

M.Sc (P) Math

Roll No.

Total Pages : 03

DMDE/M-18

4028

ADVANCED ABSTRACT ALGEBRA

MM-401

Time : Three Hours]

[Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Section I

1. (a) Prove that a group of order p^n (p is a prime and $n \geq 0$) has a composition series of length n .
(b) Let A and B be subgroups of a group G such that $[A, B, B] = \{e\}$. Prove that $[A, B]$ is a Abelian subgroup of G .
2. (a) Prove that subgroup and a factor group of a nilpotent group are again nilpotent.
(b) Define the derived series $\{\delta_i(G)\}_{i \geq 0}$ of a group G and prove that G is soluble iff \exists an integer $s \geq 0$ such that $\delta_s(G) = \{e\}$.