DIRECTORATE OF DISTANCE EDUCATION
KURUKSHETRA UNIVERSITY,
KURUKSHETRA - 136 119

Syllabus & Examination
for

M.Sc. (Previous) Mathematics

Session 2016-17

1. There will be five theory papers in each year.
2. Each theory paper will consist of four sections.
3. Paper setter will set ten questions.
4. The candidate will be required to attempt five questions in all, selecting at least one from each section. All questions will be of equal marks.
5. Duration of examination of each theory paper will be three Hours.
6. Max. Marks of each theory paper will be 100, 80 Marks for External Theory Examination and 20 Marks for Internal Assessment.

Scheme of Examination for M.Sc. Mathematics Through Distance Education with effect from the session 2016-17

M.Sc. (Previous) Mathematics

		Marks		Time
		External Theory	**Internal Assessment**	
Paper I - MM-401	Advanced Abstract Algebra	80	20	3 hrs.
Paper II - MM-402	Real Analysis	80	20	3 hrs.
Paper III -MM-403	Topology And Functional Analysis	80	20	3 hrs.
Paper IV -MM-404	Complex Analysis	80	20	3 hrs.
Paper V - MM-405	Differential Equations	80	20	3 hrs.

Paper-I : MM 401 : Advanced Abstract Algebra

External Theory Marks: 80
Internal Assessment Marks: 20
Time: 3 Hours

Note : The examiner is requested to set ten questions by dividing the paper into the sections as indicated in the syllabus. The candidates are required to attempt five questions, selecting at least one from each section.

Section-I (Two Questions):

Zassenhaus's lemma. Normal and Subnormal series. Scheiers Theorem, Composition Series. Jordan-Holder theorem.

Commutators and their properties. Three subgroup lemma of P. Hall. Central series. Nilpotent groups. Upper and lower central series and their properties. Invariant (normal) and chief series. Solvable groups. Derived series.

Section-II (Three Questions):

Field theory. Prime fields. Extension fields. Algebraic and transcendental extensions. Algebraically closed field. Conjugate elements. Normal extensions. Separable and inseparable extensions. Perfect fields. Finite fields. Roots of unity Cyclotomic Polynomial (x) Primitive elements.

Automorphisms of extensions. Galois extensions. Fundamental theorem of Galois theory. Solutions of polynomial equations by radicals, Insolvability of the general equation of degree 5 by radicals. Construction with ruler and compass.

Section-III (Two Questions):

Canonical Forms-Similarity of linear transformations. Invariant subspaces. Reduction of triangular forms. Nilpotent transformations. Index of nilpotency. Invariants of a nilpotent transformation. The primary decomposition theorem. Jordan blocks and Jordan forms. Rational Canonical form. Generalized Jordan form over any field.

Section-IV (Three Questions):

Cyclic modules. Free modules. Simple modules. Semi-simple modules. Schuler's Lemma. Noetherian and Artinian modules and rings, Hilbert basis theorem. Wedderbum-Artin theorem. Uniform modules, primary modules, and Noether- Lasker theorem. Smith normal form over a principal ideal domain and rank-Fundamental structure theorem for finitely generated abelian groups and its application to finitely generated Abelian groups.

Recommended Text :

1. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd. New Delhi,1975.
2. P.B. Bhattacharya, S.K. Jain and S.R. Nag Paul, Basic Abstract Algebra (2nd Edition), Cambridge University Press, Indian Edition, 1997.

References:

1. M. Artin, Algebra, Prentice Hall of India, 1991.
2. P.M. Cohn, Algebra, Vols. I, II & III, John Wiley & Sons, 1982,1989,1991.
3. N. Jacobson, Basic Algebra, Vols. I & II, W.H. Freeman, 1980 (a Company).
4. S. Lang, Algebra, 3rd Edition, Addison-Wesley,1993.
5. I.S. Luther and I.B.S. Passi, Algebra, Vol. I-Groups, Vol. II-Rings, Narosa Publishing House (Vol.I-1996, Vol. II-1999).
6. D.S. Malik, J.N. Mordeson, and M.K. Sen, Fundamentals of abstract Algebra, McGraw-Hill, International Edition, 1997.
7. K.B. Datta, Matrix and Linear Algebra, Prentice-Hall of India Pvt. Ltd., New Delhi, 2000.
8. S.K. Jain, A. Gunawardena and P.B. Bhattacharya, Basic Linear Algebra with MATLAB, Key College Publishing (Springer-Verlag),2001.
9. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice-Hall of India, 2000.
10. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
11. I. Stewart, Galois Theory, 2nd Edition, Chapman and Hall, 1989.

