M.Sc. (P) math

Roll No. .... Total Pages : 3

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DMDE/M–19

ADVANCED ABSTRACT ALGEBRA

Paper–MM–401

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) Let $H \Delta G$. Prove that if G has a composition series, then G/H also has a composition series.

   (b) State and prove three subgroups Lemma of P. Hall.

2. (a) Let H be a proper subgroup of a nilpotent group G. Prove that $H \subsetneq N_G(H)$

   (b) Let $H \Delta G$. Prove that if both H and G/H are solvable, then so is G.

UNIT–II

3. (a) Prove that if P is a prime field, then either $P \cong z/pz$ or $P \cong Q$.

   (b) Find the degree of the splitting field of the polynomial $(x^3 - 2)(x^2 - 4)$ over Q.

4. (a) Prove that a finite normal extension is a splitting field.

   (b) Prove that of $\propto \in K/F$ is separable, then $F(\propto)/F$ is separable.

5. (a) Find the Galois group of $x^4 - 2$ over Q.

   (b) Prove that the Galois group of $x^n - a \in F[x]$ is solvable.

UNIT–III

6. (a) Prove that if $T \in A_F(V)$ has all ch. roots in F, then T satisfies a polynomial of degree $n = \dim_F(V)$ over F.

   (b) Let $T \in A_F(V)$ be such that $T^m = I$. Prove that if ch(F) = 0, then T is diagonalizable. $(\dim_F(V) < \infty)$.

7. (a) Prove that invariants of a nilpotent linear transformation are unique.

   (b) Prove that if $f(x) \in F[x]$, then the companion matrix C(f(x)) of f(x) satisfies the polynomial f(x).

UNIT–IV

8. (a) Let I be a left ideal of a ring R with unity. Prove that I is a direct sum and of R iff I is generated by an idempotent.

   (b) Let R be a ring with unity. Prove that a left R-module is simple iff $M \cong R/I$ where I is a maximal left ideal of R.

9. (a) Prove that in an Artinian ring every nil left ideal is nilpotent.

   (b) Prove that each ideal in a Noetherian ring contains a finite product of prime ideals.

10. (a) Let M be a left module over a Noetherian ring. Prove that every non-zero sub module of M contains a uniform module.

    (b) Prove that over a PID, submodule of a finitely generated module is again finitely generated.
