads	0	1	2	3	4	5	6	7	8
Frequency	1	8	28	56	70	56	28	8	1

(8) A book has 2 mistakes per page. Using Poisson distribution. Find the probability that the page selected at random has (i) no mistake, (ii) exactly 3 mistakes.

(i) 0.315, (ii) 0.1804. **Ans.**

(9) Fit a Poission Distribution to the following set of observations.

Death	0	1	2	3	4
Frequency	122	60	15	2	1

Calculate theoretical frequency.

(10) If the mean of a Poission Distribution is 2.56, find : evaluate of other constants. Also determine the probability of exactly 2 success.

Ans.

s = 1.6	$m_3 = 2.56$	$b_2 = 3.39$
$m_1 = 0$,	$m_4 = 22.22$	$p(2) = 2532$
$m_2 = 2.56$	$b_2 = 0.39$	

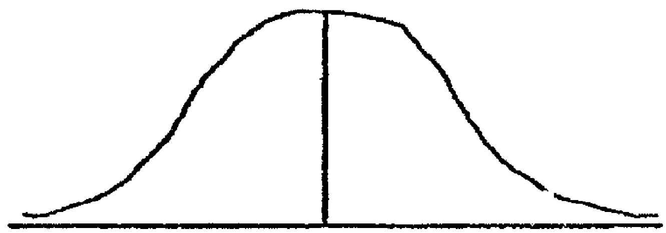
3.3.3. lkekU; forj.k (Normal Distribution) :

lkekU; forj.k ,d v[kf.Mr pj (Continuous variable) tks dn] otu dh ikf;drk (Probability) Kkr djus fy, iz;qDr fd;k tkrk gS A bl forj.k dk O;kogkfjd :i ls iz;ksx djus dk Js; ykIykl (Laplace) rFkk dkyZ xkSl (Karl Gauss) dks tkrk gS ijUrq lcls igys bl forj.k dk ckjs esa lh[kus dk Js; Mh- ek;j (D. Moiyre) dks tkrk gS ftUgksus blds ifrikfnr lu 1733 esa fd;kA

lkekU; forj.k dk xzkiQ iqjh rjg vfdr djus ls tks js[kk curh gS mls lkekU; oØ (Normal Curve) dgrs gSA ;g ,d ?k.Vkdkj (bell shaped) vkoQfr dk gksrk gS] rFkk Hkqtk{k (X-axis) d nkuksa vkj lferh; (Symmetrical) ,oa vuUrLi'kh (asymptotic) gksrk gSA

lkekU; oØ dh vkoQfr (Shape of Normal Curve)

lkekU; oØ dh vkoQfr bl idkj gksrh gSA



$\bar{x}$

(μ)

;g oØ eè; (X) rFkk ieki fopyu (Standard Deviation) ij vk/kfjr gksrk gSA

mijksDr oØ dk ge lehdj.k }kjk Hkh O;Dr dj ldrs gS tks bl idkj gS—

$$y = \frac{1}{\sigma\sqrt{2x}} e = \frac{-(x-m)^2}{2\sigma^2}$$

Where,

m = mean

s = standard deviation

X = 3.141 or 22/7

X = X– X (deviation of variable X from X)

3.3.3.1. lkekU; forj.k dh ekU;rk,¡ (Assumptions of Normal Distribution) :

lkekU; forj.k fuEufyf[kr ekU;rkvksa ij vk/kfjr gksrk gS—

(1) fofHkUu ?kVukvksa ij iHkko Mkyus okys vusd dkj.k gS ,oa mu lcdk vius egRo gSA

(2) fofHkUu ?kVukvksa dks iHkkfor djus okys dkj.k LorU=k gSA

(3) lkekU; oØ dh lHkh dkj.kjkf'k;k¡ lelr lex (universe) ij cjkcj gks Hkys gh mudk iHkko fofHkUu ?kVukvksa ij vyx&vyx iMsA

(4) lkekU; forj.k d fofHkUu dkj.kk dk iHkko bl idkj dk gk fd %

(i) vf/drj vkofÙk;k x d vkl&ikl gk] o

(ii) lekUrj ekè; l fopyu vkil e lUrfyr gk dj 'kU; gk tkr gkA

3.3.3.2. lkekU; forj.k dk egRo (Importance of Normal Curve) :

lkekU; forj.k dk lkf[;dh e fo'k"k egRo gA bl lkf[;dh dk vk/kj ekuk tkrk gA fuEufyf[kr 'kCn bld egRo ij idk'k Mkyr g %