12. J.P. Escofier, Galois Theory, GTM, Vol. 204, Springer, 2001.

13. TV. Lam, Lectures on Modules and Rings, GTM,Vol.189,Springer-Verlag, 1999.

14. D.S. Passman, A Course in Ring Theory, Wadsworth and Brook/Cole Advanced Books and Softwares, Pacific Groves, California,1991.

15. I.D. Macdmald, Theory of Groups.

16. Surjeet Singh, Modem Algebra.

PAPER-II: MM 402 : Real Analysis

External Theory Marks: 80
Internal Assessment Marks: 20
Time: 3 Hours

Note : The examiner is requested to set ten questions by dividing the paper into the sections as indicated in the syllabus. The candidates are required to attempt five questions, selecting at least one from each section.

Section-I (Three Questions):

Definition and existence of Riemann-Stieltjes integral, properties of the Integral, Integration and differentiation, the fundamental theorem of Calculus, integration of vector-valued functions, Rectifiable curves. (Scope as in Chapter 6 of 'Principles of Mathematical Analyses' by Walter Rudin (3 rd edition).

Rearrangements of terms of, a series, Riemann's theorem.(Scope as in 3.52) to 3.55 of chapter 3 of Principle of Mathematical Analysis' by Walter Rudin (3rd Edition)

Sequences and series of functions, point wise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, uniform convergence and Continuity, uniform Convergence and Riemann-Stieltjes integration uniform convergence and Differentiation , Weierstrass

Approximation Theorem. Power Series, Uniqueness theorem for power series, Abel's and Tauber's theorems. (Scope as in 7.1 to 7.27 of Chapter 7 of 'Principles of Mathematical Analysis' by Walter Rubin (3^{rd} Edition) and 8.1 to 8.5 of chapter 8 of Principles of Mathematical Analysis' by Walter Rudin (3^{rd} Edition).

Section - II (Two Questions):

Function of Several variables, Linear transformations. Derivative in an open subset of R, Chain rule. Partial derivatives, interchange of the order of differentiation, derivarives of higher crders, Taylor's theorem. Inverse function theorem. Implicit function theorem, Jacobians. Extremum problems with Constraints, Lagrange's multiplier method, Differentiation of integrals. Partitions of unity. Differential forms, Stoke's theorem,(scope as in 9.1 to 9.29 & 9.39 to 9.43 of Chapter 9 & 10.8 to 10.33 of Ch. 10 of 'Principles of Mathematical Analysis' by Walter Rudin (3^{rd} Edition).

Section-III (Two Questions):

Lebesgue outer measure. Measurable sets. Regularity. Measurable functions. Borel and Lebesgure measurability. Non-measurable sets. Integration of Non-negative functions. The General integral. Integration of Series. Reimann and Lebesgue Integral, (scope as indicated by the relevant portions of sections 2.1 to 2.5 & 3.1 to 3.4 of Measure & Integration by G.de Barra, Wiley Eastern Ltd. 1981).

Section-IV (Three Questions)

The Four derivatives. Functions of Bounded variation. Lebesgue Differentiation Theorem.

Differentiation and Integration.

Measures and outer measures, Extension of measure. Uniqueness of Extension. Completion of a measure. Measure, spaces. Integration with respect to a measure.

The I^p spaces. Convex functions, Jensen's inequality. Holder and Minkowski inequalities. Completeness of I^p convergence in measure. Almost unifonn convergence.(scope as indicated by the relevant portions of sections 4.1,4.3,4.4,5.,5.1 to 5.6, 6.1 to 6.5 & 7.1,7.2 of Measure & Integration' by G.de Barra, Wiley Eastern Ltd.,1981).

Recommended Texts:

1. Walter Rudin, Principles of Mathematical Analysis, (3rd Edition) Mc-Graw Hill, Kogokusha,1976, International Standard Edition, student edition.
2. G.de Barra, Measure Theory and Integration, Wiley Eastern Ltd.,1981.

