M.Sc. (P) Math

Roll No. .... Total Pages : 3

MDE/M-15 **4028**

ADVANCED ABSTRACT ALGEBRA

Paper : MM-401

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all, selecting at least *one* question from each section. All questions carry equal marks.

SECTION–I

1. (a) Prove that if an Abelian group has a unique composition series, then G is a cyclic p-group.

(b) Prove that centre of nilpotent group G is non-trivial and non-trivial normal subgroup of G intersects non-trivially with the centre of G.

2. (a) Prove with usual notation that

$$[Z_m(G), r_n(G)] \subseteq Z_{m-n}(G) \ \forall\, m \geq n \geq 1.$$

(b) Let H Δ G such that both H and G | H are solvable. Prove that G is also solvable.

SECTION–II

3. (a) Let K | F and L | K be finite extensions. Prove that L | F is finite and [L : F] = [L : K] [K : F].

(b) Let K be the splitting field of $f(X) \in F[X]$. Prove that if degree of $f(X)$ is n, then [K : F] divides $\underline{|n}$.

4. (a) Find the degree of the splitting field of $X^4 - 5$ over Q.

(b) Prove that if $Ch(F) = p \neq 0$, then $\alpha \in K \mid F$ is separable iff $F(\alpha) = F(\alpha^p)$.

5. (a) Find the Galois group of $X^4 - 8X^2 + 15$ over Q.

(b) Prove that if K is the splitting field of $X^n - a \in F[X]$, then $G(K \mid F)$ is a solvable group.

SECTION–III

6. (a) Prove that if $A \in M_n(f) = F_n$ has all its ch. roots in F, then A is similar to a triangular matrix.

(b) Prove that the invariants of a nilpotent linear transformation are unique.

7. (a) Let $T \in A(V)$ has all its ch. roots in F. Prove that T is diagonalizable iff $u(T - \lambda I)^m = 0$, for $u \in V$ and $\lambda \in F$, implies that $u(T - \lambda I) = 0$.

(b) Let K be an extension of F. Prove that if $A, B \in M_n(F)$ are similar in $M_n(K)$, then A and B are also similar in $M_n(F)$.

SECTION–IV

8. (a) Let I be a left ideal of a ring R with 1. Prove that if $R \cong R/I$ as left R-modules, then $I = Re$ for some idempoten $e \in I$.

(b) Prove that any two bases of a f.g. free module over a commutative ring R with 1 have the same number of elements.

9. (a) Prove that every nil ideal in a Noetherian ring is nilpotent.

(b) Let N be a sub-module of a lift R-module M. Prove that if both N and M|N are Artinian R-module, then so is M.

10. (a) Prove that every non-zero module over a Noetherian ring contains a uniform module.

(b) Find the Abelian group generated by x and y, where $3x = 4y$.