(1) lHkh ikoQfrd rF;k e lkekU; forj.k oky x.k ik, tkr gA vr% budk vè;;u iHkkoi.k fd;k tk ldrk gA

(2) ^lkf[;dh x.koÙkk fu;U=k.k* e 0;kid :i l lkekU; forj.k dk i;kx fd;k tkrk gA

(3) lkekU; forj.k ^ifrp;u fl¼kUr* (Sampling Theory) dk vk/kj gA

(4) ;g forj.k f}in dk ik;lu forj.kk dk Hkh vk/kj gA

3.3.3.3. lkekU; oo dh fo'k"krk, (Properties of Normal Distribution) :

(1) ;g oo ?k.Vk d vkdkj dk gkrk gA

(2) lfer oo (Symmetrical Curve)

(3) ble Mean = Median = Mode

(4) bldk {k=k ^,d* ekuk tkrk g ,o ekè; (Mean) v{k (Ordinate) bldk nk cjkcj Hkkxk e ckVrk gA

(5) ;g oo vk/kj j[kk dk dHkh Li'k ugh djrkA foLrkj dju ij fudV vkrk tkrk gA

(6) bl forj.k e pj (Variable) v[kf.Mr (Continuous) gkrk gA

(7) bl forj.k dk ek=k ,d Hkf;"Bd (Mode) vFkkr Unimodal gkrk gA

(8) ,l forj.k e Q_3 - Median = Median - Q_1

(9) Quartile deviation = 2/3 standard deviation.

(10) bl forj.k e $Q_1 + Q_2 = 2$ median

(11) P.E. = .845 M.D.

(12) bl forj.k e M.D. = $\frac{4}{5}$ S.D. (Where M.D. = Mean Deviation)

(13) ;g forj.k ekè; (Mean) d vkl&ikl vorj.k (Concave) gkrk gA

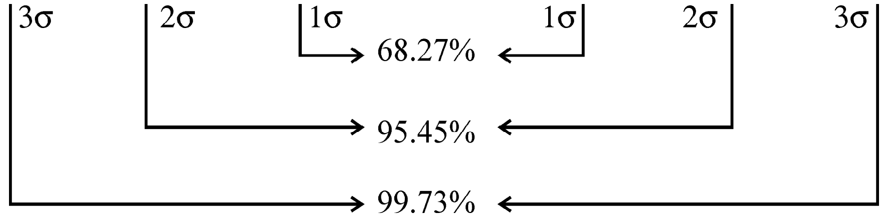
(14) ;g forj.k 3s d vkl&ikl mÙke (Convex) gkrk gA

(15) lkekU; forj.k dh {k=kiQy (Area) l lEcfU/r fo'k"krk, bl idkj g%

$\overline{X} \pm s = 68.27\%$ area

$\overline{X} \pm 2s = 95.45\%$ area

$\overline{X} \pm 3s = 99.73\%$ area



(16) lkekU; forj.k d vpy (Constants of Normal Distribution) :

Mean $= m$

Standard deviation $= s$

Variance $= s^2$

First Moment $= m_1 = 0$

Second Moment $= m_2 = s^2$

Third Moment $= m_3 = 0$

Fourth Moment $= m_4 = 3\mu_2^2 - 3s^4$

Moment Coefficient of Skewness $= b_1 = \frac{\mu_3^2}{\mu_2^3} = 0$

Moment Coefficient of Kurtosis $= b_2 = \frac{\mu_4}{\mu_2^2} = \frac{3\mu_2^2}{\mu_2^2} = 0$ (Always Meso-Kurties)

$y^1 = 0$ (Skewness)

$y_2 = 0$ (Meso Kurtosis)

3.3.4. lkekU; forj.k ,o f}in forj.k e vUrj

(Difference between Distribution and Binomial Distribution) :

lkekU; forj.k ,o v[kf.Mr ikf;drk forj.k (Continuous Probability distribution) g] tcfd f}in forj.k ,d [kf.Mr ikf;drk forj.k (Discrete Probability Distribution) g nkuk forj.kk d lehdj.k Hkh vyx&vyx gA f}in forj.k e[;r% *n* ,o *p* ij vk/kfjr gkrk g tcfd lkekU; forj.k X ,o s ij vk/kfjr gkrk gA f}in forj.k e l[;k (*n* dk eku) fuf'pr gkrh g ,o *p* dk eY; .5 d vkl&ikl gkrk g tcfd lkekU; forj.k e ,lk ugh gA