References:

1. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi,1985.
2. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York, 1975.
3. A.J. White, Real Analysis, an introduction. Addison-Wesley Publishing Co.,Inc.,1968.
4. E. Hewitt and K. Stromberg. Real and Abstract Analysis, Berlin, Springer, 1969.
5. P.K. Jain and V.P. Gupta, Lebesque Measure and Integration, New Age International (P) Ltd., Published New Delhi,1986(Reprint 2000).
6. I.P. Natanson, Theory of Functions of a Real Variable Vol. I, Frederick Ungar Publishing Co., 1961.
7. H.L. Royden Real Analysis Macmillan Pub. Co. Inc. 4th Edition, New York,1993.
8. Richard L. Wheeden and Antoni Zygmund, Measure and Integral: An Dekker 1977.
9. J.H. Williamson, Lebesque Integration, Holt, Rinehart and Winston, Inc, New York, 1970.
10. A Friedman, Foundation of Modern Analysis, Roll, Rinehart and Winston, Inc. New York, 1970.
11. P.R. Halmos, Measure Theory, Van Nostrcni, Princeton 1950.
12. T.G. Hawkins, Lebesque's Theory of Integration: Its Origins and Development, Chelsea, New York, 1979.
13. K.R. Parthasarathy, Introduction to Probability and Measure, Macmillan Company of India Ltd., Delhi, 1977.
14. R.G. Bartle, The Elements of Integration, John Wiley & Sons, Inc., New York, 1966.
15. Serge Lang, Analysis I & II, Addition- Wesley Publishing Company Inc., 1969.
16. Inder R. Rana, An Introduction to Measure and Integration, Narosa Publishing House, Delhi, 1997.

PAPER-III : MM 403 : Topology And Functional Analysis

External Theory Marks: 80
Internal Assessment Marks: 20
Time: 3 Hours

Note : The examiner is requested to set ten questions by dividing the paper into the sections as indicated in the syllabus. The candidates are required to attempt five questions, selecting at least one from each section.

Section-I (Three Questions):

Definition and examples of topological spaces. Neighbourhoods, ulterior point and interior of a set. Closed set as a complement of an open set. Adherent point and limit point of a set, closure of a set as a adherent points, derived set of a set, properties of closure operator, boundary of a set, Dense subsets.

Base and sub-base for a topology. Neighbourhood system of a point and its properties. Base for Neighbourhood system.

Relative Topology and subspace of a topological space. Alternate methods of defining a topology in terms of 'Neighbourhood system', 'Interior Operator', 'Closed sets' and Kuratowski closure operator.

First countable. Second countable and separable spaces, their relationships and hereditary property. About countability of a collection of disjoint open sets in a separable and second countable space, Lindelof.. theorems.

Comparison of Topologies on a set, About intersection, union, infimum and supremum of a collection of topologies on a set.

Definition, examples and characterizations of continuous functions, composition of continuous functions. Open and closed functions, Homeomorphism.

Tychonoff product topology in terms of standard sub-base, projection maps. Characterisation of product topology as smallest topology with projections continuous, continuity of a function from a space into a product of spaces, countability and product spaces.

To, T1, T2 Regular & T3, separation axioms, their characterization and basic properties i.e. hereditary property of To, T1, T2 Regular & T3 spaces, and productive property of T1 & T2 spaces.

Quotient topology w.r.t. a map. Continuity of function with domain a space having quitient topology. About Hausdorffness of quotient space.

Completely regular and Tychonoff (31/2), spaces, their hereditary and productive properties, Embedding lemma. Embedding theorem.

SECTION - II (Two Question):

Normal and T4 spaces: Definition and simple examples. Normality of a regular Lindel of space, Urysohn's Lemma, complete regularity of a regular normal space, T4 implies Tychonoff, Tietze's extension theorem.

Fliter on a set : Definition and examples. Collection of all filters on a set as a P.O. set, finer filter, methods of generating filters/finer filter. Ultra filter (u.f.) and its characterizations. Ultra Filter Principle (UFP) i.e. Every filter is contained in an ultra filter. Image of filter under a function.

Convergence of filters: Limit point and limit of a filter and relationship between them. Continuity in terms of convergence of filters, Hausdoffness and filters. Convergence of filter in a product space.

Compactness : Definition and examples of compact spaces and subsets, compactness in terms of finite intersection property, continuity and compact sets, compactness and separation properties, closedness of compact subset and a continuous map from a compact space into a Hausdoriff and its consequence. Regularity and normality of a compact Hausdoriff space.

Compactness and filter convergence, compactness and product space. Tychonoff product theorem using filters, Tychonoff space as a subspace of a compact Hausdorff space and its converse. (both the sections i.e. I & II are based on Chapters 1 to 5 of the Book General Topology by John L. Kelley).

