M.Sc. (P) Math

Roll No. .... Total Pages : 3

4028

MDE/M-17

ADVANCED ABSTRACT ALGEBRA

Paper–I (MM-401)

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all, selecting at least *one* question from each section. All questions carry equal marks.

SECTION–I

1. (a) Let K be a normal subgroup of a finite group G. Prove that G has a composition series containing K.

(b) State and prove Three subgroup lemma of P. Hall.

2. (a) Prove that direct product of two nilpotent groups is nilpotent.

(b) Prove that a f.g. nilpotent group generated by torsion elements is finite.

SECTION–II

3. (a) Prove that if P is a prime field then either $P \cong Q$ or $P \cong Z/\phi Z$.

(b) Let K/F be algebraic and $\sigma : K \to K$ be a F-homomorphism. Prove that $\sigma \in \text{Aut}(K)$.

4. (a) Find the degree of the splitting field of

$$f(x) = (x^2 - 1)(x^4 - 3) \text{ over } Q.$$

(b) Prove that if $a \in K/F$ is separable then $F(\alpha)/F$ is separable.

5. (a) Find the Galois group of $x^3 - 2$ over Q.

(b) Prove that the polynomial $f(x) = x^5 - 4x + 2$ is not solvable by radicals over Q.

SECTION–III

6. (a) Prove that if $T \in A_F(V)$ has all characteristic roots in F, then T satisfies a polynomial of degree $n = \dim_F(V)$ over F.

(b) Prove that if $T \in A_F(V)$ is such that $T^2 = T$, then T is diagonalizable.

7. (a) Prove that invariants of a nilpotent transformation are unique.

(b) Let K be an extension of F. Prove that if $A, B \in M_n(F) = F_n$ are similar in K_n, then, they are also similar in F_n.

SECTION–IV

8. (a) Let I be a left ideal of a ring R with unity such that $R \cong R/I$ as left R-modules. Prove that I is generated by an idempotent.

(b) Prove that rank of a finitely generated free module over a commutative ring is an invariant.

9. (a) Prove that an R-module M is Artinian iff every factor module of M is finitely co-generated.

(b) Prove that in a Noetherian ring R, intersection of all the prime ideals of R is nilpotent.

10. (a) Prove that every non-zero Noetherian module contains a uniform module.

(b) Find the Abelian group generated by x, y, where $5x = 6y$.