3.3.3.5 f}in forj.k] ik;lu forj.k ,o lkekU; forj.k e lEcU/ (Relationship between Binomial, Poission and Normal Distribution) :

bl lHkh forj.kk e ?kfu"B lEcU/ ikIr tkrk gA f}in forj.k o ik;lu forj.k e geu n[kk fd f}in forj.k e tc *p* dk eY; ;k rk 'kU; d lehi gk ;k ,d lehi gk ,o *n* dh l[;k vf/d gk rk ; forj.k ik;lu forj.k dk vkdkj y yrk gA bu fLFkfr e lekUrj ekè; (Mean) $= m = np$.

f}in forj.k e ;fn p dk eY; .5 gk ,o n dk vkdkj cMk gk rk f}in forj.k dk vkdkj rk lhfer (Symmetrical) gh gkxk ij bl dk foLrkj dk iQh cMk gkxkA bl idkj l lkekU; forj.k tlk gh cu tk,xkA bl idkj—

$$Z = \frac{X - \overline{X}}{\sigma}$$

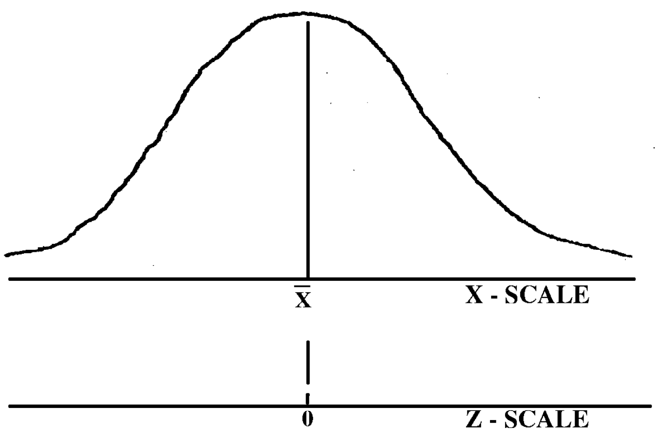
$$= \frac{X - rp}{\sqrt{npq}}$$

bl idkj ik;lu forj.k o lkekU; forj.k e Hkh lEcU/ gA vr% Z dk eku ik;lu forj.k }kjk Hkh Kkr fd;k tk ldrk gA ,lh fLFkfr e]

$Z = \frac{X - m}{\sqrt{m}}$ (ukV % ik;lu forj.k e $\overline{X} = m$,o Standard deviation = $\sqrt{m}$)

3.3.3.6. lkekU; oo }kjk {k=kiQy vFkok ikf;drk Kkr dju dh fof/ (Method of finding out the area of Probability using Normal Distribution) :

lkekU; oo dk {k=kiQy ikf;drk Kkr dju l igy mldk iekf.kd vfHkO;fDr (Standard Form) tkuuk vfuok;Z gk tkrk gA lkekU; oo bl idkj dk gkrk gA



i'u dju d fy, igy X-Scale ij Area Kkr fd;k tkrk g fiQj mldh Z dh Value Kkr dh tkrh g ,o fiQj Table }kjk Area Kkr fd;k tkrk gA igyk i'u X-Scale d i;kx dk crkrk gA

Question 1. A normal curve has $\overline{X}$ = 50 and standard deviation of (s) = 10. Show the area in graph.

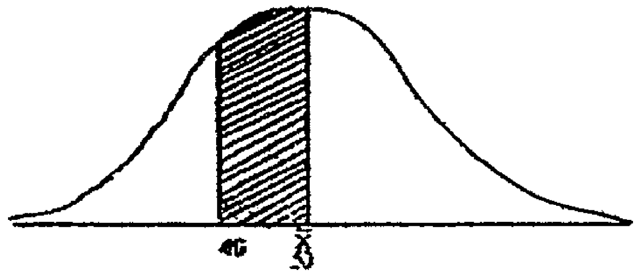
(i) between 40 and 50

(ii) between 60 and 80

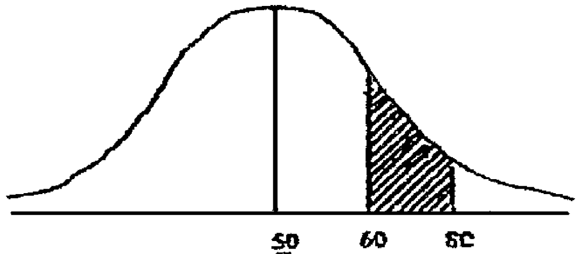
(iii) between $\overline{X} \pm 2$s

(iv) more than 70.