SECTION - III (Three question):

Normed Linear spaces, Banach spaces, completion of a normed space, finite dimensional normed spaces and subspaces, equivalent norms, F. Riesz's Lemma, bounded and continuous linear operators, normed spaces of operators, dual spaces.

Hahn Banach theorem, application to bounded linear functionals on C [a,b], adjoint operator, reflexive spaces, uniform boundedness theorem.

Strong and weak convergence, open mapping theorem, bounded inverse theorem, closed linear operators closed graph theorem.

(Scope as in relevant parts of Chapters 2 & 4 of "Introductory Functional Analysis with applications" by E. Kreyszig).

SECTION - IV (Two question):

Inner product spaces, Hilbert spaces and their examples, Pythagorean theorem, Appolloniu's identity, Schwarz inequality, continuity of innerproduct, completion of an inner product space, sub space of a Hilbert space, orthogonal complements and direct sums, projection theorem.

Orthonormal sets and sequences, Bessel's inequality, series related to orthonormal sequences and sets, total(complete) orthonormal sets and sequences, Parseval's identity, separable Hilbert spaces. Representation of functionals on Hilbert spaces, sesquilinear form, Riesz representation theorem for bounded sesquilinear forms on a Hilbert space.

Hilbert adjoint operator, its existence and uniqueness, properties of Hilbert adjoint operators, self, adjoint, unitary, normal, positive and projection operators. (Scope as in relevant parts of Chapters 3 & 9 of "" by E. Kreyszig.)

Books:

1. John L. Kelley : General Topology (Springer International Edition, Second Indian Reprint 2008).
2. E. Kreyszig : Introductory Functional Analysis with applications (John Willey 1973).

Paper-IV: MM 404 : Complex Analysis

External Theory Marks: 80
Internal Assessment Marks: 20
Time: 3 Hours

Note : The examiner is requested to set ten questions by dividing the paper into the sections as indicated in the syllabus. The candidates are required to attempt five questions, selecting at least one from each section.

Section-I (Three Questions):

Complex Integration ; Cauchy-Goursat Theorem. Cauchy's integral formula. Higher order derivatives. Morera's Theorem. Cauchy's inequality and Liouville's theorem. The fundamental theorem of algebra. Taylor's theorem-Maximum modulus principle.

Section-II (Two Question):

Schwarz Lemma. Laurent's series. Isolated singularities. Meromomorphic functions. The argument Principle. Rouche's theorem, Inverse function theorem.

Residues. Cauchy's residue theorem-Evaluation of integrals. Branches of many valued functions with special reference to agr z, log z and z^a.

Bilinear transformations, their properties and classifications. Definitions and examples of conformal mappings.

Section-III (Three Questions):

Spaces of analytic functions. Hurwitz's theorem, Montel's theorem, Riemann mapping theorem.

Weierstrass' factorization theorem. Gamma function and its properties, Riemann Zeta function, Riemann's functional equation. Range's theorem. Mittag- Leffler's theorem, Analytic Continuation. Uniqueness of direct analytic continuation along a curve. Power series method analytic continuation Schwarz Reflection principle. Monodromy theorem and its consequences.

Section-IV (Two Question):

Harmonic functions on a disk. Hamack's inequality an theorem. Dirichlet problem. Green's function.

Canonical products: Jensen's formula. Poisson-Jensen formula. Hanamard's three circles theorem. Order of an entire function. Exponent of Convergence. Borel's theorem. Hadamard's factorization theorem.

The range of an analytic function. Bloch's theorem. The little Picard theorem. Schottky's theorem. Montel Caratheodory and the Great picard theorem.

Univalent functions. Bieberbach's conjecture (statement only) and the theorem.

Recommended Text:

J.B. Conway, Function of one Complex variable. Springer-Vergtag. International student - Edition. Narosa Publishing House, 1980.

Reference:

1. H.A. Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 1990.
2. Liang-Shin Hahn & Bernard Epstein, Classical Complex Analysis, Jones, and Bartlett Publishers International, London, 1996.
3. L.V. Ahlfors, Complex Analysis, McGraw-Hill, 1979.
4. S. Lang, Complex Analysis, Addison Wesley, 1977.
5. D. Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
6. Mark J. Ablowitz and A.S. Fokas, Complex Variables : Introduction and Application, Cambridge University Press, South Asian Edition, 1998.
7. E. Hille, Analytic Function Theory (2 Vols.) Gonn & Co., 1959.
8. W.H.J. Funchs, Topics in the Theory of Functions of one Complex Variable, D. Van Nostrand Co., 1967.
9. C. Caratheodory, Theory of Functions(2 Vols.) Chelsea Publishing Company, 1964.