Solution. (i) between 40 and 50.



(ii) between 60 and 80.

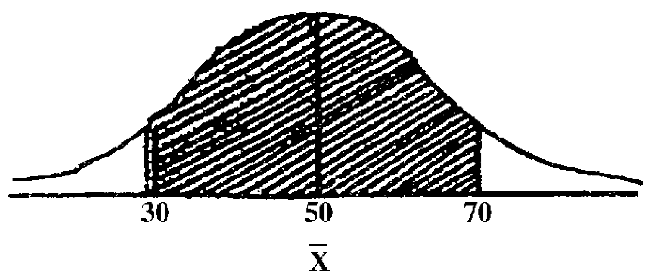


(iii) between $\overline{X} \pm 2s$;gk $\overline{X} = 50$

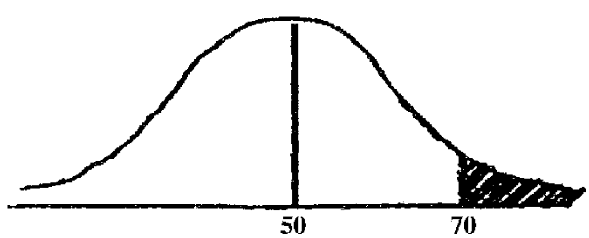
$s = 10$

rk 50 ± 210

$50 \pm 20 = 30$ and 70



(iv) more than 70.



Question 2. A normal curve has $\overline{X}$ and $s = 20$. Find the area between $x_1 = 30$ and $x_2 = 80$

Solution : We are given $\overline{X} = 40$

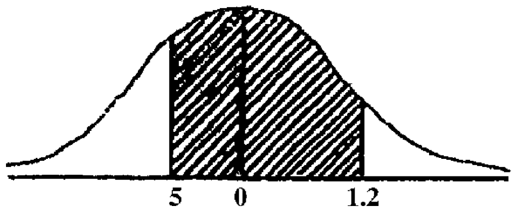
$s = 20$

To find out the area between 30 and 80 we are first of all to find out the value of *X*. The value of *z* for X = 30 is

$$Z = \frac{x - \bar{x}}{\sigma}$$

$$= \frac{30 - 40}{20} = -5$$

and the value of *z* for $\bar{X}$ = 80 is



Scale not properly taken

Required Area between $z = -.5$ to $z = 0$ and $z = 0$ to $z = +2$

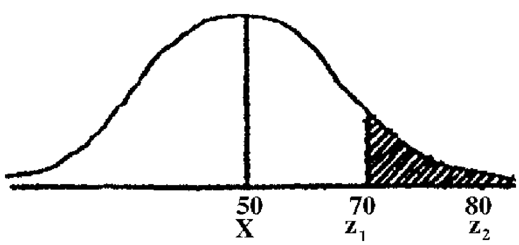
$= .1915 + .4772$

$= .6687$

(ukV % Area, Normal Curve dh Table l n[kk tkrk gA)

Question 3. The main of a distribution is 60 and standard deviation 10. Assuming the distribution to be normal what percentage of items will be between 70 and 80.

Solutoin : Drawing the normal curve.



Finding the value of Z

$$z = \frac{x - \bar{x}}{\sigma}$$

For $z = 70$ Area

$$z_1 = \frac{70 - 60}{10} = 1 \quad 0.3413$$

For $z = 80$

$$z_2 = \frac{80-60}{10} = 2 \quad 0.4772$$

Area between 70 and 80

$$= z_2 - z_1$$

$$= 0.4772 - 0.3413 = 0.1359$$

Hence 13.59% items will be between 70 and 80.

Question 4. In a normal distribution 31% of the items are less than 45 and 8% are above 64. Find $\bar{x}$ and s the distribution.

Solution : Since 31% of the items are under 45, therefore, the area to the left to ordinate at $x = 45$ will be 31% of the total, *i.e.*, 0.31.