10. M. Heins, Complex Function Theory, Academic Press, 1968.

11. Walter Rudin, Real and Complex Analysis, McGraw-Hill Book Co., 1966.

12. S. Saks and A. Zygmund, Analytic Functions, Monografie Matematyczne, 1952.

13. E.C. Titchmarsh, the Theory of Functions, Oxford University Press, London.

14. W.A. Veech, A Second Course in Complex Analysis, W.A. Benjamin, 1967.

15. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

Paper-V: MM 405 : Differential Equations

External Theory Marks: 80
Internal Assessment Marks: 20
Time: 3 Hours

Note : The examiner is requested to set ten questions by dividing the paper into the sections as indicated in the syllabus. The candidates are required to attempt five questions, selecting at least one from each section.

Section-I (Three Questions):

Preliminaries: Initial value problem and equivalent integral equation, e-approximate solution, equicontinuous set of functions.

Basic theorems: Ascoli- Arzela theorem, Cauchy —Peano existence theorem and its corollary. Lipschitz condition. Successive approximations. Picard-Lindelof theorem. Continuation of solution, Maximal interval of existence, Extension theorem. Kneser's theorem (statement only)

(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

System of Differential Equations. Maximal and Minimal solutions. Differential inequalities. A theorem of Wintner. Uniqueness theorems: Kamke's theorem, Nagumo's theorem and Osgood theorem.

(Relevant portions from the book 'Ordinary Differential Equations' by P. Hartman)

Section-II (Two Questions):

Linear differential systems: Definitions and notations. Linear homogeneous systems;

Fundamental matrix, Adjoint systems, reduction to smaller homogeneous systems. Non-homogeneous linear systems; variation of constants. Linear systems with constant coefficients. Linear systems with periodic coefficients; Floquet theory.

(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Higher order equations: Linear differential equation (LDE) of order n; Linear combinations, Linear dependence and linear independence of solutions. Wronskian theory: Definition, necessary and sufficient condition for linear dependence and linear independence of solutions of homogeneous LDE. Abel's Identity, Fundamental set, More Wronskian theory.

Dependence of solutions on initial conditions and parameters: Preliminaries, continuity and differentiability.

Section-III (Two Questions):

Autonomous systems: the phase plane, paths and critical points, Types of critical points; Node, Center, Saddle point, Spiral point. Stability of critical points. Critical points and paths of linear systems: basic theorems and their applications.

Critical points and paths of non-linear systems: basic theorems and their applications. Liapunov function. Liapunov's direct method for stability of critical points of non-linear systems. Limit cycles and periodic solutions: Limit cycle, existence and non-existence of limit cycles, Benedixson's non-existence criterion. Half-path or Semiorbit, Limit set, Poincare-Benedixson theorem. Index of a critical point. Pioncare — Benedixson theorem.

Section-IV (Three Questions):

Linear second order equations: Preliminaries, self adjoint equation of second order, Basic facts, superposition principle, Riccati's equation, Pruffer transformation, zero of a solution, Oscillatory and non-oscillatory equations. Elementary Linear oscillations Abel's formula. Common zeros of solutions and their linear dependence.

Sturm theory: Sturm separation theorem, Sturm fundamental comparison theorem and their corollaries.

Second order boundary value problems(BVP). Sturm-Liouville BVP: definitions, eigen values and eigen functions. Orthogonality of functions, orthogonality of eigen functions corresponding to distinct eigen values. Green's function. Applications of boundary value problems. Use of Implicit function theorem and Fixed point theorems for periodic solutions of linear and non-linear equations.

Recommended Text:

1. E.A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations,* Tata McGraw-Hill , 2000.
2. S.L. Ross, *Differential Equations,* John Wiley & Sons
3. P. Hartman, *Ordinary Differential Equations,* John Wiley & Sons NY, 1971.
4. S.G. Deo, V. Lakshmikantham and V. Raghavendra, *Textbook of Ordinary Differential Equations,* Tata McGraw-Hill, 2006.

References:

1. G. Birkhoff and G.C. Rota, *Ordinary Differential Equations,* John Wiley & Sons, 1978.
2. G.F. Simmons, *Differential Equations,* Tata McGraw-Hill, 1993.
3. I.G. Petrovski, *Ordinary Differential Equations,* Prentice-Hall, 1966.
4. D. Somasundaram, *Ordinary Differential Equations, A first Course,* Narosa Pub., 2001.