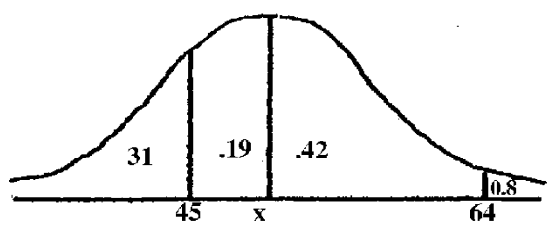
The area between mean and $x = 45$ will be 0.19 (.50 – .31). The value of Z at 0.19 is 0.5. Hence

$$\frac{45-\bar{x}}{\sigma} = -5$$

or $\quad 45 - x = -5$

(The value of – 0.5 as the area is on the left side of $\bar{x}$ ordinate)

Secondly, 8% of the items are above 64. Hence, the area right to ordinate 64 shall



be 8% or 0.8 and the area between mean ordinate and the ordinate at 64 shall be (.50 – .08) = 42

The value of Z when area is .42 is 1.4 (See table)

Thus, $\quad \frac{64-\bar{x}}{\sigma} = 1.4$

or $\quad 64 - \bar{x} = 14s \quad$ ...(ii)

Solving equation (i) and (ii) we get

$$45 - \bar{x} = .5s$$

$$65 - \bar{x} = 1.4s$$

$$-19 = -19s$$

$$\frac{-+-}{-19=-19\sigma} = \text{By Subtracting}$$

$$= \frac{1.9}{1.1} = s$$

$$10 = s$$

Putting the value in equation no. (i)

$$45 - \bar{x} = 5s$$

$$45 - \bar{x} = 5(10)$$

$$-\bar{x} = 5 - 45$$

$$-\bar{x} = -50$$

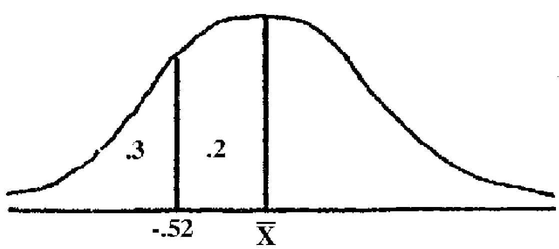
$$\bar{x} = 50$$

Thus $\bar{x}$ = 50 and s = 10.

Question 5. In a normal distribution 30% of the items are under 50 and 10% are over 86. Find mean and standard deviation of the distribution.

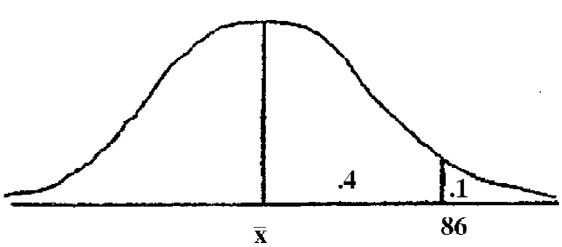
Solution : Since 30% of the items are under 50 therefore the area to the left to ordinate at x = 50 shall be 30% of the total *i.e.,* 0 – 3.

The area between mean and x = 50 and shall be 0.2 (0.5 – 0.3)



Hence $$\frac{50-\bar{x}}{\sigma} = 52 \text{ or } 50 - \bar{x} = -52s \quad ...(i)$$

Secondly, 10% of the items are over 86. Hence, the area to the right of the ordinate at 86 shall 10% *i.e.,* 0.1 only and the area between mean ordinate and the ordinate at 86 shal be 0.4 (0.5 – 0.1). The value of z when area is 0.4 is 1.28.



Thus, $\frac{86-\bar{x}}{\sigma} = 1.28$ or $86-\bar{x} = 1.28s$ ...(ii)

Solving equation (i) and (ii)

$$50-\bar{x} = 52s$$

$$86-\bar{x} = 1.28s$$

$$- \quad + \quad -$$

$$.36 = 1.80\ s \text{ by substracting}$$

$$\frac{36}{18} = s$$

$$20 = s$$

Putting the value of s in equation (i)

$$50-\bar{x} = -52\ s \qquad \text{Thus } \bar{x} = 60.4$$

$$50-\bar{x} = -52+20 \qquad s-20.0$$

$$50-\bar{x} = -10.4$$

$$-\bar{x} = -60.4$$

$$\bar{x} = 60.4$$

4. lkjk'k (Summary) :

no fun'ku pj (Random Sampling Variable) e vkofÙk forj.k nk idkj d gkr g—(i) [kf.Mr vkofÙk forj.k o (ii) v[kf.Mr vkofÙk forj.kA [kf.Mr J.kh e vk'k; ledk dh ml J.kh l g ftle iR;d en dk ;FkkFk eki fd;k tk ldA v[kf.Mr J.kh l vk'k; g ftue pjk dk eku i.k vFkok vi.k l[;k d :i e gk vkj bu oxkUrjk d :i e iLrr fd;k tk ldrk gA ,l vkofÙk forj.k tk okLrfod voykduk ;k i;kxk l iklr u fd; tkdj dN lfuf'pr iodYiukvk] ekU;rkvk vFkok ikoQfrd fu;ek d vk/kj ij xf.krh; fof/ l vu vuekfur fd; tkr g] ikf;drk vkofÙk forj.k dgykr gA bu forj.kk }kjk iklr lEHkkfor ledk d vk/kj ij foodi.k fu.k; fy, tkr gA ikf;drk vkofÙk forj.k rhu idkj d gkr g—(i) f}in forj.k] (ii) ik;lu forj.k ,o (iii) lkekU; forj.kA f}in forj.k ,d [kf.Mr vkofÙk forj.k g tk }U}kRed fodYik—lQyrk rFkk vlQyrk d ,d leg dh ikf;drk dk O;Dr djrk gA ik;lu forj.k Hkh ,d [kf.Mr ikf;drk forj.k gkrk g vkj ;g ,lh ifjfLFkfr;k e ykx gkrk g tgk ij ?kVuk d ?kVu dh ikf;drk cgr gh de gkrh gA lkekU; vkofÙk forj.k ,d lrr ikf;drk forj.k gA bl forj.k dk fodflr djus e 18oh 'krkCnh d teu [kxky'kkL=kh dky ekl dk cgr egRoi.k ;kxnku jgk gA vr% bl vkofÙk forj.k dk xkfl;u (Gaussian Curve) ;k foHkek dk ilkekU; fu;e (Normal Law of Error) dgk tkrk gA

5. iLrkfor iLrd (Recommended Books) :

(i) Fundamentals of Statistics by S.C. Gupta, Himalaya Publishing House.

(ii) Statistics for Business and Economics—By Dr. R.P. Hooda, Macnillan India Limited.

(iii) Business Statistics—By S.C. Sharma, R.C. Jain, Arya Book Depot, New Delhi.

(iv) Statistical Methods—By S.P. Gupta, Sultan Chand and Sons.

6. vH;kl d fy; i'u (Self Assessment Qusestion) :

(1) What are the conditions for the Binomial Distribution ? Discuss its Properties.

(2) What is Poission distribution ? Discuss its uses and properties.

(3) Show that Binomial distribution is the limiting form of Binomial distribution.

(4) Compare the Normal, Poisson and Binomial Distribution.

(5) The Normal rate of infection of a certain disease in animals is know to be 25%.

In an experiment with 6 animals infected with a new vaccine its was observed that none of the animals caught infection. Calculate the probability of the observed result.

$\left(\frac{729}{4096}\right)$ **Ans.**

(6) Suppose that in key punching of 80 column IBM cards, the arithmetic means number of mistake per card is 0.3. What percent of cards will have (i) no mistake, (ii) one mistake, and (iii) two mistakes.

[(i) = 74%, (ii) = 22%, (iii) = 3%] **Ans.**

(7) If 8 coins are tossed 256 times, find the expected frequencies of getting various heads.

Ans.

Heads	0	1	2	3	4	5	6	7	8
Frequency	1	8	28	56	70	56	28	8	1

(8) A book has 2 mistake per page. Using Poission distributions, find the probability that the page selected at random as (i) no mistake (ii) exactly 3 mistake.

Ans. (i) 0.315, (ii) 0.1804

(9) Fit a Poission Distribution to the following set of observations.

Deaths	0	1	2	3	4
Frequency	122	60	15	2	1

Calculate theoretical frequencies.

(10) If the mean of a Poisson Distribution is 2.56, Find the value of other constants. Also determine the probability of exactly 2 successes.

Ans.

s = 1.6,	m_3 = 2.56, b_2 = 3.39,
m_1 = 0,	m_3 = 22.22, $p(2)$ = .2532
m_2 = 2.56,	b_1 = 0.39

(11) What do you understand by the Normal Distribution ? States its main properties.

(12) Write shotes on :

(i) Importance of normal distribution.

(ii) Difference between binomial distribution and normal distribution.

(13) Mean of a variable is 50 and standard deviation 10. Find the percentage of items between :

(i) 50 and 70

(ii) 70 and 90

(iii) Above 100

(iv) Less than 45

(v) 42 and 58

(14) The marks obtained by students in an examination are normally distributed. If 10% students have marks more than 75 and 60% have marks more than 50, find the mean and value of distribution.

(15) The mean height of 1000 workers in a automobile unit is 67 inches with standard deviation of 5 inches. How many workers are expected to be above 72 inches in the unit.

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